MATHEMATICS

(For 6th class)

H2Q4P8

ਸਮੱਗਰ ਸਿੱਖਿਆ ਅਭਿਆਨ

ਪੜ੍ਹੋ ਸਾਰੇ ਵਧੋ ਸਾਰੇ

ਸਿੱਖਿਆ ਅਤੇ ਭਲਾਈ ਵਿਭਾਗ, ਪੰਜਾਬ ਦਾ ਸਾਂਝਾ ਉਪਰਾਲਾ

Punjab School Education Board

Sahibzada Ajit Singh Nagar

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FOREWORD

The Punjab School Education Board has been continuously engaged in developing syllabi, producing and renewing text books according to the changing educational needs at the state and national level.

This book has been developed in accordance to the guidelines of National Curriculum Framework (NCF) 2005 and PCF 2013, after careful deliberations in workshops involving experienced teachers and experts from the board and field as well. All efforts have been made to make this book interesting with the help of activities and coloured figures. This book has been prepared with the joint efforts of subject experts of Board, SCERT and experienced teachers/experts of mathematics. Board is thankful to all of them.

The authors have tried their best to ensure that the treatment, presentation and style of the book in hand are in accordance with the mental level of the students of class VI. The topics, contents and examples in the book have been framed in accordance with the situations existing in the young learner's environment. A number of activities have been suggested in every lesson. These may be modified, keeping in view the availability of local resources and real life situations of the learners.

I hope the students will find this book very useful and interesting. The Board will be grateful for suggestions from the field for further improvement of the book.

Chairman

Punjab School Education Board

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Contents

SYMBOLS AND THEIR MEANING

$\therefore$	=	So/Therefore
$\because$	=	Because
$\Rightarrow$	=	Implies
$>$	=	Greater than
$<$	=	Less than
$\parallel$	=	Parallel
$\perp$	=	Perpendicular
Δ	=	Triangle
$\angle$	=	Angle
:	=	Ratio
: :	=	Proportion
i.e.	=	id est (that is)
e.g.	=	exempli gratia (for example)
etc.	=	Et cetera (the rest of same type)

REVISION OF FUNDAMENTAL OPERATIONS (+, −, ×, ÷)

Before building a strong and beautiful structure it is always good to test its foundation, on which that structure is supposed to stand. With same motive, in this chapter, Let us strengthen our previous knowledge and remove deficiency if any.

Exercise (Addition)

Addition (+)

1. Solve the following:

(a) $\begin{array}{r} 5999 \\ +\ 1233 \\ \hline \\ \hline \end{array}$ (b) $\begin{array}{r} 5219 \\ +\ 3899 \\ \hline \\ \hline \end{array}$ (c) $\begin{array}{r} 2009 \\ +\ 7788 \\ \hline \\ \hline \end{array}$

(d) $\begin{array}{r} 112 \\ +\ 2709 \\ \hline \\ \hline \end{array}$ (e) $\begin{array}{r} 3486 \\ +\ 4306 \\ \hline \\ \hline \end{array}$ (f) $\begin{array}{r} 506 \\ +\ 909 \\ \hline \\ \hline \end{array}$

2. Fill boxes operating as directed:

(a) 305 + 289 = ☐

(b) 2186 + 476 = ☐

(c) 332 + 4097 + 81 = ☐

(d) 77777 + 7777 + 777 + 77 + 7 = ☐

3. Fill empty boxes:

(a) $\begin{array}{r} 4\ 9\ 3 \\ +\ 3\ 0\ 9 \\ \hline 8\ \square\ 2 \\ \hline \end{array}$ (b) $\begin{array}{r} 2\ 6\ 3\ 6 \\ +\ 5\ 9\ 9 \\ \hline \square\ 2\ 3\ \square \\ \hline \end{array}$ (c) $\begin{array}{r} 9\ 7\ 3\ 9 \\ +\ 6\ 5\ 2\ 8 \\ \hline \square\ \square\ 2\ 6\ \square \\ \hline \end{array}$

4. Fill empty boxes operating as directed:

(a) 2017 + 928 + 74 = 3 ☐ ☐ 9

(b) 5077 + 537 + 98 = ☐ 7 1 ☐

(c) 3344 + 403 + 37 = ☐ ☐ 8 4

5. In a cricket match Virat scored 129 runs and Dhawan scored 97 runs. How many runs are made by both of them together.

Exercise (Subtraction)

1. Solve the following:

(a) $\begin{array}{r} 532 \\ -289 \\ \hline \\ \hline \end{array}$ (b) $\begin{array}{r} 643 \\ -478 \\ \hline \\ \hline \end{array}$ (c) $\begin{array}{r} 912 \\ -289 \\ \hline \\ \hline \end{array}$

(d) $\begin{array}{r} 604 \\ -467 \\ \hline \\ \hline \end{array}$ (e) $\begin{array}{r} 7800 \\ -471 \\ \hline \\ \hline \end{array}$ (f) $\begin{array}{r} 10000 \\ -9999 \\ \hline \\ \hline \end{array}$

2. Solve the following:

(a) 795 – 199 = ...

(b) 996 – 848 = ...

(c) 776 – 499 = ...

3. Fill empty boxes :

(a) $\begin{array}{rrr} 4 & \square & 6 \\ -2 & 7 & \square \\ \hline \square & 2 & 7 \\ \hline \end{array}$ (b) $\begin{array}{rrrr} 3 & \square & 6 & 8 \\ - \square & 7 & 4 & 5 \\ \hline 1 & 6 & \square & 3 \\ \hline \end{array}$ (c) $\begin{array}{rrrr} 3 & \square & 6 & 0 \\ - \square & 8 & 9 & 4 \\ \hline 0 & 2 & \square & 6 \\ \hline \end{array}$

4. 196 litre of water has been used out of tank having 807 litre water. How much water has been left in tank?

5. Solve the following:

(a) 2048 + 3088 – 4017 = ...

(b) 48 + 37 – 23 + 49 – 63 = ...

(c) –103 + 63 + 36 – 37 + 269 = ...

Exercise (Multiplication)

1. Fill in the blanks :

(a) 7 × 0 = ... (b) 6 × 1 = ...

(c) 9 × 1 = ... (d) 71547×1 = ...

(e) 963×0 = ... (f) $23 \times 47 \times 0 \times 32$ = ...

(g) 1×1 = ... (h) 0×0 = ...

2. Fill in the blanks:

(a) 20×30 = ... (b) 40×300 = ...

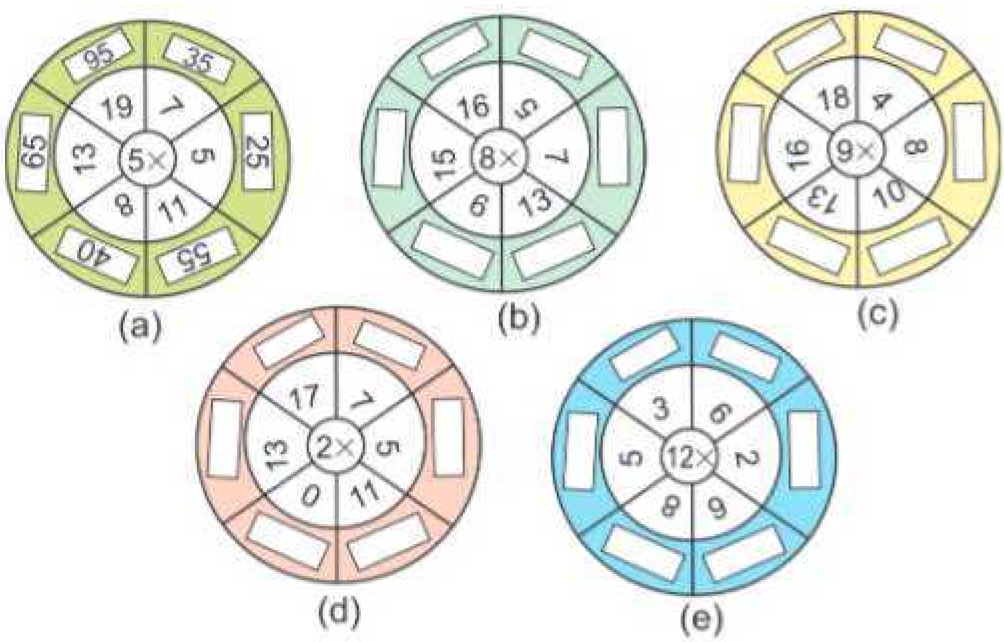
(c) 40×2000 = ... (d) 90×9000 = ...

(e) 800×700 = ... (f) 12×200 = ...

(g) 8×11000 = ... (h) 7×1200 = ...

3. (a) 309 × 42 (b) 567 × 56 (c) 407 × 43 (d) 165 × 14

4. Fill the empty boxes:



5. Isha saves ₹ 48290 every month. How much money will she have after 2 years ?

6. Surinder bought 15346 chairs for auditorium. If each chair costs ₹398. How much money did Surinder paid?

7. A milk booth sells 448 litres of milk daily. How many litres of milk will it sell in 4 weeks?

8. Multiply :

(a) 3125×533 (b) 2391×236

(c) 4332×805 (d) 9219×78

(e) 473×999 (f) 234×11

Exercise (Division)

1. Fill in the blanks :

(a) 725 ÷ 1 = ... (b) 725 ÷ 725 = ...

(c) 0 ÷ 725 = ... (d) 823 ÷ 1 = ...

(e) 823 ÷ 823 = ... (f) 0 ÷ 823 = ...

(g) 0 ÷ 99999 = ... (h) .... ÷ 35 = 0

(i) 87450 ÷ .... = 1 (j) 8129 ÷ .... = 8129

2. Find quotient and remainder for the following :

(a) 1652 ÷ 7 (b) 5893 ÷ 6 (c) 7406 ÷ 6

(d) 11982 ÷ 5 (e) 28359 ÷ 12 (f) 12321 ÷ 11

3. Solve the following (Find Quotient and Reminder)

(a) 714 ÷ 7 (b) 618 ÷ 6 (c) 2416 ÷ 8

(d) 142114 ÷ 7 (e) 1384 ÷ 6 (f) 17126 ÷ 8

(g) 2107 ÷ 9 (h) 3046 ÷ 13 (i) 27661 ÷ 12

4. Fill empty boxes: (Remember : Dividend = Divisor × Quotient + Remainder)

	Dividend	Divisor	Quotient	Reminder
Example	138	11	12	6
(a)	158	13	12	☐
(b)	2168	☐	135	8
(c)	1689	14	☐	9
(d)	1414	14	☐	0
(e)	90	☐	12	6

5. Divide the largest 3-digit number by the largest 2 digit number and find the quotient and remainder.

6. A factory manufactured 936243 clips in 21 days. How many clips are manufactured in one day?

ANSWER KEY

Exercise (Addition)

1. (a) 7232 (b) 9118 (c) 9797 (d) 2821 (e) 7792 (f) 1415
2. (a) 594 (b) 2662 (c) 4510 (d) 86415
3. (a) 0 (b) 3, 5 (c) 1, 6, 7
4. (a) 0, 1 (b) 5, 2 (c) 3, 7
5. 226

Exercise (Subtraction)

1. (a) 243 (b) 165 (c) 623 (d) 137 (e) 7329 (f) 1
2. (a) 596 (b) 148 (c) 277
3. (a) $\begin{array}{r} 4\ 0\ 6 \\ -\ 2\ 7\ 9 \\ \hline 1\ 2\ 7 \\ \hline \end{array}$ (b) $\begin{array}{r} 3\ 3\ 6\ 8 \\ -\ 1\ 7\ 4\ 5 \\ \hline 1\ 6\ 2\ 3 \\ \hline \end{array}$ (c) $\begin{array}{r} 3\ 1\ 6\ 0 \\ -\ 2\ 8\ 9\ 4 \\ \hline 0\ 2\ 6\ 6 \\ \hline \end{array}$
4. 611 litre
5. (a) 1119 (b) 48 (c) 228

Exercise (Multiplication)

1. (a) 0 (b) 6 (c) 9 (d) 71547
   (e) 0 (f) 0 (g) 1 (h) 0
2. (a) 600 (b) 12000 (c) 80000 (d) 810000
   (e) 560000 (f) 2400 (g) 88000 (h) 8400
3. (a) 12978 (b) 31752 (c) 17501 (d) 2310
4. (b) $8 \times 5 = 40$ (c) $9 \times 4 = 36$
   $8 \times 7 = 56$ $9 \times 8 = 72$
   $8 \times 13 = 104$ $9 \times 10 = 90$
   $8 \times 6 = 48$ $9 \times 13 = 117$
   $8 \times 15 = 120$ $9 \times 16 = 144$
   $8 \times 16 = 128$ $9 \times 18 = 162$
   (d) $2 \times 7 = 14$ (e) $12 \times 6 = 72$

$2 \times 5 = 10$	$12 \times 2 = 24$
$2 \times 11 = 22$	$12 \times 9 = 108$
$2 \times 0 = 0$	$12 \times 8 = 96$
$2 \times 13 = 26$	$12 \times 5 = 60$
$2 \times 17 = 34$	$12 \times 3 = 36$

5. 1158960
6. 6107708
7. 12544
8. (a) 1665625 (b) 564276 (c) 3487260
   (d) 719082 (e) 472527 (f) 2574

Exercise (Division)

1. (a) 725 (b) 1 (c) 0 (d) 823
   (e) 1 (f) 0 (g) 0 (h) 0
   (i) 87450 (j) 1
2. (a) Q = 236 ; R = 0 (b) Q = 982 ; R = 1
   (c) Q = 1234; R = 2 (d) Q = 2396 ; R = 2
   (e) Q = 2363 ; R = 3 (f) Q = 1120 ; R = 1
3. (a) Q = 102; R = 0 (b) Q = 103; R = 0 (c) Q = 302; R = 0
   (d) Q = 20302; R = 0 (e) Q = 230; R = 4 (f) Q = 2140; R= 6
   (g) Q = 234; R =1 (h) Q = 234; R = 4 (i) Q = 2305; R = 1
4. (a) 2 (b) 16 (c) 120
   (d) 101 (e) 7
5. Q = 10 ; R = 9
6. 44583

Y7A3J4

1

KNOWING OUR NUMBERS

Objectives

In this chapter you will learn

(i) To read and write numbers in Indian as well as International system of Numeration.

(ii) To solve day to day life practical mathematical problems.

(iii) To perform basic operations on numbers and adopt them in routine life.

(iv) To make comparison between numbers.

(v) To use Brackets while solving mathematical problems.

(vi) To use Roman numeration system along with Hindu Arabic system of Numeration.

1.1 Introduction

We have enjoyed working with numbers in our previous classes. We have added, subtracted, multiplied and divided them. We also looked for patterns in number sequences and done many other interesting things with numbers. In this chapter we shall move forward on such interesting things with a bit of review and revision as well.

1.2 Few Basic terms

* **Natural Numbers :** Since our childhood, we are using numbers 1, 2, 3, 4, 5, ... etc. to count and calculate. These counting numbers are called natural numbers. Whatever, there is in nature, we count with these numbers, hence they are called both **Counting Numbers** as well as **Natural Numbers**.

- Smallest or first Natural Number is '1'.
- By adding 1 to any natural number we can get next natural number.
- There is no largest natural number.
- Set of natural numbers is represented by 'N'.

* **Digits :** To represent any number, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. These ten symbols are called digits.

1.3 Comparing Numbers

While comparing two numbers, we have to remember the following steps :

Step 1 : If the number of digits in the given numbers are not same, then the number with less number of digits will be smaller.

Step 2 : If the number of digits in both the numbers are same then,

(a) First compare the digits at the first place from the left. The number with the greater digit is greater than the other number.

(b) If the numbers have same digits at the first place, then compare the digits at the second place from left. The number with the greater digit is the greater one.

(c) Continue the process till you get unequal digit at the corresponding place.

(d) Apply the same process while comparing more than two numbers.

Example 1 : **Compare the following**

(a) 235 and 1023

(b) 47321 and 39874

(c) 56398 and 56412

Solutions :

(a) 235 is the three digit number and 1023 is the four digit number So, number with more digits is greater.

Therefore 1023 is greater than 235.

(b) 4̲7321 is a five digit number.

3̲9874 is also a five digit number.

Compare the first digit from left side of both numbers. We observe these are 4 and 3 and $4 > 3$.

Therefore 47321 is greater than 39874.

(c) 56398 is a five digit number

56412 is also a five digit number.

We observe that first two digits from left side of both numbers are same. Both number start with 56.

Third digit from left side is different in both numbers and that are 3 and 4. and $3 < 4$.

Therefore 56412 is greater than 56398.

Example 2 : **Find the greatest and smallest among following numbers:**
1903, 9301, 1930, 9031, 9310

Solution : All the given numbers:

1903, 9301, 1930, 9031, 9310 are four digit numbers.

Let us examine the digit on extreme left side of each number.

First digit on the left side of two numbers is 1 and the first digit on the left side of other number is 9.

Smaller number will be observed between 1903 and 1930. And the greater number will be observed among 9301, 9031 and 9310. Then by observing second and third digit from left side we conclude that 1903 is the smallest and the 9310 is the greatest number.

1.3.1 Making Different numbers by shifting digits

Think about it, if the digits are shifted from one place to other in a number. Let us have an example 327. We can frame six different numbers (including 327) by just shifting of digits.

For Example : 327, 372, 237, 273, 732, 723

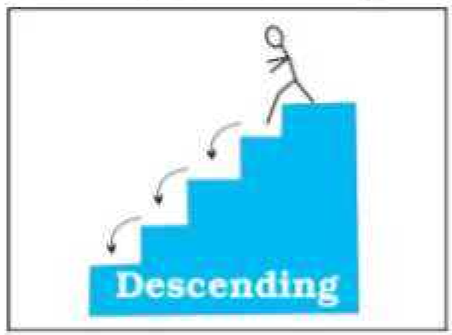
* Among these six numbers can you identify the largest and smallest number?

* Try to write all possible three digit numbers using digits 2, 3 and 5.

* Try to write all possible four digit numbers by using digits 3, 5, 7, 9 also, without repeating any digit. Write the greatest and smallest number among them.

1.3.2 Ascending (Increasing) and Descending (Decreasing) order.

Ascending order means the arrangement of numbers from the smallest to the greatest.



Descending order means the arrangement of numbers from the greatest to the smallest.

Example 3. **Arrange the numbers in ascending order 653, 1135, 47629, 2546, 7320**

Solution : The 3-digit number is the smallest number and the 5-digit number is the greatest number in the given numbers, number next to the smallest number is 1135, the next 4-digit number is 2546 followed by 7320.

Ascending order of given numbers is 653, 1135, 2546, 7320, 47629.

Example 4. **Use the given digits without repetition and make the greatest and smallest 4 digit number.**

(a) 2, 3, 1, 7 **(b) 4, 9, 0, 2**

Solution : (a) Digits to be used are 2, 3, 1, 7

Let us first arrange these digits

in ascending or descending order as per your choice.

Ascending order : 1, 2, 3 7

Now greatest 4 digit number with these digits = 7321

and the smallest 4 digit number with these digits = 1237

(b) Given digits are 4, 9, 0, 2

Let us arrange in ascending order = 0. 2, 4, 9

Now Greatest 4 digit number with these given digits = 9420

Now smallest 4 digits number with these given digits = 2049

Be Careful students 0249 is not the 4 digit number, because zero is not significant when it occupies the extreme left position. Here 0249 is the 3 digit number.

Example 5. **Using any one digit twice make the greatest and smallest 4 digit number.**

(a) 5, 2, 8 **(b) 7, 0, 2**

Solution : (a) Digits given are 5, 2, 8

Let us arrange them in ascending order as 2, 5, 8

Greatest 4 digit number repeating one digit = 8852

(We shall repeat the digit with highest face value i.e. 8)

Smallest 4 digit number repeating one digit = 2258

(We shall repeat the digit with lowest face value)

(b) Digits given are 7,. 0, 2

Let us arrange them in ascending order as 0, 2, 7

Greatest 4 digit number repeating one digit = 7720

(We shall repeat the digit with highest face value i.e. 7)

Smallest 4 digit number repeating one digit = 2007

(We shall repeat the digit with lowest face value i.e 0, but we can't place zero at extreme left place)

1.3.3 Place value and face value

The place value of a digit depends on its position, whereas the face value does not depend on its position. For example in the number 9678, the face value of 8 is 8. Similarly the face value of 7, 6 and 9 are also 7. 6 and 9 respectively.

However when we are concerned with place value.

The digit 8 has the place value	=	$8 \times 1 = 8$	(8 lies at units place)
The digit 7 has the place value	=	$7 \times 10 = 70$	(7 lies at tens place)
The digit 6 has the place value	=	$6 \times 100 = 600$	(6 lies at hundreds place)

The digit 9 has the place value = $9 \times 1000 = 9000$ (9 lies at thousands place)

In expanded form 9678 will be written as

$9678 = 9 \times 1000 + 6 \times 100 + 7 \times 10 + 8 \times 1$

It is evident from this that a number is the sum of the place values of all its digits.

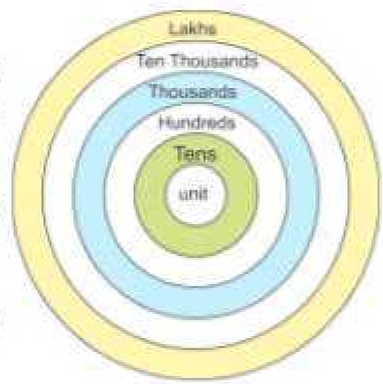
Place value of a digit = Face value × Position value

It is to be noted that position values of units, tens, hundreds, thousands, ten thousands, lakhs and so on..... are respectively 1, 10, 100, 1000, 10000, 100000, ... and so on.

Place value of 0 is 0 itself, where ever it may be.

ACTIVITY

Students! Let us play with numbers.



Draw concentric circles on a card board (as shown) or on ground. With marker write unit, tens, hundreds, thousands, ten thousands and so on....

Have some marbles and be ready to play. Throw these marbles gently on the card board with number circles.

Let us suppose that the marbles settle themselves as shown in the figure. Now can you identify the number.

Place value of Marbles in Ten thousands Circle = 1×10000

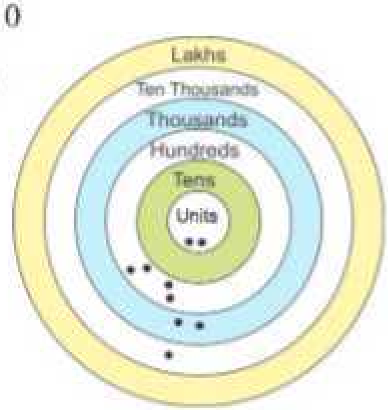
= 10000

Place value of Marbles in Thousands Circle = $2 \times 1000 = 2000$

Place value of Marbles in Hundreds Circle = $4 \times 100 = 400$

Place value of Marbles in Tens Circle = $0 \times 10 = 0$

Place value of Marbles in Units Circle = $2 \times 1 = 2$



Number = Sum of place values of all digits

= 10,000 + 2,000 + 400 + 0 + 2

= 12402

Now Let us prepare the place value chart

F.V. = Face Value

Roll No.	Name of Student	Lakhs F.V. × 100000	Ten Thousands F.V. × 10000	Thousands F.V. × 1000	Hundreds F.V. × 100	Tens F.V. × 10	Units F.V. × 1	Number Add all place values
1.	---	---	10000	2000	400	0	2	10,000+2000+ 400+0+2 =12402
2.								

Example 6. **Read and expand the numbers in the following table.**

Number	Number Name	Expansion
27000	Twenty Seven Thousand	$2 \times 10,000 + 7 \times 1000$
37600	Thirty Seven Thousand Six hundred	$3 \times 10000 + 7 \times 1000 + 6 \times 100$
56740		
69563		
42639		
29308		
20005		
19075		

1.4. Reading and Writing Numbers in Indian system of Numeration

According the 2011 census population of Punjab was approximately 2 Crore 77 lakh 43 thousand. In our daily routine life we need to speak numbers like thousands. Lakhs, Crores etc. It is the Indian system of Numeration.

In order to read numbers in the Indian system of Numeration, we make groups (periods) of place values like - 'Ones' , Thousands, Lakhs, Crores etc, separated by commas.

- The First three digits from the right of a number make unit period (or unit group)
- The next two digits from the right make thousands period.
- The next two digits from right make Lakhs period.
- The next two digits from right make crores period and so on.....

The digits in the same group or period are read together and the name of the period (except units) is read along with them.

Thus the number 8,76,54,321 is read as 'Eight Crores Seventy Six Lakhs Fifty Four Thousand Three hundred twenty one'.

Commas after each period are put to have a clear instant look.

Example 7. **Read the following numbers**

(a) 534632 **(b) 90763021**

Solution : (a) Given number is 534632.

Firstly place commas from the right side making group of 3, 2, 2 and so on ......

So given number is 5,34,632

It is clearly Five lakh Thirty Four thousand Six hundred thirty two.

(b) Given number is 90763021.

Firstly place commas from the right side making group of 3, 2, 2, 2 and so on....

So given number is 9,07,63,021

It is clearly Nine Crore Seven Lakh Sixty three thousand twenty one.

Example 8. **Write the number names as numerals.**

(a) Four lakh thirty two thousand six hundred seventy three.

(b) Six crore fifty three lakh twenty one thousand nine hundred seventy two.

Solution : (a) Four lakh thirty two thousand six hundred seventy three.

Crores	Lakhs	Thousands	Ones
0	04	32	673

= 4, 32, 673

(b) Six crore fifty three lakh twenty one thousand nine hundred seventy two.

= 6, 53, 21, 972

Example 9. **Find the difference of the place value and the face value of the digit 7 in 9745623.**

Solution : Given number is 9745623

Place value of 7 is 7,00,000

Face value of 7 is 7

Required difference is = 700000 – 7

= 699993

Example 10. **How many four digit numbers are there in all.**

Solution : Largest four digit number is 9999

Largest three digit number is 999

Total number of four digit numbers

= Largest four digit numbers – Largest three digit numbers

= 9999 – 999

= 9000

Example 11. **Write each of the numbers arranged in the following place value chart in words:**

	Crores		Lakhs		Thousands		Ones		
	Ten Crores 10,00,00,000	Crores 1,00,00,000	Ten Lakhs (10,00,000)	Lakhs (1,00,000)	Ten Thousands (10,000)	Thousands (1000)	Hundreds (100)	Tens (10)	Ones (1)
(i)		4	7	5	0	0	2	9	8
(ii)		7	8	0	5	1	0	2	4

Solution : (i) Given number is 4,75,00,298

Four crores seventy five lakh two hundred ninety eight

(ii) Given number is 7,80,51,024

Seven crore eighty lakh fifty one thousand twenty four.

Example 12. **Read the following numbers using place value chart.**

(a) 593268 **(b) 32067308**

Solution :

Crores		Lakhs		Thousands		Ones			Numbers
Ten Crores	Crores	Ten Lakhs	Lakhs	Ten Thousands	Thousands	Hundreds	Tens	Ones	
			5	9	3	2	6	8	Five lakh ninety three thousand two hundred sixty eight
	3	2	0	6	7	3	0	8	Three crore twenty lakh sixty seven thousand three hundred eight

1.5 International system of Numeration

In last section we have discussed about Indian system of Numeration. Now we shall learn about the International system of Numeration, which is followed by the most of countries of the world. In this system also, a number is split up into groups or periods. Starting from extreme right the number is split up into groups of three each. These groups are called Ones, Thousands, Millions and Billions. These groups are further sub-divided as follow:

Billions			Millions			Thousands			Ones		
Hundred Billions	Ten Billions	Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units

Example 13. **Insert Commas suitably and write the names according to:**

(a) Indian System of Numeration

(b) International System of Numeration 79530257

Solution : Indian system of Numeration

7,95,30,257

Seven crore ninety five lakh thirty thousand two hundred fifty seven.

International system of numeration

79,530,257

Seventy nine million five hundred thirty thousand two hundred fifty seven.

Example 14. **Write each of the numbers arranged in the following place value chart in words.**

Billions			Millions			Thousands			Ones		
Hundred Billions	Ten Billions	Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units
		2	3	4	1	2	9	8	3	4	6
				3	9	1	7	0	3	1	7

Solution : (i) Given number is 2, 341, 298, 346

Two billion three hundred forty one million two hundred ninety eight thousand three hundred forty six

(ii) Given number is 39, 170, 317

Thirty nine million one hundred seventy thousand three hundred seventeen.

• Relation Between Indian and International System of Numeration

Number	Indian System	International System
1	One	One
10	Ten	Ten
100	One Hundred	One Hundred
1000	One Thousand	One Thousand
10000	Ten Thousand	Ten Thousand
100000	One Lakh	One Hundred thousand
1000000	Ten Lakh	One Million
10000000	One Crore	Ten Million
100000000	Ten Crore	One Hundred Million
1000000000	One Arab	One Billion

Exercise 1.1

1. Write the smallest and the greatest number:
   (a) 30900, 30594, 30945, 30495
   (b) 10092, 10029, 10209, 10920
2. Arrange the numbers in ascending order:
   (a) 6089, 6098, 5231, 3953
   (b) 49905, 6073, 58904, 7392
   (c) 9801, 25751, 36501, 38802
3. Arrange the numbers in descending order:
   (a) 75003, 20051, 7600, 60632
   (b) 2934, 2834, 667, 3289
   (c) 1971, 45321, 88715, 92547
4. Use the given digits without repetition and make the greatest and smallest 4 digit number:
   (a) 6, 4, 3, 2 (b) 9, 7, 0, 3
   (c) 5, 4, 0, 3 (d) 3, 2, 7, 1
5. Using any one digit twice make the greatest and the smallest 4 digit number:
   (a) 2, 3, 7 (b) 5, 0, 3 (c) 2, 3, 0
   (d) 1, 3, 4 (e) 2, 5, 8 (f) 1, 2, 3
6. Read the following numbers using place value chart:
   (a) 638975 (b) 84321 (c) 29061058 (d) 60003608
7. Insert commas suitably and write the names according to Indian system of Numeration:
   (a) 98606873 (b) 7635172 (c) 89700057
   (d) 89322602 (e) 4503217 (f) 90032045
8. Insert commas suitably and write the names according to International system of Numeration:
   (a) 89832081 (b) 6543374 (c) 88976306
   (d) 9860001 (e) 90032045 (f) 4503217
9. Write the number names as numerals:
   (a) Seven lakh fifty four thousand
   (b) Nine crore fifty three lakh seventy four thousand five hundred twenty three
   (c) Six hundred forty seven thousand five hundred twenty five
   (d) Seventy two million three hundred thirty two thousand one hundred twelve.
   (e) Fifty eight million four hundred twenty three thousand two hundred two.
   (f) Twenty three lakh thirty thousand ten.
10. How many eight digit numbers are there in all.

11. Fill in the blanks:

(a) 1 Lakh = ... ten thousand

(b) 1 Million = ... hundred thousand

(c) 1 Crore = ... ten lakh

(d) 1 Crore = ... million

(e) 1 Million = ... lakh

1.6 Numbers in Length, Weight and Capacity

(Use Numbers in Practical Life):-

In previous classes we have learnt about units of measurement like length, weight and capacity. In this section we will learn to operate upon them in context of day to day practical problems of life.

- **Length :** We express the length of a pencil or paper in centimetres. Whereas to express the thickness of a paper clip, we find centimetres too big. We use smaller units i.e. millimetre. On the other hand to measure height of a building. We need bigger units that is metre. And we need bigger units to express distance between two cities, which is kilometre.

For example the distance between Bathinda and Chandigarh is about 230 kilometres.

		10 millimetres	=	1 centimetre
1 metre	=	100 centimetre	=	1000 millimetre
1 kilometre	=	1000 metres	=	1000000 millimetres

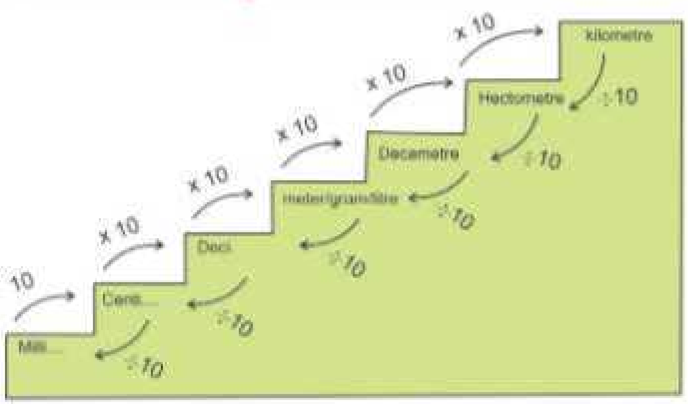
- **Weight :** We buy things like turmeric, ginger and garlic in grams whereas for buying things like flour, potatoes and rice we use bigger units like Kilograms. But medicine we consume is in milligrams.

1 kilogram	=	1000 gram
1 gram	=	1000 miligram

- **Capacity :** Usually when we drink water in a glass, it is approximately 250 to 300 mℓ. When we fill a bucket with water to have bath, it is nearly 20 to 25 litre.

1 litre	=	1000 millilitre

To Interchange the Units of Measurement



Fill all blanks with metre or gram or litre as follow:

milli .... : millimetre
milligram
millilitre

To Remember

King	Harry	Died,	Mother	Did not	Cry	Much
↓	↓	↓	↓	↓	↓	↓
Kilo	Hecto	Deca	Metre/ Gram/ Litre	Deci	Centi	Milli

Example 15. **In an election, the successful candidate registered 5765 votes and his nearest rival secured 3427 votes. By what margin did the successful candidate won the election?**

Solution : Number of votes secured by winning candidates = 5765

Number of votes secured by nearest rival = 3427

Winning Margin = 5765 – 3427

= 2338 Votes

Example 16. **A town newspaper is published everyday. One copy of that newspaper contain 13 pages and 11980 copies are printed daily. How many total pages are printed daily?**

Solution : Each copy of newspaper has 13 pages.

Hence 11980 copies will have 11980 × 13 pages

Now

$$\begin{array}{r} 11980 \\ \times 13 \\ \hline 35940 \\ +\ 119800 \\ \hline 155740 \\ \hline \end{array}$$

Hence everday 1,55,740 pages are printed.

Example 17. **A Shopkeeper has 48000 sheets of paper. He wants to bundle them in the form of reams. One ream of paper contain 480 sheets. How many reams will be made?**

Solution : Total number of sheets = 48000

Number of sheets in 1 Ream = 480

Number of Reams = Total Number of Sheets ÷ Sheets in one Ream

= 48000 ÷ 480

= 100 Ream

$$\begin{array}{r} 100 \\ 480\overline{)48000} \\ -480 \\ \hline 00 \\ -0 \\ \hline 00 \\ -0 \\ \hline 0 \\ \hline \end{array}$$

Example 18. A vessel has 4 litre and 650mℓ of curd. In how many glasses, each of 25mℓ capacity, can it be distributed ?

Solution : Volume of vessel having curd = 4 litre 650 mℓ

$= 4 \times 1000m\ell + 650m\ell$

$= 4000m\ell + 650m\ell$

$= 4650m\ell$

Capacity of One Glass = 25mℓ

Number of Glasses = Volume of Total Curd ÷ Capacity of One Glass

$= 4650\ m\ell \div 25m\ell$

$= 186$

$$\begin{array}{r} 186 \\ 25\overline{)4650} \\ -25 \\ \hline 215 \\ -200 \\ \hline 150 \\ -150 \\ \hline 0 \end{array}$$

4 litre 650 mℓ curd will be distributed in 186 glasses each of capacity 25mℓ.

Exercise 1.2

1. Convert the following measurements as directed:
   (a) 5 km into metre
   (b) 35 kilometre into metre
   (c) 2000 milligram into gram
   (d) 500 decigram into gram
   (e) 2000 millilitre into litre
   (f) 12 kilolitre into litre
2. In an election, the successful candidate registered 6317 votes whereas his nearest rival could attain only 3761 votes. By what margin did the successful candidates defeat his rival?
3. A monthly magazine having 37 pages is published on 20th day of each month. This month 23791 copies were printed. Tell us how many pages were printed in all?
4. A shopkeeper has 37 reams. One ream contain 480 pages and he wants to make quires of all these sheets to sell in retail. One quire of sheets contain 24 sheets. How many quires will be made?
5. Veerpal serves milk to the guests in glasses of capacity 250 mℓ each Suppose that the glasses are filled to capacity and there was 5 litre milk that got consumed. How many guests were served with milk?
6. A box of medicine contain 2,00,000 tablets each weighing 20mg. What is the total weight of tablets in box?
7. A bookstore sold books worth Rupees Two lakh eighty five thousand eight hundred ninety one in the first week of June. They sold books worth Rupees Four lakh seven hundred sixty eight in the second week of June. How much was the total sale for two weeks together?

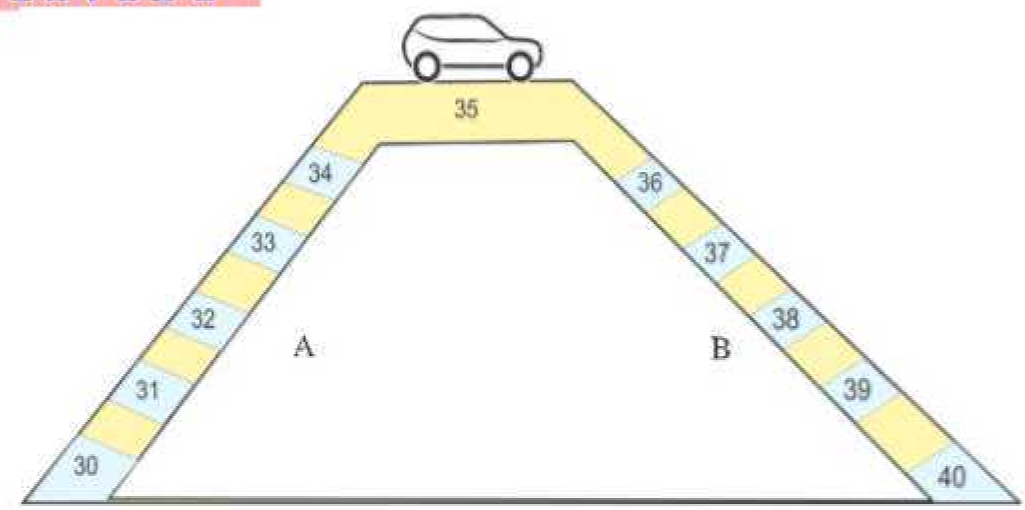
8. A famous cricket player has so far scored 6978 runs in test matches. He wishes to complete 10,000 runs. How many more runs he need?

9. Surinder has ₹ 78592 with him. He placed an order for purchasing 39 radio sets at ₹1234 each. How much money will remain with him after the purchase?

10. A vessel has 3 litre 650 mℓ of curd. In how many glasses each of 25 mℓ capacity can it be distributed?

1.7 Estimation and Approximation

Numbers are very commonly used in our daily life. We need to answer many questions like 'How many'? But we need not to answer the exact number. We instantly answer a rough estimation. For example you tell one of your friends that you attended a marriage party yesterday and there was a big gathering of 600 people. 600 is your rough estimation. You have not counted them. There are many situations where it is sufficient to tell the estimated figure instead of telling the exact one. So we must know how to estimate (or round off) a number.



Let us try to understand strongly the concept of rounding off numbers with an activity. Prepare a two way inclined plane as shown in picture above.

A car on inclined at any point in portion A slips to lowest level at number 30 in above example. It means 31,32, 33, 34 when rounded off to nearest tens, they are rounded to 30.

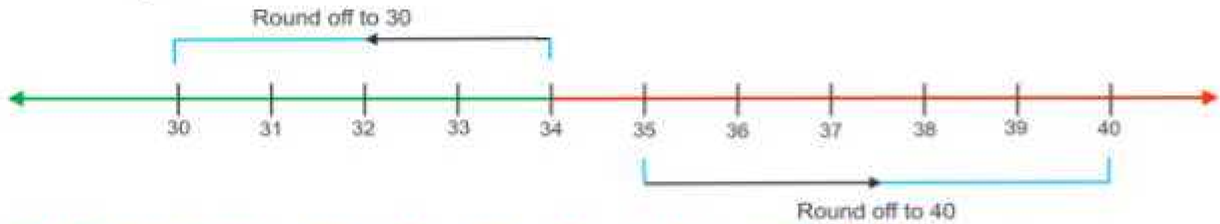
But when car is at any point in Portion B, slips to 40. It means 35, 36, 37, 38, 39 are rounded off to nearest tens are rounded to 40.

Rule I: Estimating or Rounding off numbers to Nearest Tens :

Follow the following rules to round off to nearest tens.

(a) If ones place digit is less than 5, replace ones digit by 0 and the other digits remain as they are.

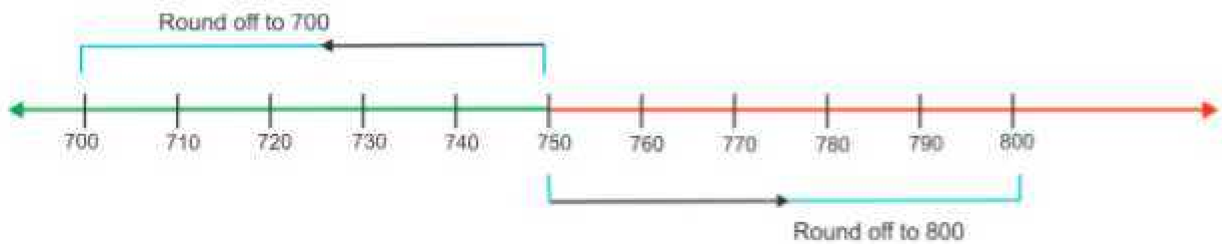
(b) If ones digits is more than or equal to 5, increase the tens digit by 1 and replace, ones digit by 0.



Rule II. Estimating or Rounding off numbers to Nearest Hundreds.

Follow the following rules to round off to nearest hundreds.

(a) If tens digit is less than 5, replace tens and ones digit by 0 and other digits remaining same.



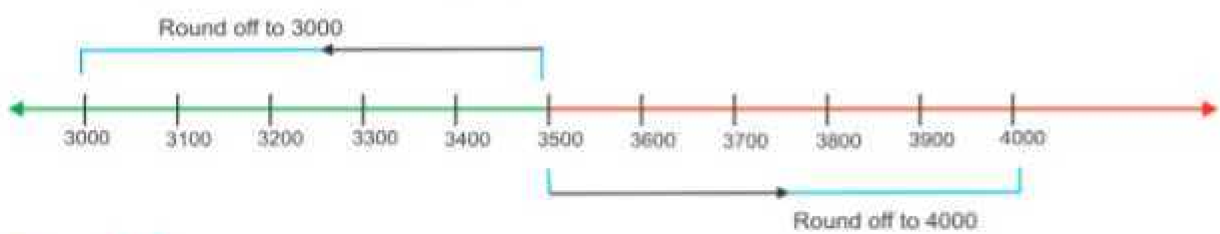
(b) If tens digit is more than or equal to 5, then increase the hundreds digit by 1 and replace each digit on its right by 0.

Rule III. Rounding off numbers to Nearest Thousands

Follow the following rules to round off to nearest thousands

(a) If hundreds digit is less than 5, replace hundreds, tens and ones digit by 0 and other digits remaining same.

(b) It hundreds digit is more than or equal to 5, then increase the thousands digit by 1 and replace each digit on its right by 0.



Example 19. **Round off 36182 and 36827 to the nearest tens, nearest hundreds and nearest thousands.**

Solution : Given numbers are 36182 and 36827

(i) Rounded off to nearest tens = 36180

Rounded off to nearest tens = 36830

(ii) Rounded off to nearest hundreds = 36200

Rounded off to nearest hundreds = 36800

(iii) Rounded off to nearest Thousands = 36000

Rounded off to nearest Thousands = 37000

1.7.1 To estimate sum, difference, product and quotient

There are many situations where we have to estimate the sum or difference or product or quotient of numbers. Upto which digit they should be rounded, it depends upon the necessity and requirement of data. Sometimes high degree of accuracy is required. sometimes we need to get result very quickly, although degree of accuracy is low. So all these things matter. Let us explain with examples.

Example 20. **Estimate the sum 5290 + 17986 by rounding off to**

(i) Hundreds Place **(ii) Thousands Place**

Solution : (i) While rounding off to hundreds place

5290 + 17986 = 5300 + 18000

= 23300

(ii) While rounding off to thousands place

5290 + 17986 = 5000 + 18000

= 23000

You can verify by performing actual addition that 23300 is more close, hence more reasonable.

Example 21. **Estimate 5673 – 436**

Solution : To begin with we round off to thousands

5673 rounds off to 6000

436 rounds off to – 0

Estimated difference = 6000

This is not a reasonable estimate. As the difference is greater than both numbers.

To get a closer estimate, Let us try rounding each number to hundreds place.

5673 rounds off to 5700

436 rounds off to 400

Estimated Difference 5700 – 400 = 5300

This is better and more meaningful estimate.

Example 22. **Estimate the Products**

(a) 87 × 313 **(b) 898 × 785**

(c) 63 × 182 **(d) 81 × 479**

By General Rule

General rule is rounding off each factor to its greater place, then multiplying the rounded off numbers.

Solution : (a) 87 × 313

87 rounds off to tens place = 90

313 rounds off to hundreds place = 300

Estimated product = 90×300

= 27000

(b) 898 × 785

898 rounds off to hundreds place = 900

785 rounds off to hundred place = 800

Estimated product = 900 × 800

= 720000

(c) 63 × 182

63 rounds off to tens place = 60

182 rounds off to hundreds place = 200

Estimated product = 60 × 200

= 12000

(d) 81 × 479

81 rounds off to tens place = 80

479 rounds off to hundreds place = 500

Estimted product = 80 × 500

= 40000

Example 23. **Find estimated quotient 2437 ÷ 125 by general rule**

Solution : 2437 ÷ 125

2437 rounded off to thousands place = 2000

125 rounded off to hundreds place = 100

Esttimated Quotient = 2000 ÷ 100

= 20

Exercise 1.3

1. Estimate each of the following using general rule:

   (a) 837 + 987 (b) 783 – 427 (c) 1391 + 2783

   (d) 28292 – 21496

2. Estimate the product using general rule:

   (a) 898 × 785 (b) 9 × 795 (c) 87 × 317

   (d) 9250 × 29

3. Estimate by rounding off to nearest hundred:

   (a) 439 + 334 + 4317 (b) 108734 – 47599

4. Estimate by rounding off to nearest tens:

   (a) 439 +334 + 4317 (b) 108734 – 47599

1.8. Use of Brackets

Brackets are symbols used in pairs to group things together and write statements in explicit form   Most commonly used brackets are

( ) : **Parentheses or common brackets**

{ } : **Curly Brackets**

[ ] : **Square Brackets or Box Brackets**

In this section, we shall learn about the use of parentheses only.

Consider the statement '4 is multiplied by the sum of 3 and 7'.

Using Parentheses this statement is written as $4 \times (3 + 7)$

Following Examples illutrates the use of brackets.

Example 24. Write expression for each of the following statements using brackets and then simplify.

(a) Seven is multiplid by the sum of three and four.

(b) Sum of nine and four is multiplied by six.

(c) Divide the difference of eighteen and six by four.

Solution : (a) $7 \times (3 + 4) = 7 \times 7 = 49$

(b) $(9 + 4) \times 6 = 13 \times 6 = 78$

(c) $(18 - 6) \div 4 = 12 \div 4 = 3$

1.8.1 Expanding Brackets

To expand brackets, we need to use distributive law. Distributive law states that a number outside a bracket performs the same operation with each term inside the bracket as follow:

$a(b+c) = ab + ac$

$a(b-c) = ab - ac$

Example 25. Solve by expanding brackets

(a) $7 \times (3 + 4)$ (b) $(9 + 4) \times 6$ (c) $(20 - 8) \div 4$

Solution : (a) $7 \times (3 + 4) = 7 \times 3 + 7 \times 4$

$= 21 + 28$

$= 49$

(b) $(9 + 4) \times 6 = 9 \times 6 + 4 \times 6$

$= 54 + 24$

$= 78$

(c) $(20 - 8) \div 4 = (20 \div 4) - (8 \div 4)$

$= 5 - 2$

$= 3$

Example 26. Simplify:

(a) 8×107 (b) 14×108

Solution : (a) $8 \times 107 = 8\times (100 + 7)$

$= 8 \times 100 + 8 \times 7$

$= 800 + 56$

$= 856$

(b) $14 \times 108 = (10 + 4) \times 108$

$= 10 \times 108 + 4 \times 108$

$= 10 \times (100 + 8) + 4 \times (100 + 8)$

$= 10 \times 100 + 10 \times 8 + 4 \times 100 + 4 \times 8$

$= 1000 + 80 + 400 + 32$

$= 1000 + 400 + 80 + 32$

$= 1512$

Exercise 1.4

1. Simplify each of following:

(a) 13×104 (b) 102×105 (c) 6×107

(d) 16×106 (e) 201×205 (f) 22×102

(g) $6 \times (4 + 3)$ (h) $(17 - 9) \times 3$ (i) $(20 + 4) \div 2$

1.9 Roman Numerals

We have already learnt the Indian system of numeration as well as International system of Numeration. There is another numeration system which was developed by Romans and widely used between 900 BC and 300BC. It was originated in ancient Rome. In Roman Numeration system only symbols were used to express numbers. There are 7 basic symbols in Roman system which are used to represent different numbers.

Roman Symbols	I	V	X	L	C	D	M
Numeral corresponding in Hindu Arebic System	1	5	10	50	100	500	1000

- A common belief is that the symbols of Roman system were taken from the pictures of Hands and Fingers.

This system of Numeration is still used in many places like : In front of your class rooms (Class VI etc.), numbers on clock faces, parts of books, to denote historical events such as : world war I, world war II, etc.

Remember Corresponding to '0' of Hindu Arabic system, there is no symbol for 0 (Zero) in Roman Numeration system.

Using these 7 symbols, we can write number by following certain rules which are given below:

Rule 1 : If a symbol is repeated its value, it is added as many times as it occours.

For Example : II = 1 + 1 = 2

XXX = 10 + 10 + 10 = 30

CC = 100 + 100 = 200

It must be noted that symbol I, X. C. M never repeated more than three times, and V, L. D are never repeated.

Rule 2 : Any smaller Roman numeral that comes after a larger numeral is added to it.

For Example: VI = 5 + 1 = 6

VII = 5 +1 + 1 = 7

XIII = 10 + 1 + 1 + 1 = 13

XVII = 10 + 5 + 1 + 1 = 17

LXXV = 50 + 10 + 10 + 5 = 75

Rule 3 : Any smaller Roman numeral that comes before a larger numeral is subtracted from it.

For Example : IV = 5 – 1 = 4

IX = 10 – 1 = 9

XL = 50 – 10 = 40

XC = 100 – 10 = 90

Rule 4 : The symbols V, L and D are never written to the left of a larger value symbol.

i.e. V, L, D are never subtracted.

I (V) X (L) C (D) M

- I, X, C comes before larger value numerals i.e. I, X, C can be subtracted from larger value numerals.
- I, X, C can be subtracted from next two respective numerals.

i.e. I can be subtracted from V and X only

X can be subtracted from L and C only

C can be subtracted from D and M only

- V, L, D are never subtracted.

Rule 5: If a smaller Roman numeral comes between two larger numerals then the smaller numeral is subtracted from the larger numeral following it:

For Example : XIX = X + IX = 10 + (10 – 1) = 19

LXIV = L + X + IV = 50 + 10 + (5–1) = 64

Rule 6 : If a bar is placed over a numeral then it is multiplied by 1000.

For Example: $\overline{V}$ = 5000, $\overline{X}$ = 10000

Let us write few numbers using these rules:

1	=	I	
2	=	II	
3	=	III	
4	= 5–1 =	IV	(∵ I cannot be repeated more than 3 times)
5	=	V	
6	=	VI	
7	=	VII	
8	=	VIII	
9	= 10–1 =	IX	(∵ I cannot be repeated more than 3 times)
10	=	X	
20	=	10 + 10 = XX	
30	=	10 + 10 + 10 = XXX	
40	=	50 – 10 = XL	(∵ X cannot be repeated more than 3 times)
45	=	40 + 5 = XLV	
49	=	40 + 9 = XLIX	(It can't be written as 50 – 1= IL as I cannot be subtrated from L, I can be subtracted from V and X)
50	=	L	
60	=	50 + 10 = LX	
70	=	50 + 10 + 10 = LXX	
80	=	50 + 10 + 10 + 10 = LXXX	
90	=	100 – 10 = XC	
100	=	C	
400	=	500 – 100 = CD	
500	=	D	
900	=	1000 – 100 = CM	
1000	=	M	

Example 27. **Which of the following are meaningless.**

(a) XXXX (b) LXIX (c) VL (d) LIV (e) IL

Solution : (a) XXXX

Since X cannot be repeated more than 3 times so XXXX is meaningless.

(b) LXIX = LX + IX = 60 + 9 = 69

So LXIX is meaningful.

(c) VL

Since V can never be subtracted.

So VL is meaningless.

(d) LIV = L + IV = 50 + 4 = 54

So LIV is meaningful.

(e) IL

Since I can be subtracted only from V and X not from L.

So IL is meaningless.

Example 28. Write the following in Hindu Arabic Numerals.

(a) LXXI (b) CXLV (c) CCXLI

(d) CLXVII (e) MCCXLI

Solution : (a) LXXI = L+X+X+I = 50 + 10 + 10 + 1 = 71

(b) CXLV = C+XL+V = 100 + 40 + 5 = 145

(c) CCXLII = C + C + XL + II = 100 + 100 + 40 + 2 = 242

(d) CLXVII = C + L + X + VII = 100 + 50 + 10 + 7 = 167

(e) MCCXLI = M + CC + XL + I = 1000 + 200 + 40 + 1 = 1241

Example 29. Express each of the following as a Roman Numeral.

(a) 49 (b) 82 (c) 198

(d) 541 (e) 826

Solution : (a) 49 = 40 + 9 = XLIX

(b) 82 = 50 + 30 +2 = LXXXII

(c) 198 = 100 + 90 + 8= CXCVIII

(d) 541 = 500 + 40 + 1 = DXLI

(e) 826 = 500 + 300 + 20 + 6 = DCCCXXVI

Exercise 1.5

1. Which of the following are meaningless:

(a) IC (b) VD (c) XCVII

(d) IVC (e) XM

2. Write the following in Hindu Arabic Numerals:

(a) XXV (b) XLV (c) LXXIX (d) XCIX

(e) CLXIV (f) DCLXII (g) DLXIX (h) DCCLXVI

(i) CDXXXVIII (j) MCCXLVI

3. Express each of the following as Roman numerals:

(a) 29 (b) 63 (c) 94 (d) 99

(e) 156 (f) 293 (g) 472 (h) 638

(i) 1458 (j) 948 (k) 199 (l) 499

(m) 699 (n) 299 (o) 999 (p) 1000

Multiple Choice Questions

1. The number of digits are ....
   (a) 9 (b) 10 (c) 8 (d) Infinite
2. The greatest 4 digit number using 1, 5, 2, 9 once is
   (a) 9215 (b) 9512 (c) 5912 (d) 9521
3. The smallest 4 digit number using 2, 0, 3, 7 once is
   (a) 0237 (b) 2037 (c) 7320 (d) 7023
4. Which of the following are in ascending order?
   (a) 217, 271, 127, 721 (b) 217, 127, 721, 271
   (c) 127, 217, 271, 721 (d) 721, 271, 217, 127
5. The face value of digit 4 in 23468 is :
   (a) 4 (b) 400 (c) 40 (d) 468
6. The place value of digit 2 in 4123 is
   (a) 23 (b) 2 (c) 20 (d) 200
7. The difference between place value and face value of 5 in 76542 is:
   (a) 537 (b) 45 (c) 0 (d) 495
8. $5 \times 10000 + 3 \times 100 + 2 \times 10 + 2 =$ .... = ...
   (a) 5322 (b) 53022 (c) 50322 (d) 53202
9. Four lakh two thousand three hundred fifty one = ...
   (a) 42351 (b) 402351 (c) 420351 (d) 4002351
10. How many four digit numbers are there?
    (a) 9999 (b) 9900 (c) 9000 (d) 9990
11. Seventeen million twenty four thousand fifty four = ...
    (a) 172454 (b) 170024054
    (c) 170240054 (d) 17024054
12. 1 Crore = ... million
    (a) 1 (b) 10 (c) 100 (d) 1000
13. Rounded off 7213 to nearest thousands.
    (a) 7200 (b) 7000 (c) 7210 (d) 7213
14. Rounded off 45553 to nearest hundreds.
    (a) 45500 (b) 45550 (c) 45600 (d) 45650
15. Solve; $(9-4) \times 6 =$ ....
    (a) 30 (b) 54 (c) 78 (d) 64
16. Which of the following number does not have symbol in Roman numerals?
    (a) 0 (b) 1 (c) 10 (d) 1000

17. How many symbols are used in Roman Numerals?

(a) 5 (b) 8 (c) 9 (d) 7

18. Which of the following are meaningless?

(a) LXIX (b) XC (c) IL (d) LI

19. CLXVI = ...

(a) 164 (b) 144 (c) 176 (d) 166

20. XCIX + XLVI =

(a) CVL (b) CLV (c) CXLV (d) CXLIV

Learning Outcomes

After completion of this chapter students are now able to

(i) Read and the write numbers in Indian as well as International sytem of Numeration.

(ii) Solve day to day life practical mathematical problems.

(iii) Perform basic operations on numbers and adopt them in routine life.

(iv) Make comparison between numbers.

(v) Use brackets while solving mathematical problems.

(vi) Use Roman Numeration system along with Hindu Arabic system of Numeration.

ANSWER KEY

Exercise 1.1

1. (a) Smallest = 30495 Greatest = 30945

   (b) Smallest = 10029 Greatest = 10920

2. (a) 3953, 5231, 6089, 6098 (b) 6073, 7392, 49905, 58904

   (c) 9801, 25751, 36501, 38802

3. (a) 75003, 60632, 20051, 7600 (b) 3289, 2934, 2834, 667

   (c) 92547, 88715, 45321, 1971

4. (a) 6432, 2346 (b) 9730, 3079

   (c) 5430, 3045 (d) 7321, 1237

5. (a) 7732, 2237 (b) 5530, 3005

   (c) 3320, 2003 (d) 4431, 1134

   (e) 8852, 2258 (f) 3321, 1123

6. (a) Six Lakh thirty eight thousand nine hundred seventy five
   (b) Eighty four thousand three hundred twenty one
   (c) Two crore ninety lakh sixty one thousand fifty eight
   (d) Six crore three thousand six hundred eight

7. (a) 9,86,06,873
   Nine crore eighty six lakh six thousand eight hundred seventy three
   (b) 76,35,172
   Seventy six lakh thirty five thousand one hundred seventy two
   (c) 8,97,00,057
   Eight crore ninety seven lakh fifty seven.
   (d) 8,93,22,602
   Eight crore ninety three lakh twenty two thousand six hundred two.
   (e) 45,03,217
   Forty five lakh three thousand two hundred seventeen.
   (f) 9,00,32,045
   Nine crore thirty two thousand forty five.

8. (a) 89,832,081
   Eighty nine million eight hundred thirty two thousand eighty one
   (b) 6,543,374
   Six million five hundred fourty three thousand three hundred seventy four.
   (c) 88,976,306
   Eighty eight million nine hundred seventy six thousand three hundred six.
   (d) 9,860,001
   Nine million eight hundred sixty thousand one
   (e) 90,032,045
   Ninety million thirty two thousand forty five.
   (f) 4,503,217
   Four million five hundred three thousand two hundred seventeen.

9. (a) 7,54,000 (b) 9,53,74,523 (c) 647,525
   (d) 72,332, 112 (e) 58,423,202 (f) 23.30.010

10. 90000000

11. (a) Ten (b) Ten (c) Ten
   (d) Ten (e) Ten

Exercise 1.2

1. (a) 5000 (b) 35000 (c) 2
   (d) 50 (e) 2 (f) 12000

2. 2556	**3.** 880267	**4.** 740	**5.** 20
6. 4 kg	**7.** 686659	**8.** 3022	**9.** 30466
10. 146			

Exercise 1.3

1.	(a) 1800	(b) 400	(c) 4000	(d) 10000
2.	(a) 720000	(b) 8000	(c) 27000	(d) 270000
3.	(a) 5000	(b) 61100		
4.	(a) 5090	(b) 61130		

Exercise 1.4

1.	(a) 1352	(b) 10710	(c) 642	(d) 1696
	(e) 41205	(f) 2244	(g) 42	(h) 24
	(i) 12			

Exercise 1.5

1. a, b, d, e

2.	(a) 25	(b) 45	(c) 79	(d) 99
	(e) 164	(f) 662	(g) 569	(h) 766
	(i) 438	(j) 1246		
3.	(a) XXIX	(b) LXIII	(c) XCIV	(d) XCIX
	(e) CLVI	(f) CCXCIII	(g) CDLXXII	(h) DCXXXVIII
	(i) MCDLVIII	(j) CMXLVIII	(k) CXCIX	(l) CDXCIX
	(m) DCXCIX	(b) CCXCIX	(c) CMXCIX	(p) M

Multiple Choice Questions

(1) b	(2) d	(3) b	(4) c	(5) a
(6) c	(7) d	(8) c	(9) b	(10) c
(11) d	(12) b	(13) b	(14) c	(15) a
(16) a	(17) d	(18) c	(19) d	(20) c

E2F5X1

2

WHOLE NUMBERS

Objectives

In this chapter you will learn

(i) To understand extended number system from natural numbers to whole numbers.

(ii) To represent whole numbers on number line and operate on number line.

(iii) To understand properties of whole numbers.

(iv) To understand goemetrical pattern from whole numbers.

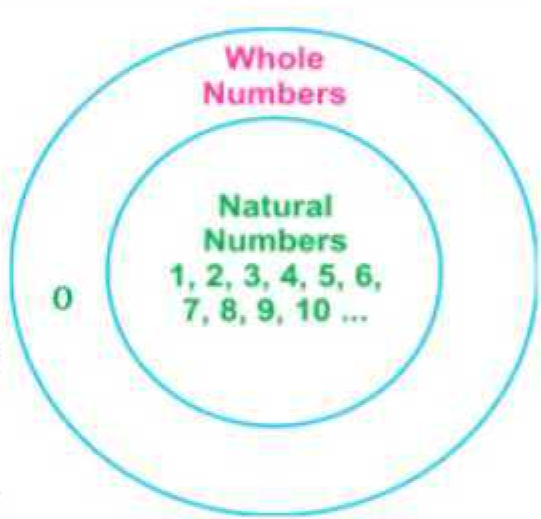
2.1 Introduction

We have already learnt about the natural numbers i.e 1, 2, 3, 4, 5, ... Thus counting numbers are called Natural numbers. If forty students are present in class 6th and all of these students went to play ground to play games, then how many students are left behind in the classroom? Your answer will be no student is present in classroom. In this situation we say that zero ('0') student is present in class.

The counting numbers or natural numbers i.e. 1, 2, 3, 4, 5, ... together with the number '0' are called whole numbers.

Whole numbers are represented by W

Thus W = {0, 1, 2, 3, 4, 5, 6, 7, 8 .... }



2.2. Relation between Natural Numbers and Whole Numbers

N = {1, 2, 3, 4, 5,6 ,7, 8, 9, 10, 11 .... }

W = {0, 1, 2, 3, 4. 5. 6, 7, 8. 9, 10, 11, ... }

All natural numbers are contained in collection of whole numbers.

- All natural numbers are whole numbers.
- All whole numbers are not natural numbers. (because '0' is a whole number but not natural

number)

- Smallest Natural number is 1.
- Smallest Whole number is 0.
- Greatest natural number cannot be written because by adding 1 to any natural number, we get larger natural number.
- Greatest whole number cannot be obtained because by adding 1 to any whole number, we get larger whole number.

2.3. Successor and Predecessor of a Whole Number

Successor : The successor of a whole number is the number obtained by adding 1 to it. Clearly the successor of 0 is 1; sucessor of 1 is 2, successor of 2 is 3 and so on......

Every whole number has successor.

Predecessor : The predecessor of a whole number is one less than the given number. Clearly the predecessor of 2 is 1, Predecessor of 1 is 0. But 0 does not have any predcessor in whole numbers. Every whole number other than zero has predecessor.

Example 1. **Write the successor of**

(a) 40099 **(b) 1000**

Solution : (a) Successor of 40099 = 40099 + 1

= 40100

(b) Sucessor of 1000 = 1000 + 1

= 1001

Example 2. **Write the predecessor of**

(a) 10000 **(b) 20099**

Solution : (a) Predecessor of 10000 = 10000 – 1

= 9999

(b) Predecessor of 20099 = 20099– 1

= 20098

2.4 Representation of Whole numbers on Number Line

Draw a line. Mark a point on it. Label it '0'. Mark a second point to the right of 0. Label it 1. The distance between these points labelled as 0 and 1 is called **unit distance**. On this line, mark a point to the right of 1 and at unit distance from 1 and label it 2. In this way go on labeling points at unit distance as 3, 4, 5, ... on the line.

You can go upto any whole number on the right in this manner. This is the number line for whole numbers.

Looking at above number line of whole numbers we observe that

(a) No whole number is on the left of zero ('0') and every whole number on right of '0' is greater than 0.

(b) A whole number on the right of given whole number is greater than the given whole number. e.g : 5>3, 7>6, and so on.

(c) A whole number on the left of given whole number (except 0) is less than the given whole number, e.g : 2<3, 5<7 and so on.

Number Line can be drawn in vertical form also.

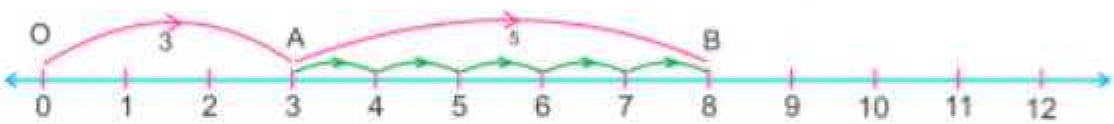
2.4.1 Addition of Whole Numbers on number line

In order to add two whole numbers on the number line, we follow the following steps:

1. Draw a number line and mark whole numbers on it.
2. Mark the first given number on the number line.
3. Move as many units as the second number to the right of the first number.
4. The number obtained in step 3 represents the sum of two whole numbers.
5. Similarly, sum of three, four and five whole numbers can be found out.

Example 3. **Represent 3+5 on the number line.**

Solution : We draw a number line and move 3 steps from 0 to the right and mark this point as A. Now starting from A, we move 5 steps towards right and arrive at B.



OA = 3, AB = 5, OB = 8

Hence, OB = 3 + 5 = 8

2.4.2. Subtraction of Whole Numbers on number line

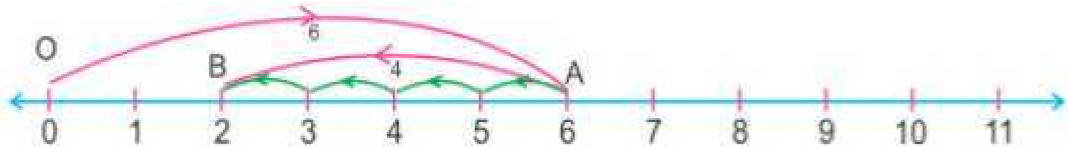
In order to subtract one whole number from another whole number on number line following steps are followed:

1. Draw a number line and mark whole number on it.
2. Mark the first given number on the number line.
3. Starting from first number move as many units as the second number to the left of first number.
4. The number obtained in step 3 represents the required difference of the given whole numbers on the number line.

Example 4. **Represent 6-4 on the number line.**

Solution : We draw a number line.

Starting from point 0 (i.e. zero), we move 6 steps to the right and arrive at A. Now starting from A we move 4 units left of A and arrive at B.



OA = 6. AB = 4

Clearly OB = 6 – 4 = 2

2.4.3 Product of Whole Numbers on number line

For multiplying the two whole numbers on number line, following steps are followed:

1. Draw a number line and mark whole numbers on it.
2. Starting from 0, we move to the right of 0 and count the units same as second number and it is considered as one jump.
3. Similar jumps are made equal to first number to reach at final point.
4. The final number represent the product of two whole numbers.

Example 5. **Find 3 × 4 using number line.**

Solution : We draw a number line.

Starting from 0 we move 4 units to the right of 0 to arrive at A. We make two more such same moves starting from A (total 3 moves of 4 unit each) to reach finally at C which represents 12.

Hence 4 × 3 = 12

2.4.4 Division on number line

For division of two whole numbers, following steps are followed :

1. Draw a number line and mark whole numbers on it.
2. Starting from 0 we move to the right of 0 and reached at first number.
3. From First number we jumps towards zero taking one jump value equal to second number.
4. The number of jumps taken to reach at zero is quotient.

Example 6. **Find 12 ÷ 3 by number line.**

Solution : Draw a number line.

Starting from 0, we move 12 units to the right of 0 to arrive at A. Now from A take moves of 3 units to the left of A till we reach at '0'. We observe that there are 4 moves.

So 12 ÷ 3 = 4

Exercise 2.1

1. Answer the following questions.
   (a) Write the smallest whole number.
   (b) Write the smallest natural number.
   (c) Write the successor of 0 in whole numbers.
   (d) Write the predecessor of 0 in whole numbers.
   (e) Largest whole number.
2. Which of the following statements are True (T) and which are False (F)?
   (a) Zero is the smallest natural number.
   (b) Zero is the smallest whole number.
   (c) Every whole number is a natural number.
   (d) Every natural number is a whole number.
   (e) 1 is the smallest whole number.
   (f) The natural number 1 has no predecessor in natural numbers.
   (g) The whole number 1 has no predecessor in whole numbers.
   (h) Successor of the largest two digit number is smallest three digit number.
   (i) The successor of a two digit number is always a two digit number.
   (j) 300 is the predecessor of 299.
   (k) 500 is the successor of 499.
   (l) The predecessor of a two digit number is never a single digit number.
3. Write the successor of each of following:
   (a) 100909 (b) 4630999
   (c) 830001 (d) 99999
4. Write the predecessor of each of following:
   (a) 1000 (b) 208090
   (c) 7654321 (d) 12576
5. Represent the following numbers on the number line.
   2, 0, 3, 5, 7, 11, 15
6. How many whole numbers are there between 22 and 43?
7. Draw a number line to represent each of following on it.
   (a) $3 + 2$ (b) $4 + 5$ (c) $6 + 2$
   (d) $8 - 3$ (e) $7 - 4$ (f) $7 - 2$

(g) 3×3 (h) 2×5 (i) 3×5

(j) $9 \div 3$ (k) $12 \div 4$ (l) $10 \div 2$

8. Fill in the blanks with the appropriate symbol < or >:

(a) 25 .... 205 (b) 170 107

(c) 415 ....514 (d) 10001 9999

(e) 2300014 .... 2300041 (f) 99999 888888

2.5. Properties of Whole numbers

We have already learnt about four fundamental operations addition, subtraction, multiplication and division on numbers. Now let us study the properties of these operations on whole numbers.

⇒ Closure Property

- Closure property holds under addition of whole numbers. As we know that sum of two whole numbers is also a whole number.

e.g: $7 + 5 = 12$ is a whole number

$5 + 6 = 11$ is a whole number

$0 + 4 = 4$ is a whole number

Hence **Whole number + Whole number = Whole number**

- Closure Property holds under multiplication of whole numbers also. As we know that product of two whole numbers is also a whole number.

e.g.: $7 \times 3 = 21$ is a whole number

$4 \times 6 = 24$ is a whole number

$0 \times 3 = 0$ is a whole number

Hence **Whole number × Whole number = Whole number**

- Closure Property does not hold under subtraction of whole numbers. As difference of two whole numbers is not always a whole number.

e.g. $7 - 9 = ?$ is not a whole number.

$3 - 7 = ?$ is not a whole number.

$7 - 4 = 3$ is a whole number.

- Closure property does not hold under division of whole numbers. As quotient of two whole numbers is not always a whole number.

e.g. $8 \div 4 = 2$ is a whole number.

but $5 \div 7 = \frac{5}{7}$ is a not a whole number

$6 \div 5 = \frac{6}{5}$ is not a whole number

Division by Zero

Division by a number means subtracting that number repeatedly.

Let us find $8 \div 2$

8	6	4	2
-2	-2	-2	-2
6	4	2	0

Subtracting 2 again and again from 8
We reached '0' after 4 steps
$\therefore \quad 8 \div 2 = 4$

Hence $8 \div 2 = 4$

Let us try $2 \div 0$ now

2	2	2	2
-0	-0	-0	-0
2	2	2	2

In every move we get 2 again!
Will this ever stop?
No.
We say $2 \div 0$ is not defined

Hence $2 \div 0$ is not defined

Note :- *Divison by zero is not defined*

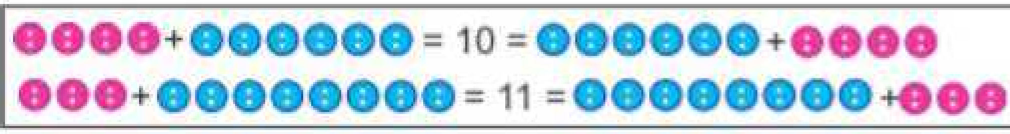
⇒ Commutative Property

- Commutativity holds under addition of whole numbers. You can add two whole numbers in any order. i.e. $a + b = b + a$

  e.g. $4 + 6 = 10 = 6 + 4$

  $3 + 8 = 11 = 8 + 3$

Button Activity



- Commutativity holds under multiplication of whole numbers also. You can multiply two whole numbers in any order. i.e. $a \times b = b \times a$

  e.g. $4 \times 5 = 20 = 5 \times 4$

  $3 \times 6 = 18 = 6 \times 3$

- But Subtraction is not commutative for whole numbers. In case of subtraction, If we change the order of whole numbers, result will not be same. $a - b \neq b - a$ (a and b are whole numbers)

  e.g. $10 - 3 = 7$

  $3 - 10 \neq 7$

- Similarly Division is not commutative for whole numbers.

  e.g. $12 \div 4 = 3$

  but $4 \div 12 \neq 3$

  $a \div b \neq b \div a$ (a and b are two different numbers, $a \neq 0$, $b \neq 0$)

⇒ Associativity

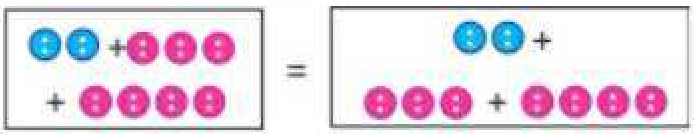
- **Addition** is associative for whole numbers if a, b, c are three whole numbers then $(a + b) + c = a + (b + c)$

  e.g.: $(2 + 3) + 4 = 5 + 4 = 9$

  & $2 + (3+4) = 2 + 7 = 9$

  ⇒ $(2 + 3) + 4 = 2 + (3 + 4)$

Button Activity



- **Multiplication** is also associative for whole numbers If a, b, c, are three whole numbers

  then $(a \times b) \times c = a \times (b \times c)$

  e.g. : $(2\times3) \times4 = 6 \times 4 = 24$

  $2 \times (3 \times 4) = 2 \times 12 = 24$

  Hence $(2\times3) \times 4 = 2\times (3 \times 4)$

- **Subtraction and Division** are not associative for whole numbers. If a, b, c, are three whole numbers then

  $(a - b) - c \neq a - (b - c)$

  and $(a \div b) \div c \neq a \div (b \div c)$

⇒ Existence of Identity

- **Additive Identity:**

  If a is any whole number then

  $a + 0 = a = 0 + a$

  In other words, the sum of any whole number and zero is the number itself. The whole number 0 (zero) is called the **additive identity**.

- **Multiplicative Identity**

  $a \times 1 = a = 1 \times a$

  In other words, the product of any whole number and 1 is the number itself. The whole number 1 (one) is called the **multiplicative identity**.

- Identity element does not exist under subtraction and division of whole numbers as subtracting and division are not commutative.

Distributive of Multiplication over Addition

If a, b, c are any three whole numbers,

then (i) $a \times (b+c) = a \times b + a \times c$ (ii) $(b + c) \times a = b \times a + c \times a$

In other words, the multiplication of whole numbers distributes over their addition.

Verification : In order to verify this property, we take any three whole numbers a, b, c and find the values of the expression $a\times(b+c)$ and $a\times b + a\times c$ as shown below.

Whole Numbers a, b, c	**Expression $a\times(b+c)$**	**Expression $a\times b + a\times c$**	**Is $a\times(b+c) = a\times b+a\times c$?**
2, 3, 5	$2 \times (3+5) = 2\times8 = 16$	$2\times3+2\times5$ $= 6 +10$ $= 16$	Yes
3, 7, 15	$3 \times (7+15) = 3\times22 = 66$	$3 \times 7 + 3\times 15$ $= 21 + 45 = 66$	Yes
0, 4, 9	$0\times(4+9) = 0\times 13 = 0$	$0 \times 4+ 0 \times 9$ $= 0 + 0$ $= 0$	Yes

We see that the expression $a \times (b+c)$ and $a\times b + a\times c$ are equal in each case.

Distributivity of Multiplication over Subtraction

Multiplication of whole numbers is also distributive over their subtraction. In other words, if a, b, c, are whole numbers such that $b>c$, then

(i) $a \times (b - c) = a \times b - a \times c$

(ii) $(b - c) \times a = b \times a - c \times a$

Example 7: **Using suitable arrangement of terms find the product of**

(a) 25 × 4 × 384 **(b) 25 × 9 × 40 × 22637**

Solution : (a) $25 \times 4 \times 384 = (25 \times 4) \times 384 = 100 \times 384 = 38400$

(b) $25 \times 9 \times 40 \times 25637 = (25 \times 40) \times (9 \times 25637)$

$= 1000 \times 230733$

$= 230733000$

Example 8: **Find the product using properties of whole numbers:**

(a) 187 × 107 **(b) 42 × 96**

Solution : (a) $187 \times 107 = 187 \times (100+7)$

$= 187 \times 100 + 187 \times 7$

$= 18700 + 1309$

$= 20009$

Distributive property of multiplication over addition

(b) $42 \times 96 = 42 \times (100-4)$

$= 42 \times 100 - 42 \times 4$

$= 4200 - 168$

$= 4032$

Distributive property of multiplication over subtraction

Example 9: **Simplify using distributive property of multiplication.**

(a) $15 \times 32 + 15 \times 68$ (b) $125 \times 215 - 125 \times 15$

Solution : (a) $15 \times 32 + 15 \times 68 = 15 \times (32 + 68)$

$= 15 \times 100$

$= 1500$

Using Distributive property of multiplication over addition

(b) $125 \times 215 - 125 \times 15$

$= 125 \times (215 - 15)$

$= 125 \times 200$

$= 25000$

Using Distributive property of multiplication over subtraction

Example 10: **Divide 4567 by 2354 by actual division and check the result by division algorithm.**

Solution :

$$\begin{array}{r} 234\,)\,4567\,(\,19 \\ -234\downarrow \\ \hline 2227 \\ -2106 \\ \hline 121 \\ \hline \end{array}$$

Here Dividend = 4567, Divisor = 234

Quotient = 19, Remainder = 121

Check/ Verification : By Division Algorithm

Dividend = (Divisor×Quotient) + Remainder

$4567 = (234 \times 19) + 121$

or $4567 = 4446 + 121$

or $4567 = 4567$

Which is true

So, result is verified.

Example 11: **What is the largest 4 digit number divisible by 13?**

Solution : Largest 4 digit number = 9999

Let us divide it by 13.

$$\begin{array}{r} 13\,)\,9999\,(\,769 \\ -91\downarrow \\ \hline 89 \\ -78\downarrow \\ \hline 119 \\ -117 \\ \hline 2 \end{array}$$

On dividing 9999 by 13 we get remainder 2. We subtract 2 from 9999 to get number exactly divisible by 13.

So, 9999 – 2 = 9997 is the largest 4 digit number which is divisible by 13.

Exercise 2.2

1. Find the sum by suitable arrangement of terms:
   (a) 837 + 208 + 363 (b) 1962 + 453 + 1538 + 647
2. Find the product by suitable arrangement of terms:
   (a) 2 × 1497 × 50 (b) 4 × 263 × 25
   (c) 8 × 163 × 125 (d) 963 × 16 × 25
   (e) 5 × 171 × 60 (f) 125 × 40 × 8 × 25
   (g) 30921 × 25 × 40 × 2 (h) 4 × 2 × 1932 × 125
   (i) 5462 × 25 × 4 × 2
3. Find the value of each of the following using distributive property:
   (a) (649×8) + (649×2) (b) (6524 × 69) + (6524 × 31)
   (c) (2986 × 35) + (2986 × 65) (d) (6001 × 172) – (6001 ×72)
4. Find the value of the following :
   (a) 493 × 8 + 493 × 2 (b) 24579 × 93 + 7 × 24579
   (c) 3845 × 5 × 782 + 769 × 25× 218
   (d) 3297 × 999 + 3297
5. Find the product using suitable properties:
   (a) 738 × 103 (b) 854 × 102 (c) 258 × 1008
   (d) 736 × 93 (e) 816 × 745 (f) 2032 × 613
6. A taxi driver filled his car petrol tank with 40 litres of petrol on monday. The next day, he filled the tank with 50 litres of petrol. If the petrol costs ₹ 78 per litre, how much he spend in all on petrol?
7. A vendor supplies 32 litres of milk to a hotel in morning and 68 litres of milk in the evening. If the milk costs ₹ 35 per litre, how much money is due to the vendor per day?
8. We know that 0×0=0. Is there any other whole number which when multiplied by itself gives the product equal to the number itself? Find out the number.
9. Fill in the blanks:
   (a) 15 × 0 = ... (b) 15 + 0 = ...
   (c) 15 – 0 = ... (d) 15 ÷ 0 = ...
   (e) 0 × 15 = ... (f) 0 + 15 = ...
   (g) 0 ÷ 15 = ... (h) 15 × 1 = ...
   (i) 15 ÷ 1 = ... (j) 1 ÷ 1 = ...

10. The product of two Whole numbers is zero. What do you conclude. Explain with example.

11. Match the following:

(i)	$537 \times 106 = 537 \times 100 + 537 \times 6$	(a) Commutativity under multiplication
(ii)	$4 \times 47 \times 25 = 4 \times 25 \times 47$	(b) Commutativity under addition
(iii)	$70 + 1923 + 30 = 70 + 30 + 1923$	(c) Distributivity of multiplication over addition.

2.6 Patterns in Whole numbers

In this section, we shall try to arrange numbers in elementary shapes made up of dots. The shapes we take are (1) a line (2) a rectangle (3) a square and (4) a triangle. Every number should be arranged in one of these shapes. No other shape is allowed.

2.6.1 Representing whole numbers by line segments

If '•' represents 1, then 2, 3, 4, 5, ... can be represented by line segments as follow:

The number 2 is shown as 

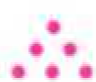
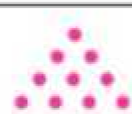
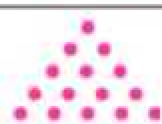
The number 3 is shown as 

The number 4 is shown as 

and so on.....

2.6.2 Triangular Numbers

Since whole numbers can be represented by triangles. Such numbers are called triangular numbers. 1, 3, 6, 10, 15 are some triangular numbers Let '•'represent 1. Following table shows the representation of whole numbers by triangles.

Triangular Number	Representation	Pattern
1		First triangular number = $\frac{1\times 2}{2}=1$
3		Second triangular number = $\frac{2\times 3}{2}=3$
6		Third triangular number = $\frac{3\times 4}{2}=6$
10		Fourth triangular number = $\frac{4\times 5}{2}=10$
15		Fifth triangular number = $\frac{5\times 6}{2}=15$

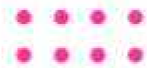
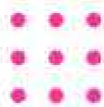
1 is both triangular and square number.

By observing the above pattern possessed by triangular numbers, we can formulate the following rule:

nth triangular number $= \dfrac{n \times (n+1)}{2}$

2.6.3 Representły Whole Numbers by squares and Rectangles

Some whole numbers can be represented by squares and some by rectangles as shown below.

Square Number	Representation	Rectangular number	Representation
1		6	
4		8	
9		10	

Now complete the table

Number	Line	Rectangle	Square	Triangle
2	Yes	No	No	No
3	Yes	No	No	Yes
4	Yes	Yes	Yes	No
5	Yes	No	No	No
6				
7				
8				
9				
10				
11				
12				
13				

2.7. Patterns observations

Observation of patterns can guide you in simplifying process. Study the following:

(a) 237 + 9 = 237 + 10 –1 = 247 – 1 = 246

(b) 237 – 9 = 237 – 10 +1 = 227 + 1 = 228

(c) 237 + 99 = 237 + 100 –1 = 337 –1 = 336

(d) 237 – 99 = 237 – 100 +1 = 137 + 1 = 138

Does this pattern help you to add or subtract numbers of the form 9, 99, 999 .....?

Here is one more pattern

(a) 84 × 9 = 84 × (10–1) = ? (b) 84 × 99 = 84 × (100 –1) = ?

(c) $84 \times 999 = 84 \times (1000-1) = ?$

Do you find a shortcut to multiply a number by numbers of the form 9, 99, 999, ...

Such shotcuts enable you to do sums verbally.

The following pattern suggests a way of multiplying by 5 or 25 or 125.

(a) $96 \times 5 = 96 \times \frac{10}{2} = \frac{960}{2} = 480$

(b) $96 \times 25 = 96 \times \frac{100}{4} = \frac{9600}{4} = 2400$

(c) $96 \times 125 = 96 \times \frac{1000}{8} = \frac{96000}{8} = 12000$

Exercise 2.3

1. If the product of two whole numbers is zero. Can we say that one or both of them will be zero? Justify through examples.
2. If the product of two whole numbers is 1. Can we say that one or both of them will be 1? Justify through examples.
3. Observe the pattern in the following and fill in the blanks:

$1 \times 1 = 1$

$11 \times 11 = 121$

$111 \times 111 = 12321$

$1111 \times 1111 = ...$

$11111 \times 11111 = ...$

4. Observe the pattern and fill in the blanks:

$1 \times 9 + 1 = 10$

$12 \times 9 + 2 = 110$

$123 \times 9 + 3 = 1110$

$1234 \times 9 + 4 = 11110$

$12345 \times 9 + 5 = ...$

$123456 \times 9 + 6 = ...$

5. Represent numbers from 24 to 30 according to rectangular, square or triangular pattern.
6. Study the following pattern:

$1 = 1 \times 1 = 1$

$1+3 = 2 \times 2 = 4$

$1+3+5 = 3 \times 3 = 9$

$1+3+5+7 = 4 \times 4 = 16$

Hence find the sum of

(a) First 12 odd numbers (b) First 50 odd numbers.

Multiple Choice Questions

1. The smallest whole number is

(a) 0 (b) 1 (c) 2 (d) 3

2. The smallest natural number is

(a) 0 (b) 1 (c) 2 (d) 3

3. The successor of 38899 is

(a) 39000 (b) 38900 (c) 39900 (d) 38800

4. The predecessor of 24100 is

(a) 24999 (b) 24009 (c) 24199 (d) 24099

5. The statement $4 + 3 = 3 + 4$ represents

(a) Closure (b) Associative

(c) Commutative property (d) Identity

6. Which of the following is the additive identity?

(a) 0 (b) 1 (c) 2 (d) 3

7. The multiplicative identity is ....

(a) 0 (b) 1 (c) 2 (d) 3

8. $15 \times 32 + 15 \times 68 =$

(a) 1400 (b) 1600 (c) 1700 (d) 1500

9. The largest 4 digit number divisible by 13 is

(a) 9997 (b) 9999 (c) 9995 (d) 9991

10. The successor of 3 digit largest number is

(a) 100 (b) 998 (c) 1001 (d) 1000

Learning Outcomes

After completion of this chapter, the students are now able to

(i) Represent whole numbers on number line.

(ii) Operate upon whole numbers with the help of number line as well as arithmetically.

(iii) Use various properties of whole numbers.

(iv) Make different geometrical pattern for given whole number.

ANSWER KEY

Exercise 2.1

1. (a) 0 (b) 1 (c) 1 (d) Not Possible (e) Not Possible
2. (a) F (b) T (c) F (d) T (e) F (f) T
   (g) F (h) T (i) F (j) F (k) T (l) F
3. (a) 100910 (b) 4631000 (c) 830002 (d) 100000
4. (a) 999 (b) 208089 (c) 7654320 (d) 12575
6. 20
8. (a) $<$ (b) $>$ (c) $<$ (d) $>$ (e) $<$ (f) $>$

Exercise 2.2

1. (a) 1408 (b) 4600
2. (a) 149700 (b) 26300 (c) 163000 (d) 385200 (e) 51300
   (f) 1000000 (g) 61842000 (h) 1932000 (i) 1092400
3. (a) 6490 (b) 652400 (c) 298600 (d) 600100
4. (a) 4930 (b) 2457900 (c) 19225000 (d) 3297000
5. (a) 76014 (b) 87108 (c) 260064 (d) 68448 (e) 607920 (f) 1245616
6. ₹ 7020 7. ₹ 3500
9. (a) 0 (b) 15 (c) 15 (d) Not Defined (e) 0
   (f) 15 (g) 0 (h) 15 (i) 15 (j) 1
11. (i) $\longrightarrow$ c (ii) $\longrightarrow$ a (iii) $\longrightarrow$ b

Exercise 2.3

3. $1111 \times 1111 = 1234321$
   $11111 \times 11111 = 123454321$
4. $12345 \times 9 + 5 = 111110$
   $123456 \times 9 + 6 = 111111\ 0$
6. (a) $12 \times 12 = 144$ (b) $50 \times 50 = 2500$

Multiple Choice Questions

(1) a (2) b (3) b (4) d (5) c
(6) a (7) b (8) d (9) a (10) d

F8V4G8

3

PLAYING WITH NUMBERS

Objectives

In this chapter you will learn

- To understand about factors and multiples.
- To provide information of prime-composite numbers, even-odd numbers etc.
- To provide information of divisiblility by different numbers.
- To acquire knowledge of HCF and LCM and their practical uses in life

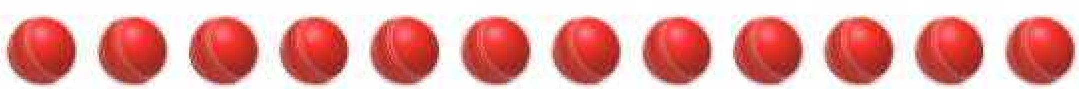
3.1 Introduction

In previous classes, we have studied about factors, multiples, prime and composite numbers. In this chapter, we shall review these concepts and extend our study to include some new properties with suitable examples.

3.2 Factors

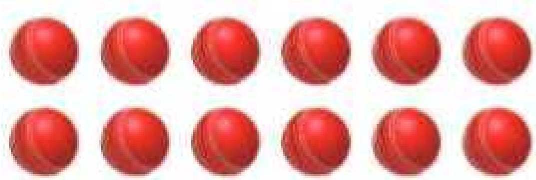
Vidhita arranges 12 balls in such a way that there are equal number of balls in each row.

→ 1 row with 12 balls



***Total number of balls** = 1 × 12 = 12*

→ 2 rows with 6 balls each

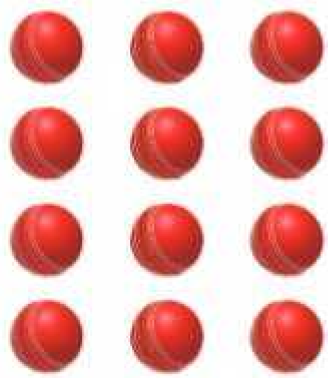


Total number of balls = 2 × 6 = 12

→ 3 rows with 4 balls each

Total number of balls = 3 × 4 = 12

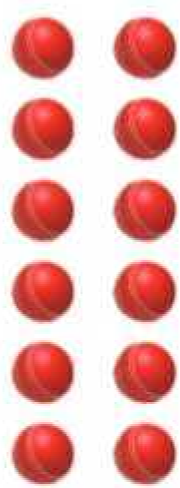
→ 4 rows with 3 balls each



Total number of balls = 4 × 3 = 12

→ 5 rows with equal number of balls in each row and having total 12 balls is not possible

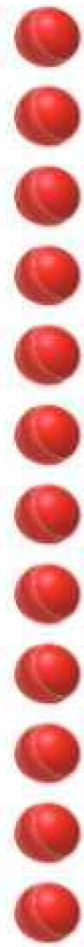
→ 6 rows with 2 balls each



Total number of balls = 6 × 2 = 12

→ 7, 8, 9, 10 or 11 rows with equal number of balls in each row having total 12 balls is not possible.

→ 12 rows with 1 ball each



Total number of balls = 12 × 1 = 12

Here, we observe that 12 can be written as the product of two numbers in different ways.

$12 = 1 \times 12$, $12 = 2 \times 6$, $12 = 3 \times 4$,

$12 = 12 \times 1$, $12 = 6 \times 2$, $12 = 4 \times 3$

Thus 1, 2, 3, 4, 6 and 12 exactly divide 12. So the numbers 1. 2. 3. 4. 6 and 12 are the factors of 12.

* If a = b×c then b and c are factors of a and a is multiple of b and c.

Factors of a number exactly divide that number without leaving any remainder.

A factor of a number is an exact divisor of that number.

Number	Factors
2	1, 2
6	1, 2, 3, 6
10	1, 2, 5, 10
20	1, 2, 4, 5, 10, 20
24	1, 2, 3, 4, 6, 8, 12, 24

We conclude from the table:

* 1 is a factor of every number.
* Every number is factor of itself.
* Every number (other than 1) has atleast two factors, 1 and itself.
* Every factor of a number is always less than or equal to the number.
* A number has always finite number of factors.

Example 1: **Find all the factors of 15.**

Solution : $15 = 1 \times 15$, $15 = 15 \times 1$

$15 = 3 \times 5$, $15 = 5 \times 3$

So, 1, 3, 5 and 15 are factors of 15.

Example 2: **Find all the factors of 36.**

Solution :

$36 = 1 \times 36$ $36 = 9 \times 4$

$36 = 2 \times 18$ $36 = 12 \times 3$

$36 = 3 \times 12$ $36 = 18 \times 2$

$36 = 4 \times 9$ $36 = 36 \times 1$

$36 = 6 \times 6$

Note:- There is no need of taking pairs 9×4, 12×3, 18×2, and 36×1. As they are repeating them selves in reverse order

So, 1, 2, 3, 4, 6, 9, 12, 18, and 36 are factors of 36.

3.3 Multiples

In class 5[th], we have studied about multiples that "Multiples of a number are obtained by multiplying it by any natural number".

Number	Multiples
1	1, 2, 3, 4, 5, ...
2	2, 4, 6, 8,10, ...
5	5, 10, 15, 20, 25, ...
8	8, 16, 24, 32, 40, ...
15	15, 30, 45, 60, 75, ...

We conclude from the table that:

* Every number is a multiple of itself.
* Every multiple of a number is greater than or equal to the number.
* The smallest multiple of a natural number is the number itself.
* There are infinite multiples of a number. So the largest multiple can not be defined.

Example 3: **Find the first six multiples of 4.**

Solution : First 6 multiples of 4 are

$4 \times 1 = 4,\quad 4 \times 2 = 8\quad 4 \times 3 = 12,\quad 4 \times 4 = 16,\quad 4 \times 5 = 20,\quad 4 \times 6 = 24$

Example 4 : **Find the first five multiples of 13.**

Solution : First 5 multiples of 13 are 13, 26, 39, 52, 65.

3.3.1 Perfect Number

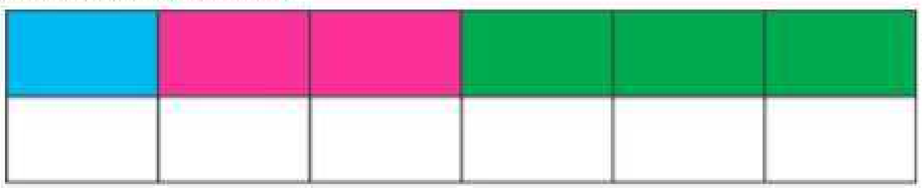
If the sum of all the factors of a number is two times the number then the number is called a perfect number.

The factors of 6 are 1, 2, 3 and 6

Also, $1 + 2 + 3 + 6 = 12 = 2 \times 6$

i.e sum of all factors of 6 = $2 \times$ Number

So, 6 is the perfect number.



$1 + 2 + 3$
$= 6$

* Other Perfect number are 28, 496 and 8128.

Note:- Factors of 6 except the number and the number along itself make a rectangle as shown.So, it is the perfect number.

3.3.2 Even numbers

All numbers which are multiples of 2 are called even numbers.

Or

Those numbers which are divisible by 2 are called even numbers e.g. 2, 4, 6, 8, 10,.....

* A number is an even number if 2 is a factor of it.
* All the even numbers end in 0, 2, 4, 6 or 8.
* If 2 is added to any even number then we get next consecutive even number.

3.3.3 Odd numbers

All numbers which are not multiples of 2 are called odd numbers. e.g. 1, 3, 5, 7, 9, 11, ...

* All the odd numbers end in 1, 3, 5, 7, 9.
* If 2 is added to any odd number then we get next consecutive odd number.

Note:- A number is either even or odd. A number cannot be both odd as well as even.

3.4 Prime and Composite Numbers

In the previous section we have learnt about factors and multiples of a number.

Let us consider the following table before discussing prime and composite numbers.

Numbers	Factors	Numbers of Factors
1	1	1
2	1, 2	2
3	1, 3	2
4	1, 2, 4	3
5	1, 5	2
6	1, 2, 3, 6	4
7	1, 7	2
8	1, 2 4, 8	4
9	1, 3, 9	3
10	1, 2, 5, 10	4
11	1, 11	2
12	1, 2, 3, 4, 6, 12	6

From the above table, we can divide the natural numbers in the following three categories:

(i) The numbers which have only **one factor**.

(ii) The numbers which have **exactly two factors (1 and itself)**

(iii) The numbers which have **more than two factors**.

We conclude that

(a) The natural number 1 is the only number which has exactly one factor, that number itself.

(b) The natural numbers 2, 3, 5, 7, 11, ... etc. have exactly two factors. 1 and the number itself. Such numbers are called **Prime Numbers**.

(c) The natural numbers 4, 6, 8, 9, 10, ... etc. have more than two factors. Such numbers are called **Composite Numbers.**

* 1 is the only number which is neither Prime nor Composite Number.
* 2 is the smallest prime number.
* 2 is the only even number which is prime. All other even numbers are composite numbers
* All prime numbers are odd except 2.
* All odd numbers are not prime numbers.

SIEVE Method

To find Prime and Composite Numbers

The Greek Mathematician Eratosthenes, found a very simple method for finding the prime and composite numbers in 3rd century B.C. He designed a table popularly known as **Sieve of Eratosthenes**.

In this table, He used natural numbers from 1 to 100.

The following steps are used to find prime and composite numbers form 1 to 100.

SIEVE OF ERATOSTHENES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Step 1:- As we know 1 is neither a prime nor a composite number. Thus put it in a square box.

Step 2:- Encircle 2 and cross out every multiple of 2 like 4, 6, 8, 10,12, ... etc.

Step 3:- Encircle next number 3 and cross out every multiple of 3 like 6, 9, 12, 15, 18, ... etc. Number already crossed need not be crossed again.

Step 4:- Encircle the next number 5 and cross out every multiple of 5 like 10, 15, 20, 25, ... etc.

Step 5:- Encircle the next number 7 and cross out every multiple of 7 like 14, 21, 28, 35, ... etc. continue this process till every number is either encircled or crossed out.

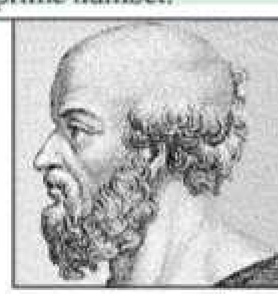
All the number that are encircled are the **prime numbers** and the number that are crossed out are the **composite numbers**.

Rule to check whether a number between 100 and 200 is prime or not: If the given number is divisible by any prime number less than 15 i.e. 2, 3, 5, 7, 11, 13 then it is composite other wise it is prime number.

→**Between 200 and 400:** If a number is divisible by any prime number less than 20, then it is composite number otherwise it is prime number.

Greek Mathematician Eratosthenes

He is best known for being the first person to calculate the **circumference of the Earth** and **tilt of the Earth's axis** with remarkable accurary. He was the founder of scientific chronology. He introduced the Sieve of Eratosthenes, an efficient method of finding prime numbers.



Eratosthenes (276 BC - 194BC)

3.4.1 Twin Primes

The pair of prime numbers having a difference of two are known as **twin primes**. The twin primes have only one composite number between them.

The twin primes from 1 to 100 are (3,5), (5,7), (11,13), (17,19),(29,31), (41,43), (59,61) and (71,73)

Key Point

Numbers between any twin primes (above 5) is a multiple of 6.

3.4.2.Prime Triplet

A set of three consecutive prime numbers that differ by 2 is called a prime triplet. The only prime triplet is (3, 5, 7).

Gold Bach's Conjecture:- In 1742, a famous mathematician Goldbach gave a rule for which he could not provide a proof. So far no proof has been provided by anybody to contradict it by finding even one example.

"Every even number greater than 4 can be expressed as the sum of two odd prime numbers."

For example : $6 = 3 + 3$, $10 = 3 + 7$ or $5 + 5$, $18 = 7 + 11$, $24 = 11 + 13$, $36 = 17 + 19$ etc.

Example 5:- Which of the following are prime numbers?

(i) 37 (ii) 117 (iii) 191 (iv) 221

Solution : (i) Given number = 37

It is divisible by 1 and itself.

So it has exactly two factors

∴ 37 is a prime number

(ii) Given number = 117

We find that 117 is divisible by 3

∴ It has more than two factors.

∴ So it is not a prime number.

(iii) Given nuumber = 191

We find that 191 is not divisible by any of the numbers 2, 3, 5, 7, 11 and 13. So it is a prime number.

(iv) Given number = 221

We find that 221 is divisible by 13.

∴ It has more than two factors.

So it is not a prime number

Example 6: Express each of the following numbers as a sum of two odd primes:

(i) 20 (ii) 32 (iii) 48

Solution : (i) $20 = 3 + 17$

$= 7 + 13$

(ii) $32 = 3 + 29$

$= 13 + 19$

(iii) $48 = 5 + 43$

$= 7 + 41$

$= 11 + 37$

$= 17 + 31$

$= 19 + 29$

Exercise 3.1

1. Write down all the factors of each of the following:-

   (i) 18 (ii) 24 (iii) 45 (iv) 60 (v) 65

2. Write down the first six multiples of each of the following:-

   (i) 6 (ii) 9 (iii) 11 (iv) 15 (v) 24

3. List all the numbers less than 100 that are multiples of

   (i) 17 (ii) 12 (iii) 21

4. Which of the following are prime numbers ?

   (i) 39 (ii) 127 (iii) 177 (iv) 201 (v) 237 (vi) 361

5. Express each of the following as sum of two odd prime numbers:-

   (i) 16 (ii) 28 (iii) 40

6. Write all the prime numbers between the given numbers:-

   (i) 1 to 25 (ii) 85 to 105 (iii) 120 to 140

7. Is 36 a perfect number?

8. Find the missing factors:-

   (i) $5 \times ..... = 30$ (ii) $..... \times 6 = 48$ (iii) $7 \times ..... = 63$

   (iv) $..... \times 8 = 104$ (iv) $..... \times 7 = 105$

9. List all 2-digit prime numbers, in which both the digits are prime numbers.

3.5 Common Factors and Multiples

In the previous section, we have learnt about the factors and the multiples of a number. In this section, we shall discuss the common factors or common multiples of two or more numbers.

Let's consider some examples:-

Example 7: **Find the common factors of 12 and 18.**

Solution : The factors of 12 = 1, 2, 3, 4, 6 and 12.

The factors of 18 = 1, 2, 3, 6, 9 and 18.

∴ Common factors of 12 and 18 are 1, 2, 3 and 6.

Example 8: **Find the common factors of 15, 24 and 30**

Solution : The factors of 15 = 1, 3, 5, 15

The factors of 24 = 1, 2, 3, 4. 6. 8. 12, 24

The factors of 30 = 1, 2, 3, 5, 6. 10. 15, 30

∴ Common factors of 15, 24 and 30 are 1 and 3.

Example 9: **Find the first four common multiples of 4 and 6.**

Solution : The multiples of 4 = 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52.

The multiples of 6 = 6, 12, 18, 24, 30, 36, 42, 48, 54

∴ The first four common multiples of 4 and 6 are 12, 24, 36 and 48.

Example 10: **Find the common factors of 16 and 25.**

Solution : The factors of 16 = 1, 2, 4, 8, 16

The factors of 25 = 1, 5, 25

∴ Common factors of 16 and 25 = 1

3.5.1 Co-Prime Numbers

Two numbers are said to be co-prime if they do not have a common factor other than 1.

In above example, common factors of 16 and 25 is 1. So these are called co-prime numbers. Other examples are (5, 7) ; (8, 9), (12, 13) etc.

- Two co-prime numbers need not to be both prime numbers.
- Two prime numbers are always co-prime.

3.6 Tests of Divisibility

To find whether a number is divisible by another number, we perform actual division and check whether the remainder is zero or not. But this is very time-consuming process. There are certain divisibility tests of numbers 2, 3, 4, 5, 6, 8, 9, 10 and 11 to check the divisibility whether a number is divisible by any of these numbers or not. In this section, we shall learn about these tests:

Divisibility by 2:- A number is divisible by 2, if its **units digit is even** i.e. 0, 2, 4, 6 or 8.

For example:-

(a) 2164, 12562, 83490 etc are divisible by 2.

(b) 6193, 82937, 14051 etc are not divisible by 2.

Divisibility by 4:- A number is divisible by 4, if the number **formed by its last two digits** is divisible by 4 or if the number ends in two zeros.

For example:-

(a) 61<u>24</u>, 256<u>32</u>, 843<u>00</u>, 124<u>96</u> etc. all are divisible by 4. As number formed by their last two digits are divisible by 4.

(b) 127<u>31</u> , 51<u>67</u> , 423<u>42</u> etc are not divisible by 4 because their last two digits are not divisible by 4.

Divisibility by 8:- A number is divisible by 8, if its **last three digits** is divisible by 8 or if the number ends in three zeros.

For example:-

(a) 214832 , 51616 , 2400 etc are divisible by 8.

(b) 613513 , 52642 , 1678093 etc are not divisible by 8 because last three digits are not divisible by 8.

Divisibility by 3:- A number is divisible by 3, if the **sum of its digits is divisible by 3**.

For example:-

(a) 258 is divisible by 3.

As Sum of digits = 2 + 5 + 8 = 15 is divisible by 3.

(b) 51062 is not divisible by 3.

As Sum of digits = 5 + 1 + 0 + 6 + 2 = 14, is not divisible by 3.

Divisibility by 9:- A number is divisible by 9, if the **sum of its digits is divisible by 9**.

For example:-

(a) 62154 is divisibly by 9.

As Sum of its digits = 6 + 2 + 1 + 5 + 4 = 18, is divisible by 9.

(b) 23509 is not divisibly by 9.

As Sum of its digits = 2 + 3 + 5 + 0 + 9 = 19, is not divisibly by 9.

Divisibility by 5:- A number is divisible by 5, if its **last digit is 0 or 5**.

For example:-

(a) 51680 , 235045 , 91435 etc. are divisible by 5, as their last digit is 0 or 5.

(b) 216803 , 52361 etc are not divisible by 5, as their last digit is not 0 or 5.

Divisibility by 10:- A number is divisible by 10, if its **last digit is 0**.

For example:-

(a) 62560 , 315680 , 25600 etc. are divisible by 10, As their last digit is 0.

(b) 2153 , 68024 , 519831 etc are not divisible by 10.

Divisibility by 6:- A number is divisible by 6, if the number is divisible **by 2 and 3**, both

Or

An **even number which is divisible by 3, will be divisible** by 6.

For example:- 25824 is divisible by 6

As it is an even number and sum of its digits = 2 + 5 + 8 + 2 + 4 = 21, is divisible by 3.

Divisibility by 11:- A number is divisible by 11, if the difference of the sum of its digits in odd places and sum of its digits in even places is either 0 or a multiple of 11.

For example:-

(a) 435204 is divisible by 11.

Since sum of digits in odd places = 4 + 5 + 0 = 9 and sum of digits in even places = 3 + 2 + 4 = 9 their difference = 9 – 9 = 0

(b) 6574312 is not divisible by 11

Since sum of digits in odd places = 6 + 7 + 3 + 2 = 18 and sum of digits in even places = 5 + 4 + 1 = 10 their differences = 18 – 10 = 8, which is not divisible by 11.

3.7 Some General Properties of Divisibility

Property 1:- Let a, b, c be three numbers. If a is divisible by b and b is divisible by c then a is divisible by c.

Or

If a number is divisible by another number, then it is divisible by each of the factors of that number.

For example:- 48 is divisible by 12 and 12 is divisible by 2, 3 and 6. So 48 is also divisible by 2, 3 and 6.

Consequences:-

* Since 4 is divisible by 2, So every number which is divisible by 4 is also divisible by 2.
* Since 6 is divisible by 2 and 3 both, So every number which is divisible by 6 is also divisible by 2 and 3 also.
* Since 9 is divisible by 3, So every number divisible by 9 is also divisible by 3.

Property 2:- If a and b are two co-prime numbers such that a number c is divisible by both a and b then c is also divisible by a × b.

Or

If a number is divisible by each of the two or more co-prime numbers then it is divisible by their product

For example:- Let us take two co-prime numbers 3 and 4.

3 is a factor of 72 and 4 is also a factor of 72. So 3 × 4 = 12 is, also a factor of 72.

Property 3:- If two numbers b and c are divisible by a then (b + c) is also divisible by a.

Or

If a number is a factor of each of two given numbers then it is a factor of their sum.

For example:- 45 and 70 both are divisible by 5. The sum of these two numbers is 45 + 70 = 115.

⇒ 115 is also divisible by 5.

Property 4:- If two numbers b and c are divisible by a then (b – c) or (c – b) is also divisible by a

Or

If a number is a factor of each of the two given numbers then it is a factor of their difference.

For example:- 84 and 45 both are divisible by 3.

The difference of these two numbers is 84 – 45 = 39

⇒ 39 is also divisible by 3.

Exercise 3.2

1. Find the common factors of the followings:-

   (i) 16 and 24 (ii) 25 and 40 (iii) 24 and 36

   (iv) 14, 35 and 42 (v) 15, 24 and 35

2. Find first three common multiples of the followings:-

   (i) 3 and 5 (ii) 6 and 8 (iii) 2, 3 and 4

3. Which of the following numbers are divisible by 2 or 4?

   (i) 52314 (ii) 678913 (iii) 4056784 (iv) 21536 (v) 412318

4. Which of the following numbers are divisible by 3 or 9?

   (i) 654312 (ii) 516735 (iii) 423152 (iv) 704355 (v) 215478

5. Which of the following numbers are divisible by 5 or 10?

   (i) 456803 (ii) 654130 (iii) 256785 (iv) 412508 (v) 872565

6. Which of the following numbers are divisible by 8?

   (i) 457432 (ii) 5134214 (iii) 7232000 (iv) 5124328 (v) 642516

7. Which of the following numbers are divisible by 6?

   (i) 425424 (ii) 617415 (iii) 3415026 (iv) 4065842 (v) 725436

8. Which of the following numbers are divisible by 11?

   (i) 4281970 (ii) 8049536 (iii) 1234321 (iv) 6450828 (v) 5648346

9. State True or False:-

   (i) If a number is divisible by 24, then it is also divisible by 3 and 8.

   (ii) 60 and 90 both are divisible by 10 then their sum is not divisible by 10.

   (iii) If a number is divisible by 8 then it is also divisible by 16.

   (iv) If a number is divisible by 15 then it is also divisible by 3.

   (v) 144 and 72 are divisible by 12 then their difference is also divisible by 12.

10. If a number is divisible by 5 and 9 then by which other number will that number be always divisible?

11. Which of the following pairs are co-prime?

    (i) 25, 35 (ii) 16, 21 (iii) 24, 41 (iv) 48, 33 (v) 20, 57

3.8 Prime Factorisation:- (Canonical Form)

In the previous sections, we have learnt about factors of a number, prime numbers and composite numbers. If a number is composite then it can be written as the product of two of its factors, the factors may be both prime or both composite or either prime or composite.

If composite, the factors can be split again, this process will be continued when we get all prime factors.

Thus "Prime Factorisation is the process by which a composite number is rewritten as the product of prime factors."

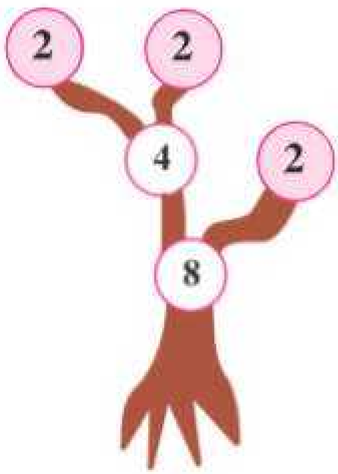
* **Fundamental Theorem of Arithmetic:-** Every Composite number can be factorised into prime factors in one and only one way apart from the order of the factors.

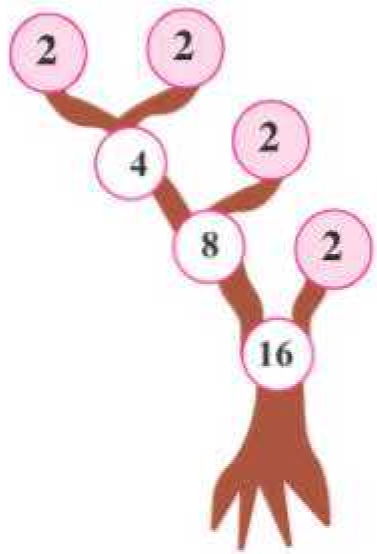
Prime factorisation can be done by two methods:-

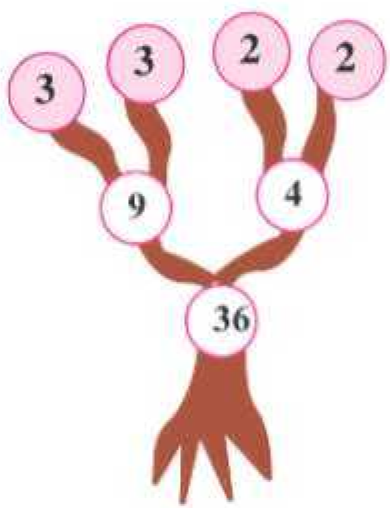
- Factor Tree Method
- Division Method

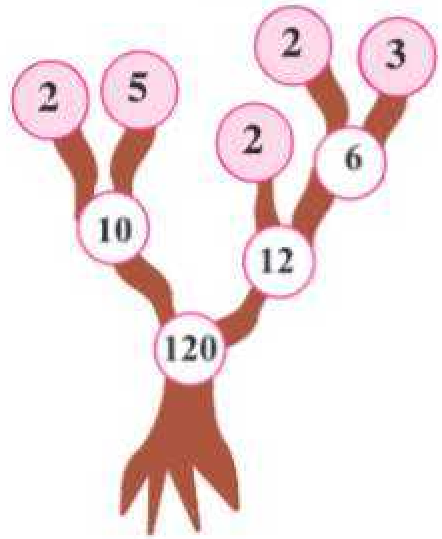
3.8.1 Factor Tree Method

In each step of the factor tree, we write the given composite number as the product of its smallest prime factors and another factor untill we get all the prime factors.





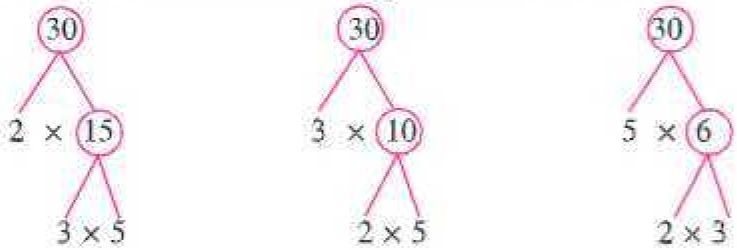




2 × 5 2 ×

3

Let us find the prime factors of 30 using the factor tree method.



So In each case, the prime factorisation of 30 is $2 \times 3 \times 5$

3.8.2.Division Method

Let us find the prime factors of 360 using the division method.

Step I:- Divide the number by any prime number which will exactly divide it.

Let us find the prime factors of 30 using the factor tree method.

Step II:- Continue dividing the quotient by any prime number till we get the quotient itself as a prime number.

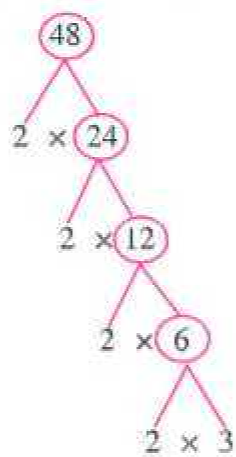
So, The prime factorisation of 360 is $2 \times 2 \times 2 \times 3 \times 3 \times 5$

2	360
2	180
2	90
3	45
3	15
	5

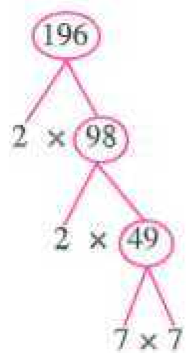
Example 11:- Find the prime factors of the following by factor tree method:-

(i) 48 (ii) 196 (iii) 150

Solution : (i)



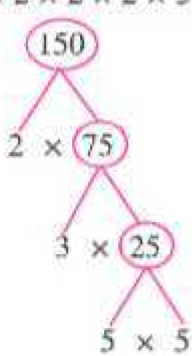
(ii)



∴ Prime factorisation of 48 $= 2 \times 2 \times 2 \times 2 \times 3$

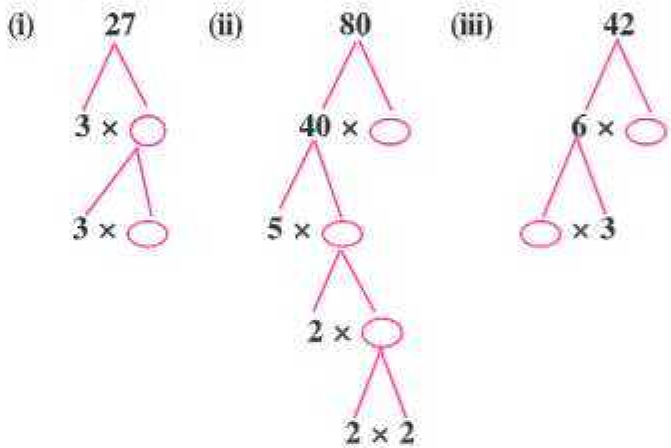
∴ Prime factorisation of 196 $= 2 \times 2 \times 7 \times 7$

(iii)

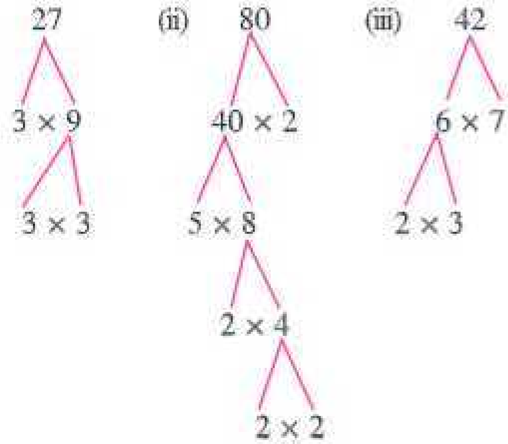


∴ Prime factorisation of $150 = 2 \times 3 \times 5 \times 5$

Example 12:- **Complete each factor tree**



Solution :



Example 13:- **Find prime factors of the following numbers:-**

(i) 216 (ii) 375 (iii) 920

Solution : (i)

2	216
2	108
2	54
3	27
3	9
	3

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

(ii)

3	375
5	125
5	25
	5

$375 = 3 \times 5 \times 5 \times 5$

(iii)

2	920
2	460
2	230
5	115
	23

$920 = 2 \times 2 \times 2 \times 5 \times 23$

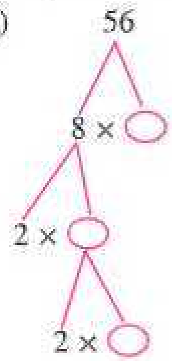
Exercise 3.3

1. Find prime factors of the following numbers by factor tree method:-

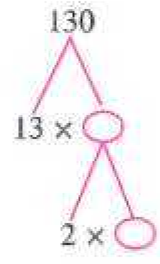
   (i) 96 (ii) 120 (iii) 180

2. Complete each factor tree:-

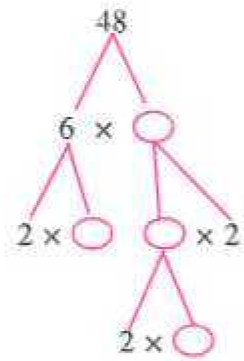
   (i)



   (ii)



   (iii)



3. Find the prime factors of the following numbers by division method:-

   (i) 420 (ii) 980 (iii) 225 (iv) 150 (v) 324

3.9. Highest common Factor (H.C.F.) Or Greatest Common Divisor (G.C.D)

In the previous sections, we have learnt about factors and multiples, common factors. In this section, we shall learn about the highest common factor among the common factors of the given numbers, which is also known as H.C.F. or G.C.D. of the numbers.

"The highest common factor (H.C.F.) of two or more numbers is the greatest or the largest among common factors".

In other words, H.C.F. of two or more numbers is the **largest number that divides all the numbers completely.**

For example:- Consider the numbers 24 and 42.

Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Factors of 42 = 1, 2, 3, 6, 7, 14, 21, 42

Common factors of 24 and 42 = 1, 2, 3, 6

out of these common factors, we find that 6 is the highest or greatest common factor.

So HCF of 24 and 42 is 6.

* HCF of two or more numbers can never be zero because 1 as a factor will be common to all numbers.
* HCF of two co-prime numbers is always 1.
* HCF is always smaller than or equal to the smallest of the given numbers.

There are two common methods to find H.C.F. of two or more numbers.

* Prime Factorisation Method

* Continued division mehtod

Here, we shall learnt about these two methods.

3.9.1. Prime Factorisation Method:- To find H.C.F., we follow the following steps:

Step 1:- Make the prime factors of each of the given number.

Step 2:- Find the common prime factors of the given numbers.

Step 3:- The product of all common factors (of step 2) is the H.C.F. of given numbers.

Example 14: **Find HCF of the following numbes:-**

(i) **36 and 48** (ii) **30 and 75** (iii) **108 and 144**

(iv) **42, 63 and 210** (v) **125, 175 and 250**

Solution : (i) First we write the prime factorisation of each of the given numbers.

2	36
2	18
3	9
	3

2	48
2	24
2	12
2	6
	3

$\therefore\ 36 = 2 \times 2 \times 3 \times 3$

and $48 = 2 \times 2 \times 2 \times 2 \times 3$

We find that 2 occurs two times and 3 occurs once as common factors.

$\therefore$ HCF of 36 and 48 $= 2 \times 2 \times 3 = 12$

(ii) First we write the prime factorisation of each of given numbers

2	30
3	15
	5

3	75
5	25
	5

$\therefore\ 30 = 2 \times 3 \times 5$

and $75 = 3 \times 5 \times 5$

We find that 3 occurs once and 5 occurs once as common factors.

$\therefore$ HCF of 30 and 75 $= 3 \times 5 = 15$

(iii) First we write the prime factorisation of each of the given numbers.

2	108
2	54
3	27
3	9
	3

2	144
2	72
2	36
2	18
3	9
	3

$\therefore\ 108 = 2 \times 2 \times 3 \times 3 \times 3$

and $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

We find that 2 occurs twice and 3 occurs twice as common factors.

$\therefore$ HCF of 108 and 144 = $2 \times 2 \times 3 \times 3 = 36$

(iv) First we write the prime factorisation of each of given numbers.

2	42
3	21
	7

3	63
3	21
	7

2	210
3	105
5	35
	7

$\therefore\ 42 = 2 \times 3 \times 7$

$63 = 3 \times 3 \times 7$

and $210 = 2 \times 3 \times 5 \times 7$

We find that 3 occurs once and 7 occurs once as common factors.

$\therefore$ HCF of 42, 63 and 210 = $3 \times 7 = 21$

(v) First we write the prime factorisation of each of given numbes

5	125
5	25
	5

5	175
5	35
	7

2	250
5	125
5	25
	5

$\therefore\ 125 = 5 \times 5 \times 5$

$175 = 5 \times 5 \times 7$

and $250 = 2 \times 5 \times 5 \times 5$

We find that 5 occurs twice as common factors.

$\therefore$ HCF of 125, 175 adn 250 = $5 \times 5 = 25$

3.9.2 Continued Division Method (Euclid's Algorithm)

Euclid, a Greek mathematician derived an interesting method to find HCF of two or more numbers. This method is known as **Euclid's algorithm** or **Long division method**.

Euclid's Algorithm (step for finding HCF)

Step 1:- From the given numbers, Identity the greater number.

Step 2:- Take the greater number as dividend and the smallest number as divisor.

Step 3:- Find the quotient and remainder.

Step 4:- If the remainder is zero then the divisor is the required HCF.

Step 5:- If the remainder is non-zero then take the remainder as new divisor and the last divisor as the new dividend.

Step 6:- Repeat the steps till the remainder obtained is zero.

Step 7:- The last divisor for which the remainder is zero is the required H.C.F.

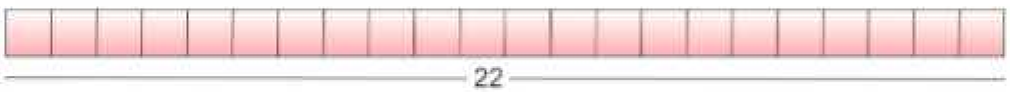
Let us perform an activity based on this method.

To Find HCF by cutting and pasting of paper.

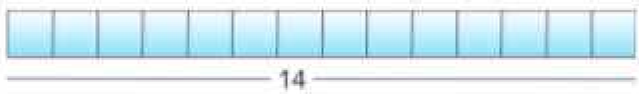
Material used:- A meaning scale, a pencil, chart paper, coloured pencils or sketch pens, eraser etc.

Procedure:- To find HCF of 14 and 22.

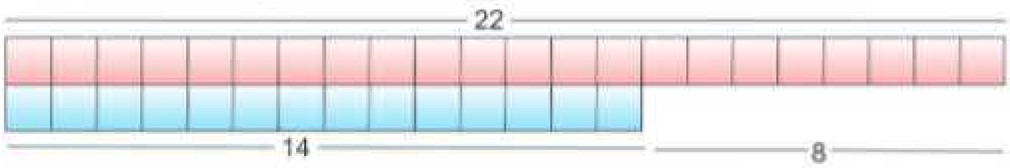
1. Take white coloured chart and cut a strip and divide it into 22 square boxes with pencil and scale and fill red colour in it.



2. Take another strip and divide it into 14 square boxes and fill blue colour in it.

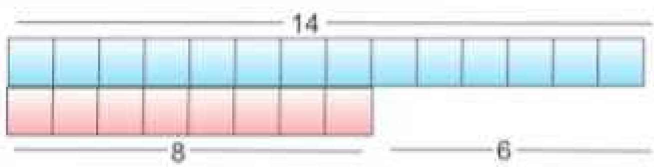


3. Divide the larger number 22 by smaller number 14 as (shown)



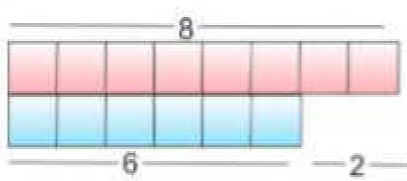
14) 22 (1
− 14
8

4. Divide the smaller number 14 by remainder 8.



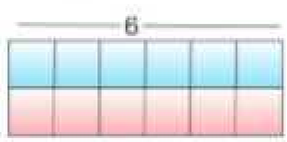
8) 14 (1
− 8
6

5. Divide 8 by 6.



6) 8 (1
− 6
2

6. Divide 6 by 2.



2) 6 (3
− 6
0

Observation:- The required HCF of 14 and 22 is 2 which is the last divisor in the above process as it leaves remainder 0.

Example 15: **Find HCF of the following by division method:-**

(i) 144, 252 (ii) 58, 70 (iii) 25, 44

Solution : (i) Given numbers are 144 and 252

```
144) 252 ( 1
   - 144
   108)144 (1
      - 108
       36 )108 (3
          - 108
             0
```

Hence, 36 is the HCF of 144 and 252.

(ii) Given numbers are 58 and 70.

```
58) 70 (1
  - 58
   12)58 (4
     - 48
      10) 12 (1
        - 10
         2 )10 (5
           -10
             0
```

Hence, 2 is H.C.F. of 58 and 70

(iii) Given numbers are 25 and 44

```
25) 44 ( 1
  - 25
   19) 25 (1
     - 19
       6 ) 19 (3
         - 18
          1 ) 6 ( 6
              6
              0
```

Hence 1, is H.C.F. of 25 and 44.

H.C.F. of More than two numbers:-

To find H.C.F. of three numbers, we proceed as follows:-

Step 1:- Find H.C.F. of any two of them.

Step 2:- Find H.C.F. of remaining number and HCF obtained in step 1.

Step 3:- HCF of step 2 is the required HCF of three numbers.

Example 16: Find H.C.F. of 50, 125 and 195.

Solution : Given numbers are 50, 125 and 195.

Consider any two numbers, say 50 and 125.

```
50 ) 125 ( 2
   - 100
     25 ) 50 ( 2
        - 50
           0
```

∴ HCF of 50 and 125 is 25.

Now, we find HCF of 25 and 195

```
25 ) 195 ( 7
   - 175
     20 ) 25 ( 1
        - 20
           5 ) 20 ( 4
             - 20
                0
```

∴ HCF of 25 and 195 is 5.

⇒ HCF of 50, 125 and 195 is 5.

3.9.3. Applications of H.C.F. (Word Problems)

In this section, we shall discuss some applications of HCF in solving some practical daily life problems. Let's illustrate these problems with following examples:-

Example 17:- Find the greatest number which divides 250 and 188 leaving the remainder 2 in each case.

Solution : Given that, required number when divides 250 and 188, the remainder is 2 in each case.

⇒ $250 - 2 = 248$ and $188 - 2 = 186$ are completely divisible by the required number.

⇒ Required number is the highest common factor of 248 and 186.

Since it is given that required number is the largest number.

∴ Required number is the HCF of 248 and 186.

```
186 ) 248 ( 1
    - 186
      62 ) 186 ( 3
         - 186
             0
```

∴ Required number (HCF) is 62

Example 18: Find the greatest number which divides 645 and 792 leaving a remainder 7 and 9 respectively.

Solution : Required greatest number = HCF of $(645 - 7)$ and $(792 - 9)$

= HCF of 638 and 783.

$$
\begin{array}{l}
638\,)\,783\,(\,1 \\
\quad -638 \\
\quad\overline{145)\,638\,(4} \\
\qquad -580 \\
\qquad\overline{58\,)145\,(2} \\
\qquad\quad -116 \\
\qquad\quad\overline{29\,)58\,(2} \\
\qquad\qquad \underline{-58} \\
\qquad\qquad \underline{\;0\;}
\end{array}
$$

$\therefore$ Required greatest number = 29

Example 19 : Find the greatest number that divides 135, 245 and 385 leaving a remainder 5 in each case.

Solution: Required Number= HCF of (135 – 5), (245 – 5) and (385 – 5)

= HCF of 130, 240 and 380

Now,

$$
\begin{array}{c|l} 2 & 130 \\ \hline 5 & 65 \\ \hline & 13 \end{array}
\qquad
\begin{array}{c|l} 2 & 240 \\ \hline 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array}
\qquad
\begin{array}{c|l} 2 & 380 \\ \hline 2 & 190 \\ \hline 5 & 95 \\ \hline & 19 \end{array}
$$

$\therefore$ $130 = 2 \times 5 \times 13$

$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$

and $380 = 2 \times 2 \times 5 \times 19$

$\therefore$ $HCF = 2 \times 5 = 10$

Hence, Required number = 10

Example 20:- Two tankers contain 434 litres and 465 litres of diesel respectively. Find maximum capacity of a container that can measure the diesel of both containers exact number of times.

Solution : We have to find, maximum capacity of a container which measure both containers.

$\Rightarrow$ We required the maximum number which divides 434 and 465 completely.

$\Rightarrow$ Required Number = HCF of 434, 465

$$
\begin{array}{c|l} 2 & 434 \\ \hline 7 & 217 \\ \hline & 31 \end{array}
\qquad
\begin{array}{c|l} 5 & 465 \\ \hline 3 & 93 \\ \hline & 31 \end{array}
$$

$434 = 2 \times 7 \times 31$

and $465 = 5 \times 3 \times 31$

$\therefore$ HCF of 434 and 465 = 31

So Required capacity of container = 31 litres

Example 21: The length, breadth and height of a room are 8m 25cm, 6m 75cm and 4m 50cm respectively. Find the longest tape which can measure the three dimensions of the room exactly.

Solution : We have to find the longest tape which measure the given dimensions of room.

So We required the maximum number which divides 8m 25cm, 6m 75cm and 4m 50cm

∴ Required length of tape = HCF of 8m 25cm, 6m 75cm and 4m 50cm

= HCF of 825 cm, 675cm and 450cm [∵ 1m = 100 cm]

Now Take any two numbers, say 825 and 675

```
675) 825 ( 1
    - 675
     150) 675 ( 4
         - 600
           75) 150 ( 2
              - 150
                  0
```

Here, HCF of 825 and 675 is 75.

Now to find HCFof 75 and 450

```
75) 450 ( 6
   - 450
       0
```

∴ HCF of 835, 675 and 450 = 75

∴ Hence, length of longest tape = 75cm

Example 22: A floor of a room is 9m × 4.75m. It is to be paved with square tiles of marble of the same size. Find the greatest measurement of each tile.

Solution: We have to find square tile of greatest measurement which paved the floor marble exactly

∴ Required size of tile = HCF of 9m and 4.75m

= HCF of 900cm and 475cm [∵ 1 m = 100cm]

```
475) 900 ( 1
    - 475
     425) 475 ( 1
         - 425
           50) 425 ( 8
              - 400
                25) 50 ( 2
                   - 50
                      0
```

∴ HCF of 900 cm and 475cm = 25cm

Hence, Side of each square tile is 25cm.

Example 23 : **Reduce $\frac{312}{507}$ to the lowest term (Simplest form).**

Solution : In order to reduce a given fraction to the lowest terms, we divide the numerator and denominator by their HCF.

Now, we find HCF of 312 and 507.

```
312) 507 ( 1
    -312
    195) 312 (1
        -195
        117 ) 195 (1
             - 117
              78)117 (1
                 - 78
                  39 ) 78( 2
                     - 78
                        0
```

Clearly HCF of 312 and 507 is 39

Now $\frac{312}{507} = \frac{312 \div 39}{507 \div 39} = \frac{8}{13}$

[Divide numerator and denomiantor by 39].

Exercise 3.4

1. Find H.C.F. of the following numbers by prime factorisation:-

   (i) 30, 42 (ii) 135, 225 (iii) 180, 192 (iv) 49, 91, 175 (v) 144, 252, 630

2. Find H.C.F. of the following numbers using division method:-

   (i) 170, 238 (ii) 54, 144 (iii) 72, 88 (iv) 96, 240, 336 (v) 120, 156, 192

3. What is the HC.F. of two prime numbers?
4. What is the H.C.F. of two consecutive even numbers?
5. What is the H.C.F of two consecutive natural numbers?
6. What is the H.C.F. of two consecutive odd numbers?
7. Find the greatest number which divides 245 and 1029, leaving a remainder 5 in each case.
8. Find the greatest number that can divide 782 and 460 leaving remainder 2 and 5 respectively.
9. Find the greatest number that will divide 398, 437 and 540 leaving remainders 7, 12 and 13 respectively.
10. Two different containers contain 529 litres and 667 litres of milk respectively. Find the maximum capacity of container which can measure the milk of both containers in exact number of times.
11. There are 136 apples, 170 mangoes and 255 oranges. These are to be packed in boxes containing the same number of fruits. Find the greatest number of fruits possible in each box.

12. Three pieces of timber 54m, 36m and 24m long, have to be divided into planks of the same length. What is the greatest possible length of each plank?

13. A room measures 4.8m and 5.04m. Find the size of the largest square tile that can be used to tile the floor without cutting any tile

14. Reduce each of the following fractions to lowest forms:-

(i) $\frac{85}{102}$ (ii) $\frac{52}{130}$ (iii) $\frac{289}{391}$

3.10 Lowest Common Multiple

In previous sections, we have learnt about the highest common factors of two or more numbers and we have learnt common multiples of two or more numbers. In this section, we shall learn about the lowest of the common multiples of given numbers which is known as L.C.M. of the numbers.

"Lowest common Multiple (L.C.M.) of two or more numbers is the smallest number which is a multiple of each of the numbers."

Or

LCM is the smallest number which is divisible by all the given numbers.

For example:- Consider number 6 and 8.

Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, ...

Multiples of 8 are 8, 16, 24, 32, 40, 48, 56, ...

Common multiples are 24, 48, ...

Clearly, 24 is the smallest among common multiples.

∴ LCM of 6 and 8 is 24.

* LCM of the numbers is exactly divisible by each number.
* LCM of two numbers is the greater number of them, if one of the numbers is multiple of other.
* LCM of given numbers is not less than any of the given numbers.

There ae two methods to find LCM of two or more numbers:-

* Prime Factorisation Method
* Common Division Method

Now, we shall learn about these two methods:-

3.10.1. Prime Factorisation Method

To find LCM, we follow the following steps:-

Step 1:- Make the prime factors of each of the given number.

Step 2:- Find the product of all different prime factors with maximum number of times each factor appear.

Step 3:- The product of those factors is the required LCM.

Example 24: Find LCM of the following numbers:-

(i) 20, 30 (ii) 36, 120 (iii) 72, 84 (iv) 40, 75, 126 (v) 108, 135, 162

Solution : (i)

2	20
2	10
	5

2	30
3	15
	5

$\therefore\ 20 = 2 \times 2 \times 5$

$30 = 2 \times 3 \times 5$

We find that in these prime factorisation 2 occurs maximum two times, 3 and 5 occurs maximum once.

$\therefore$ LCM of 20 and 30 $= 2 \times 2 \times 3 \times 5 = 60$

(ii)

2	36
2	18
3	9
	3

2	120
2	60
2	30
3	15
	5

$\therefore\ 36 = 2 \times 2 \times 3 \times 3$

$120 = 2 \times 2 \times 2 \times 3 \times 5$

In these prime factorisation, 2 occurs maximum 3 times, 3 occurs maximum 2 times and 5 occurs maximum once.

$\therefore$ LCM of 36 and 120 $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

(iii)

2	72
2	36
2	18
3	9
	3

2	84
2	42
3	21
	7

$\therefore\ 72 = 2 \times 2 \times 2 \times 3 \times 3$

$84 = 2 \times 2 \times 3 \times 7$

In these prime factorisation, 2 occurs maximum 3 times, 3 occurs maximum 2 times and 7 occurs maximum once.

$\therefore$ LCM of 72 and 84

$= 2 \times 2 \times 2 \times 3 \times 3 \times 7 = 504$

(iv)

2	40
2	20
2	10
	5

3	75
5	25
	5

2	126
3	63
3	21
	7

$\therefore\ 40 = 2 \times 2 \times 2 \times 5$

$75 = 3 \times 5 \times 5$

$126 = 2 \times 3 \times 3 \times 7$

In these prime factorisation, 2 occurs maximum 3 times, 3 and 5 occurs maximum twice and 7 occurs maximum once

$\therefore$ LCM of 40, 75 and 126 $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 = 12600$

(v) $108 = 2 \times 2 \times 3 \times 3 \times 3$

$135 = 3 \times 3 \times 3 \times 5$

$162 = 2 \times 3 \times 3 \times 3 \times 3$

$$\begin{array}{r|l} 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 162 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

In these prime factorisation 2 occurs maximum 2 times, 3 occurs maximum 4 times and 5 occurs maximum once.

$\therefore$ LCM of 108, 135 and 162 = $2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 = 1620$

3.10.2 Common Division Method

To find L.C.M. of two or more numbers, we follow the following steps:-

Step 1:- Arrange the given numbers in a row separated by commas.

Step 2:- Obtain a number which divides exactly atleast two of the given numbers.

Step 3:- Write the quotients just below them which are divisible by the chosen number and carry forward the numbers which are not divisible by that number.

Step 4:- Repeat the process till no two of the given numbers divisible by the same number.

Step 5:- The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers.

Example 25: Find the L.C.M. of the following numbers:-

(i) 45, 60 **(ii) 12, 18 and 20** **(iii) 30, 40 and 75**

(iv) 84, 90 and 120 **(v) 56, 72 and 144**

Solution : (i)

$$\begin{array}{r|l} 3 & 45, 60 \\ \hline 5 & 15, 20 \\ \hline & 3, 4 \end{array}$$

$\therefore$ LCM of 45 and 60 = $3 \times 5 \times 3 \times 4 = 180$

(ii)

$$\begin{array}{r|l} 2 & 12, 18, 20 \\ \hline 2 & 6, 9, 10 \\ \hline 3 & 3, 9. 5 \\ \hline & 1, 3, 5 \end{array}$$

$\therefore$ LCM of 12, 18 and 20 = $2 \times 2 \times 3 \times 3 \times 5 = 180$

(iii)

2	30, 40, 75
5	15, 20, 75
3	3, 4, 15
	1, 4, 5

$\therefore$ LCM of 30, 40 and 75 = $2 \times 5 \times 3 \times 4 \times 5 = 600$

(iv)

2	84, 90, 120
2	42, 45, 60
3	21, 45, 30
5	7, 15, 10
	7, 3, 2

$\therefore$ LCM of 84, 90 and 120 = $2 \times 2 \times 3 \times 5 \times 7 \times 3 \times 2 = 2520$

(v)

2	56, 72, 144
2	28, 36, 72
2	14, 18, 36
3	7, 9, 18
3	7, 3, 6
	7, 1, 2

$\therefore$ LCM of 56, 72 and 144 = $2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 2 = 1008$

3.10.3 Applications of LCM (Word Problems)

In this section, we shall discuss some applications of LCM in solving some practical daily life problems. Let's illustrate these problems with following examples:-

Example 26: Find the smallest number which is divisible by 12, 15 and 24.

Solution : We know that the smallest number divisible by 12, 15 and 24 is their LCM.

So, We calculate LCM of 12, 15 and 24

2	12, 15, 24
2	6, 15, 12
3	3, 15, 6
	1, 5, 2

$\therefore$ LCM = $2 \times 2 \times 3 \times 5 \times 2 = 120$

Hence Required number = 120

Example 27: Find the least number when divided by 6, 15 and 21 leaves remainder 4 in each case.

Solution : We know that the least number divisible by 6, 15 and 21 is their LCM.

So, the required number must be 4 more than their LCM.

We calculate LCM of 6, 15 and 21

3	6, 15, 21
	2, 5, 7

$\therefore$ LCM = $3 \times 2 \times 5 \times 7 = 210$

Hence, Required number = $210 + 4 = 214$

Example 28: Find the greatest 3-digit number exactly divisible by 18, 24 and 36

Solution : First, find LCM of 18, 24 and 36

$\therefore$ LCM = $2 \times 2 \times 3 \times 3 \times 2 = 72$

2	18, 24, 36
2	9, 12, 18
3	9, 6, 9
3	3, 2, 3
	1, 2, 1

Now the greatest 3-digit number is 999

We find that when 999 is divided by 72, the remainder is 63.

Hence, greatest number of 3 digits which is exactly divisible by 18, 24 and 36 is = $999 - 63 = 936$

$$72 \overline{)999(} \; 13$$
$$\begin{array}{r} -72 \\ \hline 279 \\ -216 \\ \hline 63 \\ \hline \end{array}$$

Example 29: Find the 4-digit smallest number which is exactly divisible by 15, 20 and 24.

Solution : First, find LCM of 15, 20 and 24

2	15, 20, 24
2	15, 10, 12
3	15, 5, 6
5	5, 5, 2
	1, 1, 2

$\therefore$ LCM = $2 \times 2 \times 3 \times 5 \times 2 = 120$

Now, 4 digit smallest number is 1000.

We find that when 1000 is divided by 120, the remainder is 40.

$$120 \overline{)1000(} \; 8$$
$$\begin{array}{r} -960 \\ \hline 40 \\ \hline \end{array}$$

$\therefore$ Smallest 4-digit number, which is exactly divisible by 15, 20 and 24 = $1000 + (120 - 40) = 1080$

Hence, required number = 1080

Example 30: In a morning walk, three persons step off together. Their steps measure 70cm, 80cm and 75cm respectively. What is the minimum distance each should walk so that all can cover the same distance in complete steps?

Solution : The distance covered by each one of them has to be same as well as minimum.So, the required minimum distance each should walk would be L.C.M. of the measure of their steps.

$\therefore$ LCM = $2 \times 5 \times 7 \times 8 \times 15 = 8400$ cm

2	70, 80, 75
5	35, 40, 75
	7, 8, 15

Hence required distance = 8400 cm or 84m

Example 31 : Four bells toll at intervals of 2, 3, 4 and 5 seconds. The bells toll together at 8 a.m. when will they again toll together?

Solution : The bells will toll together at a time which is a multiple of four intervals 2, 3, 4 and 5 seconds.

So, first we find LCM of 2, 3, 4 and 5

2	2, 3, 4, 5
	1, 3, 2, 5

$\therefore$ LCM = $2 \times 3 \times 2 \times 5 = 60$

Thus, the bells will toll together after 60 seconds or 1 minute.

First they toll together at 8a.m. then they will toll together after 1 minute i.e. 8:01a.m.

3.10.4 Relation Between H.C.F. and LCM:-

* HCF of given numbers is always a factor of LCM **or** LCM is a multiple of H.C.F.
* The product of HCF and LCM of two numbers is equal to product of both given numbers. If a and b are two numbers then $a \times b$ = HCF × LCM

For example:- Consider two numbers 12 and 18

$12 = 2 \times 2 \times 3$

and $18 = 2 \times 3 \times 3$

HCF = $2 \times 3 = 6$

and LCM = $2 \times 2 \times 3 \times 3 = 36$

Now Product of given numbers = $12 \times 18 = 216$

Product of their HCF and LCM = $6 \times 36 = 216$

Hence, **Product of two numbers = Product of their HCF and LCM**

Note:- This result is true only for two numbers

Example 32: Can two numbers have 18 as their HCF and 42 as their LCM. Give reasons in support of your answer.

Solution : We know that HCF of given numbers is a factor of their LCM.

But 18 is not a factor of 42.

So, there cannot be two numbers with HCF 18 and LCM 42.

Example 33: The HCF and LCM of two numbers are 15 and 75 respectively. If one number is 25 find other number.

Solution : I^{st} number × 2^{nd} number = HCF × LCM

$$2^{nd} \text{ number} = \frac{\text{HCF} \times \text{LCM}}{I^{st} \text{ number}} = \frac{15 \times 75}{25} = 45$$

Hence other number is 45.

Exercise 3.5

1. Find LCM of following numbers by prime factorisation method:-
   (i) 45, 60 (ii) 52, 56 (iii) 96, 360
   (iv) 36, 96, 180 (v) 18, 42, 72
2. Find LCM of the following by common division method:-
   (i) 24, 64 (ii) 42, 63 (iii) 108, 135, 162
   (iv) 16, 18, 48 (v) 48, 72, 108
3. Find the smallest number which is divisible by 6, 8 and 10.
4. Find the least number when divided by 10, 12 and 15 leaves remainder 7 in each case.
5. Find the greatest 4-digit number exactly divisible by 12, 18 and 30.
6. Find the smallest 4-digit number exactly divisible by 15, 24 and 36.
7. Four bells toll at intervals of 4, 7, 12 and 14 seconds.The bells toll together at 5a.m. When will they again toll together?
8. Three boys step off together from the same spot their steps measures 56cm, 70cm and 63cm respectively. At what distance from the starting point will they again step together?
9. Can two numbers have 15 as their HCF and 65 as their LCM. Give reasons in support of your answer.
10. Can two numbers have 12 as their HCF and 72 as their LCM. Give reasons in support of your answer.
11. The HCF and LCM of two numbers are 13 and 182 respectively. If one of the numbers is 26. Find other number.
12. The LCM of two co-prime numbers is 195. If one number is 15 then find the other number.
13. The HCF of two numbers is 6 and product of two numbers is 216. Find their LCM.

Multiple Choice Questions

1. Which number is a factor of every number?

(a) 0 (b) 1 (c) 2 (d) 3

2. How many even numbers are prime?

(a) 1 (b) 2 (c) 3 (d) 4

3. The smallest composite number is ....

(a) 1 (b) 2 (c) 3 (d) 4

4. Which of the following number is a perfect number?

(a) 8 (b) 6 (c) 12 (d) 18

5. Which of the following is not a multiple of 7?

(a) 35 (b) 48 (c) 56 (d) 91

6. Which of the following is not a factor of 36?

(a) 12 (b) 6 (c) 9 (d) 8

7. The number of prime numbers upto 25 are

(a) 9 (b) 10 (c) 8 (d) 12

8. Which mathematician gave the method to find prime and composite numbers?

(a) Aryabhatta (b) Ramayan (c) Eratosthenes (d) Goldbach

9. The statement "Every even number greater than 4 can be expressed as the sum of two odd prime numbers" is given by

(a) Goldbach (b) Eratosthenes (c) Aryabhatta (d) Ramanujan

10. Which of the following is a prime number?

(a) 221 (b) 195 (c) 97 (d) 111

11. Which of the following number is divisble by 4?

(a) 52369 (b) 25746 (c) 21564 (d) 83426

12. Which of the following is not true?

(a) If a number is factor of two numbers then it is also factor of their sum

(b) If a number is factor of two numbers then it is also factor of their difference.

(c) 15 and 24 are co-prime to each other.

(d) 1 is neither prime nor composite.

13. Which of the following pair is co-prime?

(a) (12, 25) (b) (18, 27) (c) (25, 35) (d) (21, 56)

14. Which of the following number is divisible by 8?

(a) 123568 (b) 412580 (c) 258124 (d) 453230

15. Prime factorisation of 84

(a) $2 \times 2 \times 3 \times 2 \times 7$ (b) $7 \times 2 \times 3 \times 3$

(c) $2 \times 3 \times 7 \times 2$ (d) $3 \times 2 \times 3 \times 2 \times 7$

16. HCF of 25 and 45 is

(a) 15 (b) 5 (c) 225 (d) 135

17. If LCM of two numbers is 36 then which of the following can not be their HCF?

(a) 9 (b) 12 (c) 8 (d) 18

18. The LCM of two co-prime numbers is 143. If one number is 11 then find other number.

(a) 132 (b) 154 (c) 18 (d) 13

19. Find the greatest number which divides 145 and 235 leaving the remainder 1 in each case.

(a) 24 (b) 18 (c) 19 (d) 17

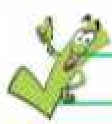
20. The greatest 4 digit number which is divisible by 12, 15 and 20

(a) 9990 (b) 9000 (c) 9960 (d) 9999

Learning Outcomes

After completion of this chapter, the students are now able to :

1. Understand about factors and multiples.
2. Give information about different types of numbers.
3. Check the divisibility of number without actual division.
4. Apply knowledge of HCF and LCM and can use them in daily life.



ANSWER KEY

Exercise 3.1

1. (i) 1, 2, 3, 6, 9, 18 (ii) 1, 2, 3, 4, 6, 8, 12, 24

(iii) 1, 3, 5, 9, 15, 45 (iv) 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

(v) 1, 5, 13, 65

2. (i) 6, 12, 18, 24, 30, 36 (ii) 9, 18, 27, 36, 45, 54

(iii) 11, 22, 33, 44, 55, 66 (iv) 15, 30, 45, 60, 75, 90

(v) 24. 48. 72, 96, 120, 144

3. (i) 17, 34, 51, 68, 85 (ii) 12, 24, 36, 48, 60, 72, 84, 96

(iii) 21, 42, 63, 84

4. (ii), (iv)

5. (i) $16 = 3 + 13 = 5 + 11$ (ii) $28 = 11 + 17$

(iii) $40 = 3 + 37 = 11 + 29 = 17 + 23$

6. (i) 2, 3, 5, 7, 11, 13, 17, 19, 23 (ii) 89, 97, 101, 103

(iii) 127, 131, 137, 139

7. No

8. (i) 6 (ii) 8 (iii) 9 (iv) 13 (v) 15

9. 23, 37, 53, 73

Exercise 3.2

1. (i) 1, 2, 4, 8 (ii) 1, 5 (iii) 1. 2. 3. 4. 6. 12 (iv) 1, 7 (v) 1

2. (i) 15, 30, 45 (ii) 24, 48, 72 (iii) 12, 24, 36

3. Divisible by 2:- (i), (iii), (iv), (v)

Divisible by 4:- (iii), (iv)

4. Divisible by 3:- (i), (ii), (iv), (v)

Divisible by 9:- (ii), (v)

5. Divisible by 5:- (ii), (iii), (v)

Divisible by 10:- (ii)

6. (i), (iii), (iv) 7. (i), (iii), (v) 8. (i), (ii), (iii), (v)

9. (i) True (ii) False (iii) False (iv) True (v) True

10. 45 11. (ii), (iii), (v)

Exercise 3.3

1. (i) $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$ (ii) $120 = 2 \times 2 \times 2 \times 3 \times 5$

(iii) $180 = 2 \times 2 \times 3 \times 3 \times 5$

2. (i) 7, 4, 2 (ii) 10, 5 (iii) 8, 3, 4, 2

3. (i) $420 = 2 \times 2 \times 3 \times 5 \times 7$ (ii) $980 = 2 \times 2 \times 5 \times 7 \times 7$

(iii) $225 = 3 \times 3 \times 5 \times 5$ (iv) $150 = 2 \times 3 \times 5 \times 5$

(v) $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$

Exercise 3.4

1. (i) 6 (ii) 45 (iii) 12 (iv) 7 (v) 18
2. (i) 34 (ii) 18 (iii) 8 (iv) 48 (v) 12
3. 1 4. 2 5. 1 6. 1 7. 16
8. 65 9. 17 10. 23 litres 11. 17 12. 6m
13. 24cm 14. (i) $\frac{5}{6}$ (ii) $\frac{2}{5}$ (iii) $\frac{17}{23}$

Exercise 3.5

1. (i) 180 (ii) 728 (iii) 1440 (iv) 1440 (v) 504
2. (i) 192 (ii) 126 (iii) 1620 (iv) 144 (v) 432
3. 120 4. 67 5. 9900 6. 1080 7. 5 : 01 : 24 cm
8. 2520cm 9. No 10. Yes 11. 91 12. 13
13 36

Multiple Choice Questions

1. b 2. a 3. d 4. b 5. b 6. d 7. a 8. c
9. a 10. c 11. c 12. c 13. a 14. a 15. c 16. b
17. c 18. d 19. b 20. c

V4M6C3

4

INTEGERS

Objectives

In this chapter you will learn

(i) To understand about the extended number system from natural numbers to integers.

(ii) To represent integers on number line and operations on number line.

(iii) To identify greater or smaller integer out of given set of integers .

(iv) To solve problems involving addition and subtraction of integers.

4.1 Introduction

We have already learnt about the natural numbers, i.e. 1, 2, 3,4, 5, ... which we also called counting numbers. We also learnt about whole numbers i.e. 0, 1, 2, 3, 4, 5, ... which is the extension of natural numbers. We have studied earlier in whole number system that sum of two whole numbers is always a whole number, but difference of two whole numbers is not always a whole number (Do you remember $7 - 5 = 2$) But what is $5 - 7 =$?. To answer this problem we need to extend our number system from whole numbers to integers. Let us look at few more real life examples:

- Sachin goes to a hill sation, the temperature of that hill station is 0°C. Further 2 degrees fall in temperature causes the temperature to be 2°C below 0°C. Can you tell the present temperature?
- Ramesh and Arjun went to a shop to buy a pen. The price of pen is ₹25. But Ramesh had only ₹20 in his pocket. He borrowed ₹5 from Arjun and bought the pen. Now Ramesh is left with no money or ₹0 in his pocket. But he has to remember the amount borrowed (should he or not?) He writes ₹5 in his note book. How will he express the money borrowed in numbers?

  In the above examples, we feel the need of introducing special numbers to deal with the situations of borrowing and going below 0.

Infact we need to extend our number system beyond whole numbers.

For Example : Earning and Spending, East and West, Deposit and Withdrawls, Above sea level and below sea level, above freezing point and below freezing point, etc. In fact for this we need negative numbers to express spending (Contrary to positive earning) and to express other similar cases.

4.2. Negative Numbers

Negative number is a real number less than zero. Negative numbers represent opposite to the positive numbers. If positive represents a movement to the right, then negative represents a movement to the left. If positive represents above sea level, then negative represents below sea level. If positive represents a deposit, negative represents a withdrawl. They are often used to represent the magnitude of a loss or deficiency. A debt owned by someone, may be considered as his negative asset. Negative numbers are used to describe values on a scale that goes below zero, such as celsius and Fahrenheit scales of Temparature.

Negative numbers are usually written with a minus sign in front of number like –1 (pronounced as : minus one or negative one). Numbers less than zero are negative and greater than zero are positive. **Zero itself is neither positive nor negative**. Zero is non-negative non positive number.

As in common sense oppsoite of opposite is the original thing, like wise negative of a negative is positive.

For Example : – (–1) = 1

In this way we got new range of numbers that we called negative numbers (negative integers) and these are:

–1, –2, –3, –4, –5, ...

4.3 Integers

The first number to be discoverd were Natural Numbers (counting numbers) ie. 1, 2, 3, 4 .... Then we included zero (0) to the set (collection) of natural numbers, we got new set of numbers known as whole numbers i.e 0, 1, 2, 3, 4.......... .Now we found that there are negative numbers too. If we include negative numbers to the set of whole numbers we get a new set of numbers called Integers as 0, ±1, ±2, ±3, ±4, ±5, ...

(........... –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5..........)

1, 2, 3, 4, 5..............

are called positive integers.

–1, –2, –3, –4, –5 ....

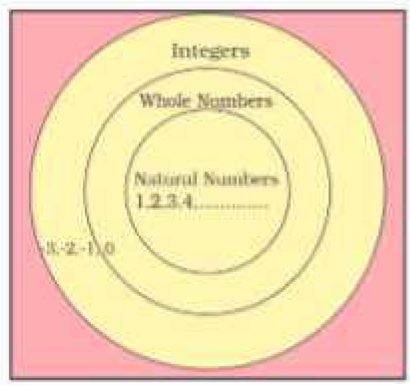
are called Negative integers.

0 (zero) is neither positive nor negative

N : {1, 2. 3, 4.........}

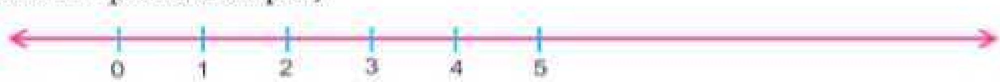
W : {0, 1, 2, 3, 4..........}

Z or I : {.........–4, –3, –2, –1, 0. 1, 2, 3, 4............}

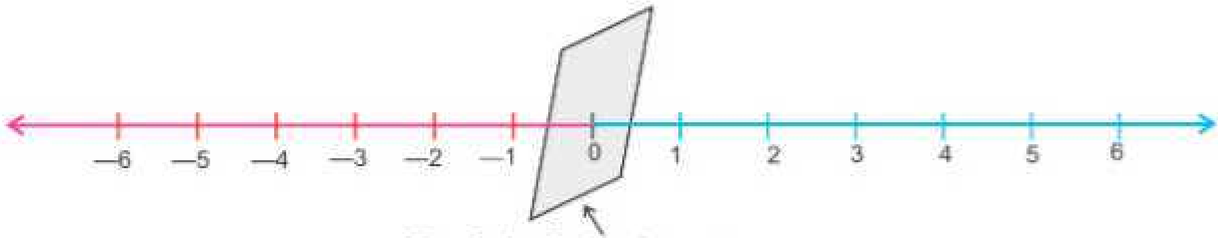


4.4 Representation of Integers on Number Line

Consider the part of a number line with whole numbers 0, 1. 2........ marked on it (we have already done it in previous chapter)



If we place mirror at 0 (Zero) facing towards the numbers 1, 2, 3. .... we get an image of the part of the line extending in opposite direction in the mirror. This part of the line are points which are images of 1, 2, 3 ...., Draw these images and mark the images of these points 1, 2, 3 .... as –1, –2, –3 .... respectively.



We got a line extending indefinitely in both directions where zero (0) lies in the centre. While facing towards zero, the numbers on the right of zero are positive integers 1, 2, 3, 4 .... and to the left of zero are negative integers (which are all images of positive integers) –1, –2, –3, –4........ . Here –1 is the image of 1, –2 is the image of 2 and so on.

4.5 Ordering of Integers

Any number on the number line is greater than any other number appearing on its left, and any number on the number line is less than any other number appearing on its right.

Some Important Observations:

(a) Every integer has its successor as well as predecessor.

(b) Every positive integer is greater than 0 and every negative integer is less than 0.

(c) The greater integer between the two given integers is the lesser integer between the negative of these integers

e..g $15 > 13$ but $-15 < -13$

(d) A number farther from 0 on the right has larger value.

(e) A number farther from 0 on the left side has smaller value.

(f) Smallest positive Integer is 1.

But largest positive integer (or Just Integer) is not possible to write in.

(g) Largest Negative integer is '–1'

but smallest Negative integer (or just integer) is not possible to write in.

(h) 0 is neither positive integer nor negative integer.

(i) Every positive integer is greater than every negative integer.

(j) 0 is greater than all negative integers.

Example 1: **Write the opposite of the following**

(a) 300 feet above sea level.

(b) Withdrawal of ₹500 from Bank Account.

Solution : (a) 300 feet below sea level.

(b) Deposit of ₹500 in Bank Account.

Example 2 : **Represent the following situations in integers.**

(a) Height of Mount Everest is 8848 m above sea level.

(b) A submarine is at a depth of 600 m below sea level.

(c) **A loss of ₹200**

(d) **Share market gained 200 points today.**

Solution : (a) +8848 (Here '+' represents above the sea level in metres)

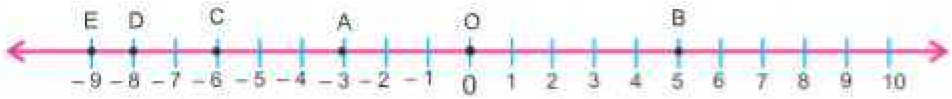
(b) –600 (Here '–' represents below the sea level in metres)

(c) –200 (Here '–' is loss in Rupees)

(d) +200 ('+' is points gained)

Example 3: **Represent the following numbers on number line : –3, +5, –6, 0, –8, –9**

Solution :



Point O represents zero, Point A represents –3, Point B represents +5, Point C reprsents –6, Point D represents –8, Point E represents –9.

Example 4: **Given figure is vertical number line.**

representing integers in which O represent zero. Answer the following.

(a) **If point D is –6, then which point is +6.**

(b) **Is A negative or a positive integer**

(c) **Write integers from B to E**

(d) **Write point on the number line having least value.**

(e) **Which number is represented by C.**

Solution : Let us write integers on this vertical number line taking O as origin.

We shall write positive integers above '0' (zero) and negative integers below '0' (zero) in sequence.

(a) Given that D is –6 and hence A is +6.

(b) A is positive integer.

(c) Integers from B to E are :4, 3, 2, 1, 0, –1,–2

(d) Point on the number line having least value is D.

(e) C represents +2.

Example 5. **In each of the following pairs, which number is to the right of the other on the number line.**

(a) **–2, 5** (b) **–7, –5**

Solution : (a) 5 lies to the right of – 2.

(b) –5 lies to the right of –7

Example 6: **Which of the following lies to the left of the other on number line ?**

(a) –10, –20 **(b) 7, –6**

Solution : (a) –20 lies to the left of –10

(b) –6 lies to the left of 7

Example 7: **Write all the integer between the given pairs.**

(a) –30 and –20 **(b) –8 and –15**

Between mean excluded "end points".

Solution: (a) The integers lying between –30 and –20 are –29, –28, –27, –26, –25, –24, –23, –22, –21

(b) The integers lying between –8 and –15 are –14, –13, –12, –11, –10, –9

Example 8: **Write four negative integers greater than –9.**

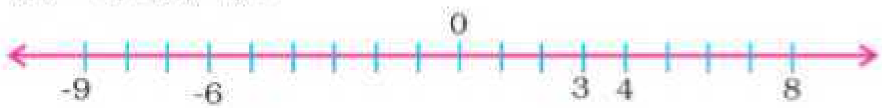
Solution : Greater integers lies on the right side on number line.

Integers on the right of (–9) on number line are : –8, –7, –6, –5.

Example 9 : **Arrange the following integers in ascending order.**

(a) –9, 3, 4, –6, 8

Solution :



Given integers in ascending order are ::

–9, –6, 3, 4, 8

Exercise 4.1

1. Write two examples from day to day life in which we can use positive and negative integers.
2. Write the opposite of the following:
   (a) A profit of ₹ 500 (b) A withdrawal of ₹70 Rs from bank account.
   (c) A deposit of ₹1000 (d) 326 B.C
   (e) 500m below Sea level (f) 25° above 0°C
3. Represent the situations mentioned in Q2 in integers.
4. Represent the following situations in Integers.
   (a) A deposit of ₹ 500.
   (b) An Aeroplane is flying at a height two thousand metre above the sea level.
   (c) A withdrawal of ₹700 from Bank Account.
   (d) A diver dives to a depth of 6 feet below ground level.

5. Represent the following numbers on number line.

(a) –5 (b) +6 (c) 0 (d) +1

(e) –9 (f) –4 (g) +8 (h) +3

6. Integers are represented on a horizontal number line as shown where A represents –2. With reference to the number line, answer the following questions :

(a) Which point represent –3?

(b) Locate the point which represents the opposite of B and name it P.

(c) Write integers for the points C and E.

(d) Which point marked on the number line has the least value?

7. In each of the following pairs, which number is to the right of other on the number line?

(a) 2, 9 (b) –3, –8 (c) 0, –5

(d) –11, 10 (e) –9, 9 (f) 2, –200

8. Write all the integers between the given pairs (write them in increasing order)

(a) 0 and –6 (b) –6 and +6 (c) –9 and –17

(d) –19 and –5

9. (a) Write five negative integers greater than '–15'.

(b) Write five integers smaller than '–20'.

(c) Write five integers greater than 0.

(d) Write five integers smaller than 0.

10. Encircle the greater integer in each given pair.

(a) –5, –7 (b) 0, –3 (c) 5, 7

(d) –9, 0 (e) –9, –11 (f) –4, 4

(g) –10, –100 (h) 10, 100

11. Arrange the following integers in ascending order:

(a) 0, –7, –9, 5, –3, 2, –4

(b) 8, –3, 7, 0, –9, –6

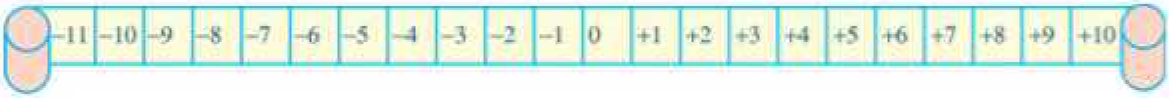
12. Arrange the following integers in descending order:

(a) –9, 3, 4, –6, 8, –3

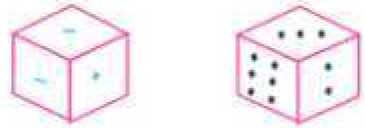
(b) 4, 8, –3, –2, 5, 0

4.6 Understanding Integers with a game

Make a number strip marked with integers –30 to +30.



Take two dice, one marked 1 to 6 and other marked with three '+' signs and three '–' signs.



Two players can play the game at a time. Players will keep different coloured buttons at the zero position on number strip.

Let player A starts the game. He throws both dice simultaneously. If on one dice there appear '+' sign and on another dice there appears 3 then it means the outcome is +3. Player A picks his button and places it on '+3'.

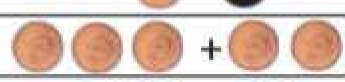
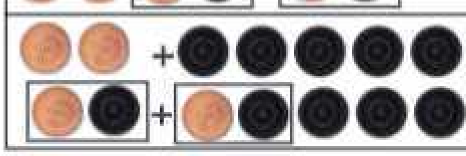
Now it is turn of B. He throws both dice simultaneously. On one die he gets '–' sign and on other dice he gets '4'. It means he has got '–4'. Player B picks his button and places it at –4.

Now it is turn of A. He throws both dice and gets '–5'. He has to move 5 steps to the left of his present position '+3'. Thus he reaches '–2' and places his button there (+3 – 5 = –2). The game continues this way. And player who reaches –30 will be considered out. And the player who reaches +30 first wins the game.

4.7 Addition of Integers (Understanding with an Activity)

Take carrom coins (Black and White) to perform this activity.

Let us assume that each white carrom coin represent +1 and each black carrom coin represent –1.

Carrom coins	Integer Represented
	= 2
	= –2
	= 0
	= (+3) + (+2) = +5
	= (–2) + (–1) = –3
	(+4) + (–2) = (+2) + 0 + = +2
	= (+2) + (–5) = 0 + 0 + (–3) = –3

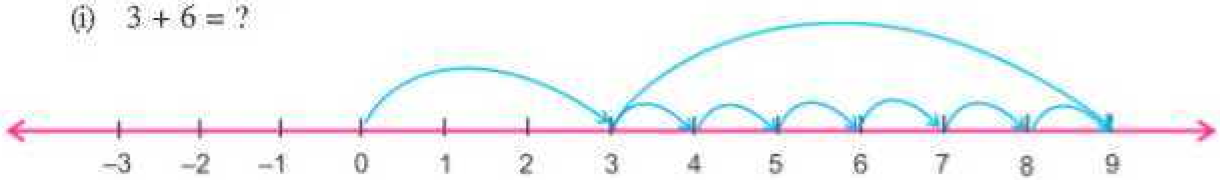
Observation:

1. We do addition when we have two positive integers like (+3) + (+2) = +5.
2. We do additon when we have two negative integers but the answer takes the negative sign (–) [minus sign] like (–2) + (–1) = –3.
3. When we have one positive and one negative integer, we must subtract, but answer will take the sign of largest integer (Ignoring the sign of integer, decide which is bigger) like (+2) +(–5) = –3.

4.7.1 Addition of Integers using Number line

It is not always easy to add integers using carrom coins. Let us try to perform these operations on number line of integers.

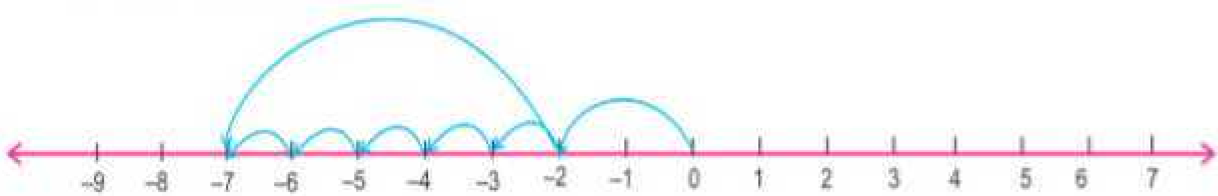
(i) 3 + 6 = ?



On number line we first move 3 steps to the right of zero (suggested by '+' sign) them move ahead 6 steps to the right of 3. We finally reach at 9. Thus '+9' is the final answer.

⇒ 3 + 6 = 9

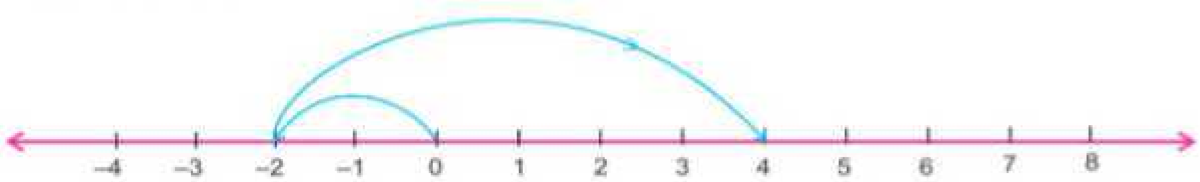
(ii) (–2) + (–5) = ?



On number line we first move 2 steps to the left of zero (left direction is suggested by minus sign then we move ahead 5 steps to the left of '–2' We finally reach at –7. Thus '–7' is the answer.

⇒ (–2) + (–5) = –7

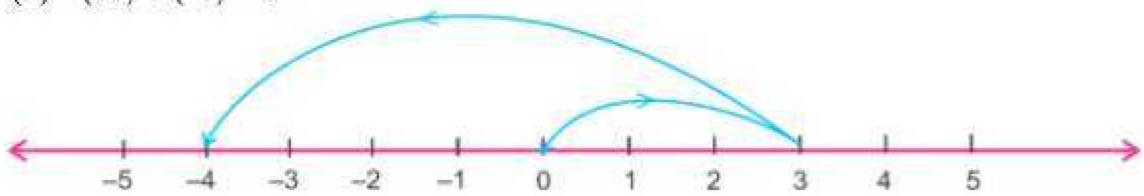
(iii) (–2) + (6) = ?



On number line we first move 2 steps to the left of zero (suggested by minus sign), then we move six steps to the right of '–2' (Direction suggested by plus sign). We finally reach at +4. Thus '+4' is the answer.

⇒ (–2) + (6) = +4

(iv) $(+3) + (-7) = ?$



On number line we first move 3 steps to the right of zero (Right direction suggested by plus sign) and reaches at +3. Then we move 7 steps to the left of '+3' (left direction suggested by minus sign) and reaches at –4. Thus answer is –4.

$\Rightarrow \quad (+3) + (-7) = -4$

Example 10. **Using number line write the integer which is**

(a) 6 less than 2 **(b) 3 less than –2**

Solution : (a) 6 less than 2 = ?

We need to find the integer which is 6 less than 2. So we shall start with '+2' and procced 6 steps to the left of '+2' as shown below.

Therefore 6 less than 2 is '–4'

(b) 3 less than –2 = ?

We need to find the integer which is 3 less than –2. So we shall start with –2 and proceed 3 steps to the left of –2 as shown below.

Therefore 3 less than –2 is '–5'.

Example 11. **Using Number line add the following integer.**

(a) $9 + (-6)$ **(b) $(-5) + 10$**

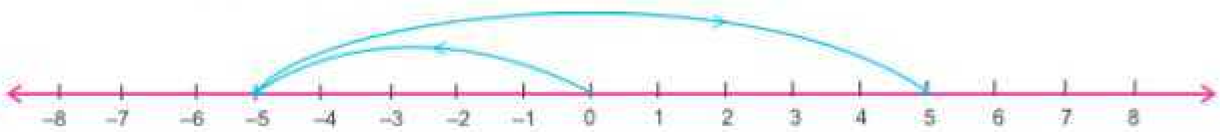
(c) $(-2) + 5 + (-3)$

Solution : (a) $9 + (-6)$

On number line we shall start from 0 and move 9 steps to the right of zero. Then we shall move six steps to the left of '+9' . We finally reach at +3. Thus +3 is the answer.

Hence $9 + (-6) = +3$

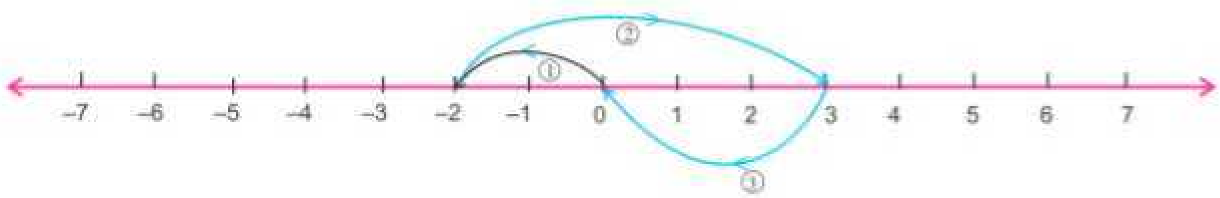
(b) (–5) + 10 = ?



On number line we shall move 5 steps to the left of zero (left side suggested by minus sign). Then we shall move 10 steps to the right of '–5' and finally reach at '+5'. Thus '+5' is the answer.

Hence (–5) + 10 = +5

(c) (–2) + 5 + (–3)



Step I : On number line we shall move 2 steps to the left of zero suggested by minus sign of '–2'.

Step II: The we shall move 5 steps to the right of '–2' suggested by plus sign of '+5' and we reach of '+3'.

Sign III : Then we shall move 3 steps to the left of '+3' as suggested by minus sign of '–3' and finally we reach at zero.

Hence answer is zero. Hence (–2) + 5 + (–3) = 0

Example 12. **Add without using number line.**

(a) 19 + (–13) **(b) 19 + 13**

(c) (–19) + (–13) **(d) (–19) + 13**

(e) 21 + (–13) + 8 + 7 + (–19) + (–11) + 2

Solution :

(a) 19 + (–13)
= + (19 – 13)
= +6

When we have one positive and one negative integer. We subtract them, but the answer takes the sign of greaterinteger (ignoring the sign of integer, decide which isgreater)

(b) 19 + 13
= 32

We simply add when we have two positive integers.

(c) (–19) + (–13)
= – (19+13)
= –32

We add when we have two negative integers. But the answer takes the minus sign.

(d) $(-19) + 13$

$= -(19 - 13)$

$= -6$

When we have one positive and one negative integer, we subtract them, But the answer takes the sign of greater integer (Ignoring the sign of integer, decide which is greater)

(e) $21 + (-13) + 8 + 7 + (-19) + (-11) + 2$

We arrange the numbers so that, the positive and negative integers are grouped together. We have

$21 + 8 + 7 + 2 + (-13) + (-19) + (-11)$

$= 38 + (-43)$

$= 38 - 43$

$= -5$

	+	–
	21	13
	8	19
	7	11
	2	
Add	+38	–43

Pick the sign. of larger number.

Example 13. **Write the successor and predecessor of the following:**

(a) –69 **(b) 59**

Solution : (a) Successor of –69 = –69 + 1 = –68

Predecessor of –69 = –69 – 1 = –70

(b) Successor of 59 = 59 + 1 = 60

Predecessor of 59 = 59 – 1 = 58

Exercise 4.2

1. Using number line write the integer which is
   - (a) 5 less than –1 (b) 5 more than –5
   - (c) 2 less than 5 (d) 3 less than –2
2. Using number line, add the following integers:
   - (a) $9 + (-3)$ (b) $5 + (-11)$
   - (b) $(-1) + (-4)$ (d) $(-5) + 12$
   - (e) $(-1) + (-2) + (-4)$ (f) $(-2) + 4 + (-5)$
   - (g) $(-3) + (5) + (-4)$
3. Add without using number line:
   - (a) $18 + 13$ (b) $18 + (-13)$
   - (c) $(-18) + 13$ (d) $(-18) + (-13)$
   - (e) $180 + (-200)$ (f) $777 + (-67)$
   - (g) $1262 + (-366) + (-962)$ (h) $30 + (-27) + 21 + (-19) + (-3) + (11) + (-9)$
   - (i) $(-7) + (-9) + 4 + 16$ (j) $37 + (-2) + (-65) + (-8)$

4. Write the successor and predecessor of the following:

(a) –15 (b) 27 (c) –79 (d) 0

(e) 29 (f) –18 (g) –21 (h) 99

(i) –1 (j) –13

5. Complete the following addition table:

+	–3	–4	–2	+1	+2	+3
–2						
–3						
0						
+1						
+2						

4.8. Additive Inverse

Two integers which when added to each other give the sum zero, are called additive inverse of each other.

e.g.: $(-3) + (3) = 0$

Here (-3) is additive inverse of 3

and (3) is additive inverse of -3

4.9 Subtraction of Integers:

Subtraction is an operation which is just the reverse of addition. We use the following rule for subtraction:

If a and b are two integers, to subtract b from a, we change the sign of b and add it in a .

e.g. : $a - b = a + (-b)$

Example 14. Using number line find the value of

(a) $-4 - (-2)$ (b) $7 - (-2)$

Solution : (a) $-4 - (-2)$

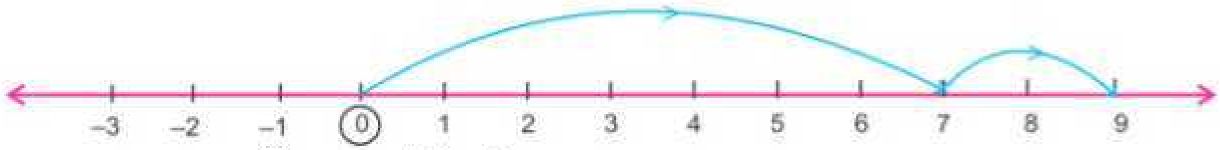
$= -4 + (\text{additive inverse of } -2) = -4 + (2)$

Hence $-4 - (-2) = -2$

(b) $7 - (-2)$

$= 7 + (\text{additive inverse of } -2)$

$= 7 + (2)$



Hence $7 - (-2) = 9$

Example 15. **Subtract the following:**

(a) 27 from 42 **(b) –13 from 91**

(c) 16 from –84 **(d) –61 from –41**

Solution : (a) $42 - (+27) = 42 + (-27)$
$= 42 - 27$
$= 15$

(b) $91 - (-13) = 91 + (13)$
$= 91 + 13$
$= 104$

(c) $-84 - (16) = -84 + (-16)$
$= -84 - 16$
$= -100$

(d) $-41 - (-61) = -41 + (61)$
$= -41 + 61$
$= +20$

Example 16. **Solve.**

(a) (–13) + 32 –8 –1

(b) 19 – (–45) – (–3)

Solution : (a) $(-13) + 32 - 8 - 1$
$= -13 + 32 - 8 - 1$
$= 32 - 13 - 8 - 1$
$= 32 - 22$
$= 10$

(b) $19 - (-45) - (-3)$
$= 19 + (45) + (3)$
$= 19 + 45 + 3$
$= 67$

Exercise 4.3

1. Fill the suitable integer in box :

(a) $2 + \square = 0$ (b) $\square + 11 = 0$

(c) $-5 + \square = 0$ (d) $\square + (-9) = 0$

(e) $3 + \square = 0$ (f) $\square + 0 = 0$

2. Subtract using number line:

(a) 5 from –7 (b) –3 from –6

(c) –2 from 8 (d) 3 from 9

3. Subtract without using number line:

(a) –6 from 16 (b) –51 from 55

(c) 75 from –10 (d) –31 from –47

4. Find:

(a) 35 – (20) (b) (–20) – (13)

(c) (–15) – (–18) (d) 72 – (90)

(e) 23 – (–12) (f) (–32) – (–40)

5. Simplify:

(a) 2 – 4 + 6 – 8 – 10 (b) 4 – 2 + 2 – 4 – 2 + 2

(c) 4 – (–9) + 7 – (–3) (d) (–7) + (–19) + (–7)

Multiple Choice Questions

1. How many integers are between –3 to 3?

(a) 5 (b) 6 (c) 4 (d) 3

2. Which of the following integer is greater than –3?

(a) –5 (b) –4 (c) 0 (d) –10

3. Which of the following integers are in ascending order?

(a) –5, –9, –7, –8 (b) –9, –8, –7, –5

(c) –5, –7, –8, –9 (d) –8, –5, –9, –7

4. Which of the following integers are in descending order ?

(a) 3, 0 , –2, –5 (b) –5, –2, 0, 3

(c) –5, 3, –2, 0 (d) –2, 0, –5, 3

5. The given number line represents:

(a) 5+1 (b) 1+5 (c) 1+1+1+1+1 (d) 5+5+5+5+5

6. 3 less than –2 =

(a) –5 (b) –6 (c) 5 (d) 6

7. (–2) + 8 =

(a) –6 (b) –10 (c) 10 (d) 6

8. Which of the following statements is true about the given number line.

(a) Value of A is greater than value of B.
(b) Value of A is greater than value of C.
(c) Value of B is less than value of C.
(d) Value of C is less than value of B.

9. (–7) + (–12) + 11 =
(a) –19 (b) 30 (c) –23 (d) –8

10. 15 – (–12) + (–27) =
(a) 0 (b) –54 (c) –24 (d) 54

Learning Outcomes

After completion of this chapter, the students are now able to

- Understand the extended number system from natural number to integers
- Represent integers on the number line and operate on number line
- Identify greater or smaller integers out of a given set of integers.
- Solve problems involving addition and subtraction of integers

ANSWER KEY

Exercise - 4.1

2. (a) A loss of ₹500 (b) deposit of ₹70 in bank account (c) withdrawal of ₹1000
(d) 326 AD (e) 500m above sea level (f) 25° below 0°C

3. (a) +500 (b) –70 (c) +1000
(d) –326 (e) –500m (f) +25

4. (a) +500 (b) +2000 (c) –700 (d) –6

6. (a) B (b) +3 (c) C = –7, E = +4 (d) C

7. (a) 9 (b) –3 (c) 0
(d) 10 (e) 9 (f) 2

8. (a) –5, –4, –3, –2, –1
(b) –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5
(c) –16, –15, –14, –13, –12, –11, –10
(d) –18, –17, –16, –15, –14, –13, –12, –11, –10, –9, –8, –7,–6

10. (a) –5 (b) 0 (c) 7 (d) 0
(e) –9 (f) 4 (g) –10 (h) 100

11. (a) –9, –7, –4, –3, 0, 2,5 (b) –9, –6, –3, 0, 7, 8

12. (a) 8, 4, 3, –3, –6, –9 (b) 8, 5, 4, 0, –2,–3

Exercise - 4.2

1. (a) –6 (b) 0 (c) 3 (d) –5
2. (a) 6 (b) –6 (c) –5 (d) 7 (e) –7 (f)–3 (g) –2
3. (a) 31 (b) 5 (c) –5 (d) –31 (e) –20 (f) 710
(g) –66 (h) 4 (i) 4 (j) –38
4. (a) –14 and –16 (b) 28 and 26 (c) –78 and –80
(d) 1 and –1 (e) 30 and 28 (f) –17 and –19
(g) –20 and –22 (h) 100 and 98 (i) 0 and –2
(j) –12 and –14
5.

+	–3	–4	–2	+1	+2	+3
–2	–5	–6	–4	–1	0	+1
–3	–6	–7	–5	–2	–1	0
0	–3	–4	–2	+1	+2	+3
+1	–2	–3	–1	+2	+3	+4
+2	–1	–2	0	+3	+4	+5

Exercise - 4.3

1. (a) –2 (b) –11 (c) 5 (d) 9 (e) –3 (f) 0
2. (a) –12 (b) –3 (c) 10 (d) 6
3. (a) 22 (b) 106 (c) –85 (d) –16
4. (a) 15 (b) –33 (c) 3 (d) –18 (e) 35 (f) 8
5. (a) –14 (b) 0 (c) 23 (d) –33

Multiple Choice Questions

(1) a (2) c (3) b (4) a (5) c
(6) a (7) d (8) c (9) d (10) a

V6Q8Q9

5

FRACTIONS

Objectives

In this chapter you will learn

- To understand about different types of fractions.
- To use fraction in practical life.
- To use fraction in different units i.e. money, length and temparature

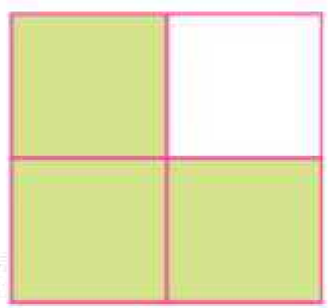
5.1 Introduction

In previous classes, we have studied that if an object is divided in equal parts then one or more parts of the object is called fraction. In our daily life, we perform many activities of a fraction like a mother is preparing breakfast for her kids and the first kid demands for half chapatti and the second kid demands for one third chapati. Their demand of half chapati and one third chapatti represents fraction of whole chapati. A fraction means a part of whole or a group.

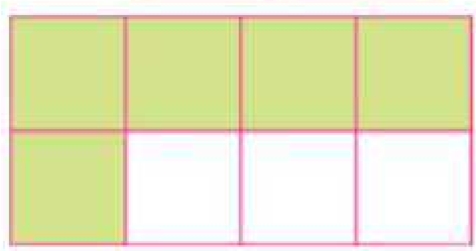
$$\text{Fraction} = \frac{\text{Parts of an object}}{\text{Total part of an object}}$$

The upper part of a fraction is called **numerator** and the lower part is called **denominator.**

Look at the following figures:



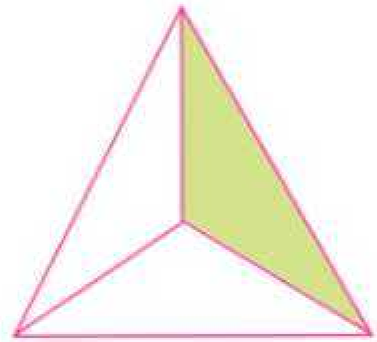
(i) A square has been divided into four equal parts. 3 parts out of 4 have been shaded i.e. three-fourth has been shaded. Mathematically, we say $\frac{3}{4}$ portion has been shaded. i.e. $\frac{3}{4} = \frac{\text{Shaded Parts}}{\text{Total number of Parts}} = \frac{\text{Numerator}}{\text{Denominator}}$



(ii) A rectangle has been divided into eight equal parts. 5 parts out of 8 have been shaded i.e. $\frac{5}{8}$ portion of rectangular sheet has been shaded and $\frac{3}{8}$ remains unshaded.

$$\frac{5}{8} = \frac{\text{Shaded Parts}}{\text{Total number of Parts}} = \frac{\text{Numerator}}{\text{Denominator}}$$

and $$\frac{3}{8} = \frac{\text{Unshaded Parts}}{\text{Total number of Parts}} = \frac{\text{Numerator}}{\text{Denominator}}$$



(iii) A triangle has been divided into three equal parts.

one part out of 3 has been shaded i.e. $\frac{1}{3}$ part has been shaded.

i.e. $$\frac{1}{3} = \frac{\text{Shaded Parts}}{\text{Total number of Parts}} = \frac{\text{Numerator}}{\text{Denominator}}$$

With the above discussion, we arrive at the definition of fractions.

"A fraction is a ratio representing part(s) of the whole".

* A fraction is of the form $\frac{a}{b}$, where a and b are whole numbers and $b \neq 0$, In $\frac{a}{b}$, a is called the numerator and b is called the denominator."

Consider the fraction $\frac{3}{5}$. This fraction is read as **"three - fifth"** which means that 3 parts out of 5 equal parts. In the fraction $\frac{3}{5}$, 3 is called the **numerator** and 5 is called the **denominator**.

Following are some more fractions:

Fraction	Meaning of the fraction	Numerator	Denominator
$\frac{2}{7}$ or Two-Seventh	Two equal parts out of seven equal parts	2	7
$\frac{3}{4}$ or Three-Fourth	Three equal parts out of four equal parts	3	4
$\frac{5}{11}$ or Five-Eleventh	Five equal parts out of eleven equal parts	5	11

Example 1: **Write the fraction for each of the following:-**

(i) Half (ii) Two-Fifth (iii) Five-Seventh

Solution : (i) Half = one out of two = $\frac{1}{2}$

(ii) Two-Fifth = Two out of five = $\frac{2}{5}$

(iii) Five-Seventh = Five out of Seven = $\frac{5}{7}$

Example 2 : **Write the numerator and the denominator for the followings:-**

(i) $\frac{7}{10}$ (ii) $\frac{3}{5}$ (iii) $\frac{9}{13}$

Solution : (i) Given fraction is $\frac{7}{10} = \frac{\text{Numerator}}{\text{Denominator}}$

$\therefore$ Numerator = 7 and Denominator = 10

(ii) Given fraction is $\frac{3}{5} = \frac{\text{Numerator}}{\text{Denominator}}$

$\therefore$ Numerator = 3 and Denominator = 5

(iii) Given fraction is $\frac{9}{13} = \frac{\text{Numerator}}{\text{Denominator}}$

$\therefore$ Numerator = 9 and Denominator = 13

Example 3: **(i) What fraction of a year is 4 months?**

(ii) What fraction of a day is 10 hours?

(iii) What fraction of a week is 2 days?

Solution : (i) We know 1 year = 12 months

$\therefore$ Required fraction = $\frac{4}{12}$

(ii) We know 1 day = 24 hours

$\therefore$ Required fraction = $\frac{10}{24}$

(iii) We know 1 week = 7 days

$\therefore$ Required fraction = $\frac{2}{7}$

Example 4: **Write the natural numbers from 1 to 15. What fraction of them are prime numbers?**

Solution : Natural numbers from 1 to 15 are:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. 12, 13, 14, 15 i.e. 15 in number

Prime numbers out of these numbers are:

2, 3, 5, 7. 11, 13, i.e. 6 in number.

$\therefore$ Required fraction = $\frac{6}{15}$

Example 5 : A bag contains 8 balls out of which 3 are blue and 5 are white. What fraction of ball represents blue and white?

Solution : Here, 3 out of 8 balls are blue

Fraction which represents blue balls = $\frac{3}{8}$

Now, 5 out of 8 balls are white

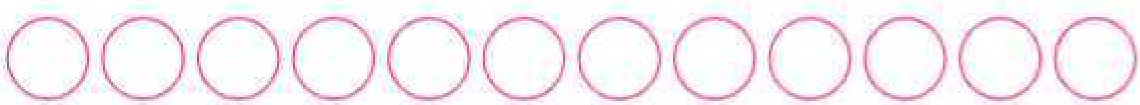
Fraction which represents while balls = $\frac{5}{8}$

5.2 Fraction and Division (Fraction as a part of collection)

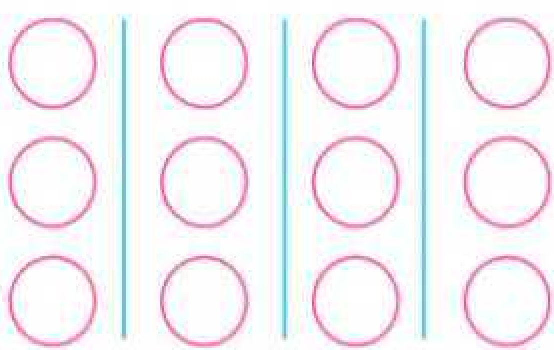
A fraction represents parts of a collection, the numerator being the number of parts we have and the denominator being the total number of parts in the collection.

For Example :

Let us take a collection of 12 balls and we want to get $\frac{1}{4}$ of the collection.

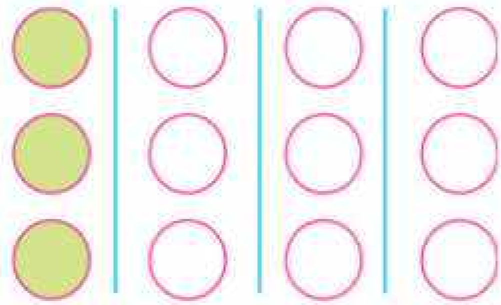


Step 1. In order to find $\frac{1}{4}$ out of the 12 balls, we divide the 12 balls into four equal groups/parts.



Each group/part contains 3 balls

Step 2. Now, we shade 1 group/part out of 4 parts.



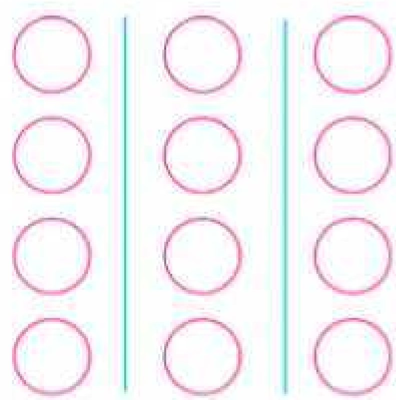
On counting, we find that the total number of shaded balls is 3.

In other words, $\frac{1}{4}$ of 12 balls = 3 balls

i.e. $\frac{1}{4} \times 12 = \frac{1 \times 12}{4} = \frac{12}{4} = 12 \div 4 = 3$ balls

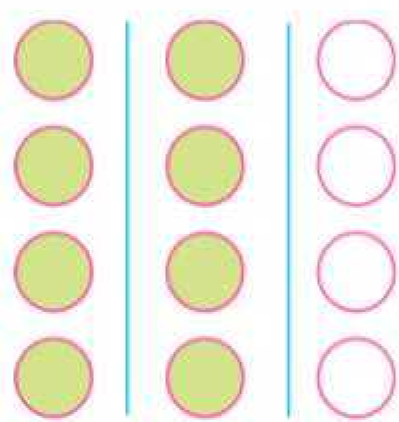
For Example : Find $\frac{2}{3}$ of 12 balls

Step 1. In order to find $\frac{2}{3}$ out of 12 balls, we divide the 12 balls into 3 equal groups/parts.



Each group/part contains 4 balls

Step 2. Now, we shade 2 groups/parts out of 3 parts.



On counting, we find that the total number of shaded balls are 8.

In other words, $\frac{2}{3}$ of 12 balls = 8 balls

i. e. $\frac{2}{3} \times 12 = \frac{2 \times 12}{3} = \frac{24}{3} = 24 \div 3 = 8$ balls

Example 6:- **(i) What is $\frac{1}{4}$ of 16?** **(ii) What is $\frac{2}{5}$ of 20?**

(iii) What is $\frac{3}{4}$ of 24?

Solution : (i) $\frac{1}{4}$ of $16 = \frac{1}{4} \times 16 = \frac{1 \times 16}{4} = \frac{16}{4} = 16 \div 4 = 4$

(ii) $\frac{2}{5}$ of $20 = \frac{2}{5} \times 20 = \frac{2 \times 20}{5} = \frac{40}{5} = 40 \div 5 = 8$

(iii) $\frac{3}{4}$ of $24 = \frac{3}{4} \times 24 = \frac{3 \times 24}{4} = \frac{72}{4} = 72 \div 4 = 18$

Example 7: **Jayant has 24 oranges. he ate $\frac{1}{6}$ of them, then**

(i) How many oranges he ate?

(ii) How many does he have left?

Solution : Total oranges = 24

(i) Oranges he ate = $\frac{1}{6}$ of $24 = \frac{1}{6} \times 24 = \frac{1 \times 24}{6} = \frac{24}{6} = 24 \div 6 = 4$ oranges

(ii) Number of oranges left out = 24 – 4 = 20 oranges

Example 8: **Jasmine has a packet of 20 biscuits. She gave $\frac{1}{4}$ of them to Harneet and $\frac{3}{5}$ of them to Sophia. The rest she keeps.**

(i) How many biscuits does Harneet have?

(ii) How many biscuits does Sophia have?

(iii) How many biscuits does Jasmine keep?

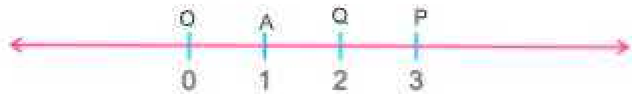
Solution : (i) Harneet have biscuits = $\frac{1}{4}$ of $20 = \frac{1}{4} \times 20 = \frac{1 \times 20}{4} = \frac{20}{4} = 20 \div 4 = 5$ biscuits

(ii) Sophia have biscuits = $\frac{3}{5}$ of $20 = \frac{3}{5} \times 20 = \frac{3 \times 20}{5} = \frac{60}{5} = 60 \div 5 = 12$ biscuits

(iii) Jasmine keep biscuits = 20 – 5 – 12 = 3 biscuits

5.3 Fractions on Number Line

In previous chapter, we have learnt about the representation of whole numbers on a number line. To represent whole numbers on a number line, we draw a straight line and mark point 0 on it. Now starting from 0, mark points, A,Q,P etc, on the line to the right of 0 at equal distance. These points represent 1, 2, 3 etc and O represents 0.

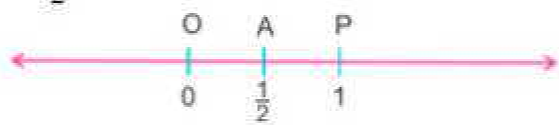


We can represent fractions on number line.

- **In order to represent $\frac{1}{2}$ on the number line:-** Draw the number line and mark a point P to represent 1.

  Now divide the gap between O and P into two equal parts. Let A be the point of division.

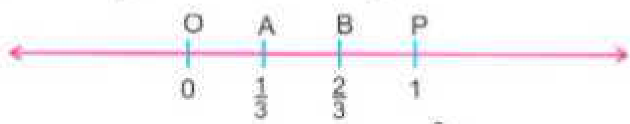
  then A represents $\frac{1}{2}$.



- **To represent $\frac{1}{3}$ on the number line:-** Draw the number line and mark a point P to represent 1.

  Now divide the gap between O and P into three equal parts. Let A and B be the points of division.

  Then A represents $\frac{1}{3}$ and B represents $\frac{2}{3}$.



* By using the same logic, point O represents $\frac{0}{3}$ and point P represents $\frac{3}{3}$

Thus we have $\frac{0}{3} = 0$ and $\frac{3}{3} = 1$

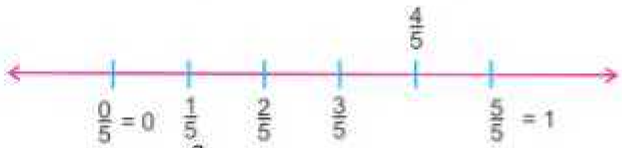
There are infinite fractions between 0 and 1.

Example 9: (i) **Represent $\frac{4}{5}$ on a number line.**

(ii) **Represent $\frac{3}{7}$ on a number line.**

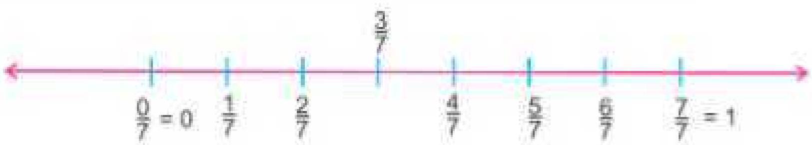
Solution : (i) In order to represent $\frac{4}{5}$ on a number line. We divide the gap between 0 and 1 into 5 equal parts, which are $\frac{0}{5}=0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$ and $\frac{5}{5}=1$ (as shown)

Then we mark fourth point from 0 to the right (as shown)



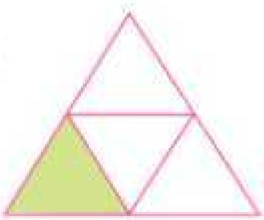
(ii) In order to represent $\frac{3}{7}$ on a number line we divide the gap between 0 and 1 into 7 equal parts, which are $\frac{0}{7}=0, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{7}{7}$ (as shown)

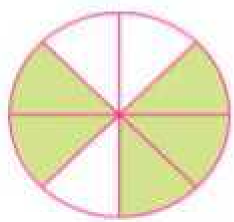
Then, we mark third point from 0 to the right (as shown)

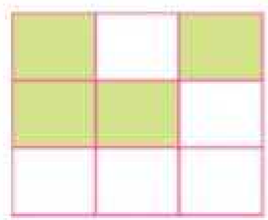


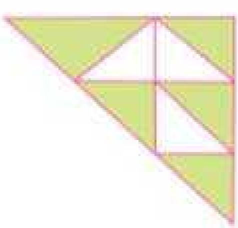
Exercise 5.1

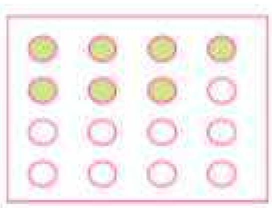
1. Write the fraction representing the shaded portion:-

(i)


(ii)


(iii)


(iv)


(v)


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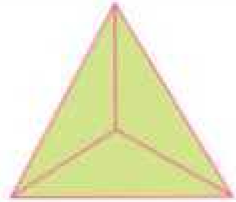
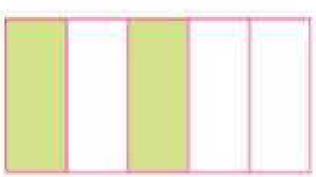
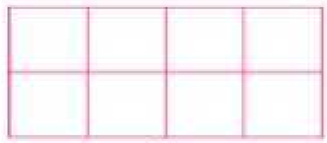


(vii)


(viii)

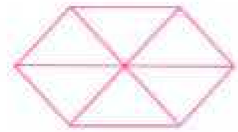

2. Colour the part according to the given fraction:-

(i) 

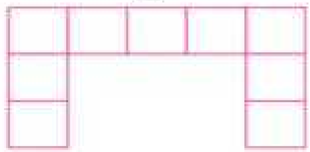
$\frac{5}{8}$

(ii) 

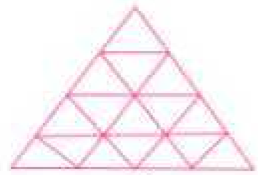
$\frac{3}{4}$

(iii) 

$\frac{2}{6}$

(iv) 

$\frac{7}{9}$

(v) 

$\frac{11}{16}$

3. Write the fraction for each of the folloiwng:-

(i) Three-Fourth (ii) Seven-Tenth (iii) A Quarter

(iv) Five-Eighth (v) Three-Twelvth

4. Write the fraction for the followings:-

(i) numerator = 5
denominator = 9

(ii) numerator = 2
denominator = 11

(iii) numerator = 6
denominator = 7

5. Write the numerator and the denominator for the followings:-

(i) $\frac{2}{3}$ (ii) $\frac{1}{4}$ (iii) $\frac{5}{11}$ (iv) $\frac{9}{13}$ (v) $\frac{17}{16}$

6. Express:-

(i) 1 day as a fraction of 1 week.

(ii) 40 seconds as a fraction of 1 minute.

(iii) 15 hours as a fraction of 1 day.

(iv) 2 months as a fraction of 1 year.

(v) 45cm as a fraction of 1 metre.

7. Write the numbers from 1 to 25.

(i) What fraction of them are even numbers?

(ii) What fraction of them are prime numbers?

(iii) What fraction of them are multiples of 3?

8. In class 6^{th}, there are 24 boys and 18 girls. What fraction of total students represent boys and girls.

9. A bag contians 6 red balls and 7 blue balls. What fraction of balls represent red and blue colour?

10. Sidharth has a cake. He cuts it into 10 equal parts. He gave 2 parts to Naman, 3 parts to Nidhi, 1 parts to Seema and the remaining four parts he kept for himself. Find

(i) What fraction of cake, he gave to Naman?

(ii) What fraction of cake, he gave to Nidhi?

(iii) What fraction of cake, he kept for himself?

(iv) Who has more cake than others?

11. In a box, there are 12 apples, 7 oranges and 5 guavas. What fraction of fruits in box represents each?

12. Dishmeet has 20 pens. He gives one-fourth to Balkirat. How many pens Dishmeet and Balkirat have?

13. Represent the following fraction on the number line?

(i) $\frac{2}{5}$ (ii) $\frac{5}{7}$ (iii) $\frac{3}{10}, \frac{5}{10}, \frac{1}{10}$ (iv) $\frac{3}{8}, \frac{5}{8}, \frac{7}{8}$

14. Find:-

(i) $\frac{3}{5}$ of 20 books (ii) $\frac{5}{8}$ of 32 pens (iii) $\frac{1}{6}$ of 36 Copies

(iv) $\frac{4}{7}$ of 21 apples (v) $\frac{3}{4}$ of 28 pencils

15. Balkirat had a box of 36 erasers. He gave $\frac{1}{2}$ of them to Rani, $\frac{2}{9}$ of them to Yuvraj and keeps the rest.

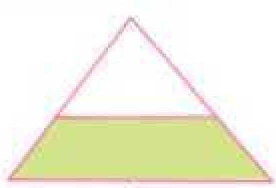
(i) How many erasers does Rani get?

(ii) How many erasers does Yuvraj get?

(iii) How many erasers does Harnik keep?

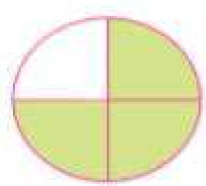
16. State True/False.

(i)



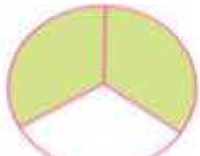
$\frac{1}{2}$

(ii)



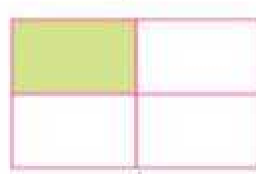
$\frac{3}{4}$

(iii)



$\frac{2}{3}$

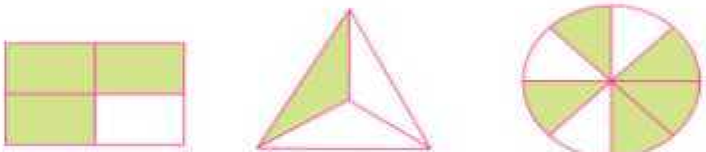
(iv)



$\frac{1}{4}$

5.4 Types of Fractions

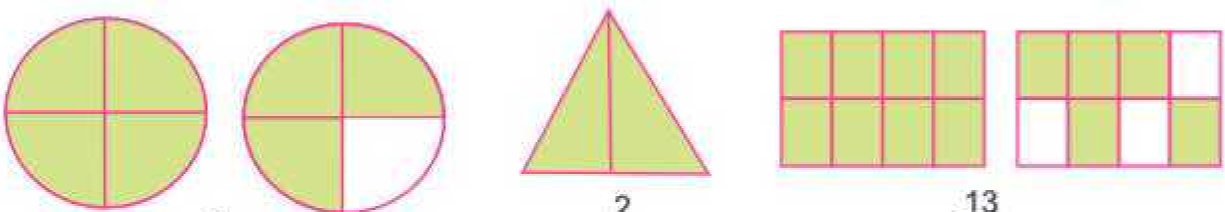
Proper Fraction:- Fractions whose numerator is less than the denominator are called the proper fractions.



$\frac{3}{4}$ $\frac{1}{3}$ $\frac{5}{8}$

These are proper fractions.

Improper Fraction:- Fractions whose numerator is either equal or greater than the denominator are called improper fractions.

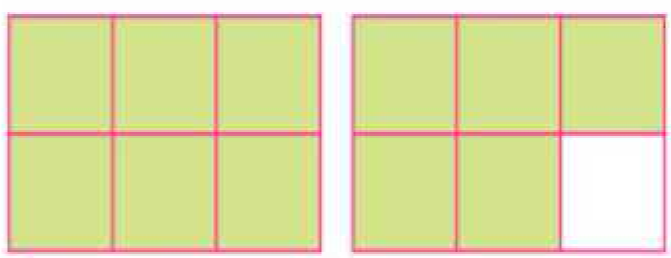


$\frac{7}{4}$ $\frac{2}{2}$ $\frac{13}{8}$

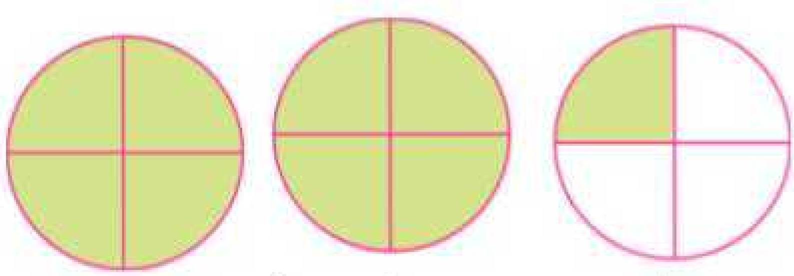
* **Improper fractions are converted to a mixed fraction.**

Mixed Fraction:- A fraction which is a combination of a whole number and a proper fraction is called a mixed fraction.

Following are mixed fractions:



This is $\frac{11}{6} = 1 + \frac{5}{6}$ is written as $1\frac{5}{6}$



This is $\frac{9}{4} = 2 + \frac{1}{4}$ and is written as $2\frac{1}{4}$

5.4.1 Conversion of Improper fractions into Mixed fractions

Step 1. Obtain the Improper fraction

Step 2. Divide the numerator by the denominator and obtain the quotient and the remainder

Step 3. Write the mixed fraction as:

$$\text{Quotient}\,\frac{\text{Remainder}}{\text{Denominator}} \quad \text{Or} \quad \text{Quotient}\,\frac{\text{Remainder}}{\text{Divisor}}$$

5.4.2 Conversion of Mixed fractions into Improper fractions

Step1. Obtain Mixed fraction which is in form of

$$\text{Quotient}\,\frac{\text{Remainder}}{\text{Divisor}} \text{ Or } (\text{Quotient}) + \frac{\text{Remainder}}{\text{Divisor}}$$

Step 2. Write the fraction having numerator equal to the (Quotient × Divisor + Remainder) and denominator same as divisor in step 1.

$$\text{Thus, Improper fraction} = \frac{\text{Quotient} \times \text{Denominator} + \text{Remainder}}{\text{Denominator}}$$

Example 10:- Classify the following as proper and Improper fractions:

$$\frac{2}{5}, \frac{7}{8}, \frac{11}{5}, \frac{6}{11}, \frac{9}{4}, \frac{5}{13}, \frac{6}{6}$$

Solution : Proper fractions : $\frac{2}{5}, \frac{7}{8}, \frac{6}{11}, \frac{5}{13}$

Improper fractions : $\frac{11}{5}, \frac{9}{4}, \frac{6}{6}$

Example 11:- Express each of the following as mixed fractions:-

(i) $\frac{16}{5}$ (ii) $\frac{19}{4}$ (iii) $\frac{28}{3}$

Solution : (i) $\frac{16}{5}$

Divisor 5) 16 (3 Quotient
− 15
1 Remainder

$$\therefore \text{Mixed fraction} = \text{Quotient}\,\frac{\text{Remainder}}{\text{Divisor}}$$

$$= 3\frac{1}{5}$$

Aliter :- $\frac{16}{5} = \frac{5+5+5+1}{5} = \frac{5}{5}+\frac{5}{5}+\frac{5}{5}+\frac{1}{5}$

$= 1+1+1+\frac{1}{5} = 3\frac{1}{5}$

(ii) $\frac{19}{4}$ Divisor 4) 19 (4 Quotient

-16

3 Remainder

$\therefore$ Mixed fraction $=$ Quotient $\frac{\text{Remainder}}{\text{Divisor}} = 4\frac{3}{4}$

(iii) $\frac{28}{3}$ Divisor 3) 28 (9 Quotient

-27

1 Remainder

$\therefore$ Mixed fraction $=$ Quotient $\frac{\text{Remainder}}{\text{Divisor}} = 9\frac{1}{3}$

Example 12: Express each of the following mixed fractions as improper fractions:-

(i) $5\frac{3}{4}$ (ii) $7\frac{2}{5}$ (iii) $8\frac{2}{7}$

Solution : (i) $5\frac{3}{4} = \frac{5\times4+3}{4} = \frac{20+3}{4} = \frac{23}{4}$

(ii) $7\frac{2}{5} = \frac{7\times5+2}{5} = \frac{35+2}{5} = \frac{37}{5}$

(iii) $8\frac{2}{7} = \frac{8\times7+2}{7} = \frac{56+2}{7} = \frac{58}{7}$

Exercise 5.2

1. Classify the following as proper and improper fractions:-

$\frac{5}{4}, \frac{9}{13}, \frac{6}{11}, \frac{3}{2}, \frac{5}{2}, \frac{6}{6}, \frac{7}{9}, \frac{2}{15}, \frac{4}{17}, \frac{7}{8}$

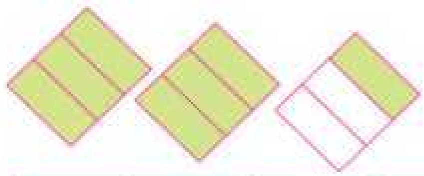
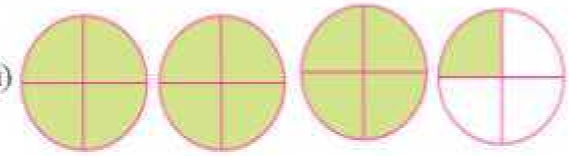
2. Express each of the following as mixed fractions, Also represent with diagrams.

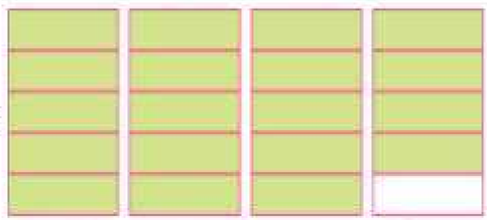
(i) $\frac{27}{5}$ (ii) $\frac{13}{4}$ (iii) $\frac{43}{8}$ (iv) $\frac{51}{7}$ (v) $\frac{20}{3}$

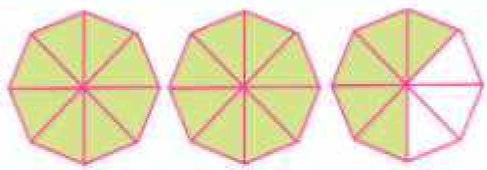
3. Express each of the following mixed fractions as improper fractions:-

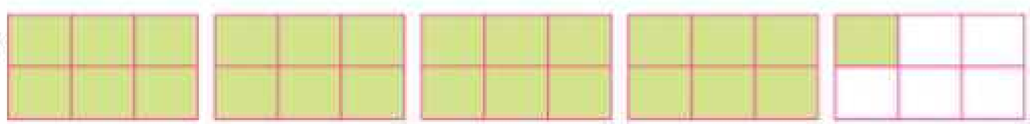
(i) $2\frac{1}{3}$ (ii) $5\frac{2}{7}$ (iii) $4\frac{3}{5}$ (iv) $3\frac{3}{4}$ (v) $9\frac{5}{8}$

4. Express the shaded portion as Improper fraction and Mixed fraction:-

(i)  (ii) 

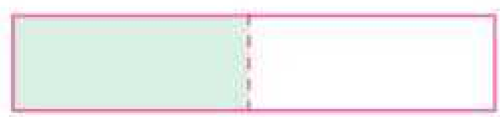
(iii) 

(iv) 

(v) 

5.5 Equivalent Fractions

- Fold the piece of paper into two equal parts. Unfold the paper Colour one of the parts.

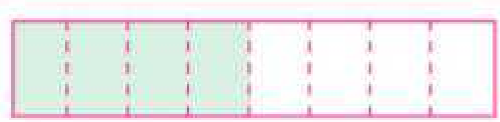


Each part represents $\frac{1}{2}$ of the paper.

- Now, fold it again and then unfold.

The coloured part $\frac{1}{2}$ now represents $\frac{2}{4}$ of the paper.

- Now, fold it again and then unfold.



The coloured part now represents $\frac{4}{8}$ of the paper.

We can observed from above that $\frac{1}{2}$, $\frac{2}{4}$ and $\frac{4}{8}$ all represent the same part of the paper.

Such fractions are called Equivalent fractions.

- **Equivalent fractions are the fractions which represent the same value.**

Look at the above activity, we have equivalent fractions, $\frac{1}{2}, \frac{2}{4}, \frac{4}{8}$

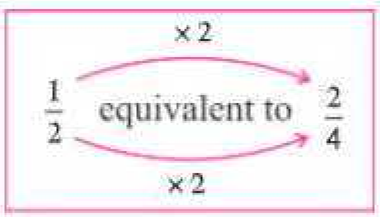
Numerator and denominator of $\frac{2}{4}$ are twice the numerator and denominator of $\frac{1}{2}$

i.e. $\frac{2}{4} = \frac{1 \times 2}{2 \times 2}$

Similarly numerator and denominator of $\frac{4}{8}$ are four times the numerator and denominator of $\frac{1}{2}$.

i.e. $\frac{4}{8} = \frac{1 \times 4}{2 \times 4}$

We observe that $\frac{2}{4}, \frac{4}{8}$ are obtained by multiplying the numerator and denominator of $\frac{1}{2}$ by 2 and 4 respectively.



Thus an equivalent fraction of a given fraction can be obtained by multiplying its numerator and denominator by the same number (other than zero),

Further $\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$

$\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$

We observe that if we divide the numerator and denominators of $\frac{2}{4}, \frac{4}{8}$ each by their common factors i.e. 2 and 4 respectively, we get an equivalent fraction $\frac{1}{2}$.

Thus an equivalent fraction of a given fraction can be obtained by dividing its numerator and denominator by their common factors (other than 1)

So in above discussion, we can find equivalent fractions of any given fraction by

* Mulitiplying its numerator and denominator by same number (other than 0)
* Dividing its numerator and denominator by their common factor (other than 1).

Example 13:- Find three equivalent fractions of the followings:-

(i) $\frac{2}{5}$ (ii) $\frac{3}{4}$ (iii) $\frac{7}{9}$

Solution : (i) Equivalent fractions of $\frac{2}{5}$ are

$$\frac{2\times2}{5\times2}=\frac{4}{10}\ ;\ \frac{2\times3}{5\times3}=\frac{6}{15}\ ;\ \frac{2\times4}{5\times4}=\frac{8}{20}$$

(ii) Equivalent fractions of $\frac{3}{4}$ are

$$\frac{3\times2}{4\times2}=\frac{6}{8}\ ;\ \frac{3\times3}{4\times3}=\frac{9}{12}\ ;\ \frac{3\times4}{4\times4}=\frac{12}{16}$$

(iii) Equivalent fractions of $\frac{7}{9}$ are

$$\frac{7\times2}{9\times2}=\frac{14}{18}\ ;\ \frac{7\times3}{9\times3}=\frac{21}{27}\ ;\ \frac{7\times4}{9\times4}=\frac{28}{36}$$

Example 14: Write the lowest equivalent fractions (Simplest form) of:-

(i) $\frac{9}{15}$ (ii) $\frac{24}{32}$ (iii) $\frac{60}{75}$ (iv) $\frac{20}{36}$ (v) $\frac{56}{84}$

Solution : (i) $\frac{9}{15}=\frac{9\div3}{15\div3}=\frac{3}{5}$

[Divide the numerator and denominator by HCF of 9 and 15 i.e. 3]

(ii) $\frac{24}{32}=\frac{24\div8}{32\div8}=\frac{3}{4}$

[Divide the numerator and denominator by HCF of 24 and 32 i.e. 8]

(iii) $\frac{60}{75}=\frac{60\div15}{75\div15}=\frac{4}{5}$

[Divide the numerator and denominator by HCF of 60 and 75 i.e. 15]

(iv) $\frac{20}{36}=\frac{20\div4}{36\div4}=\frac{5}{9}$

[Divide the numerator and denominator by HCF of 20 and 36 i.e. 4]

(v) $\frac{56}{84}=\frac{56\div28}{84\div28}=\frac{2}{3}$

[Divide the numerator and denominator by HCF of 56 and 84]

* A fraction is said to be in lowest terms or simplest form if there is no common factor (other than 1) between its numerator and denominator.

5.4.1 Checking for Equivalent Fractions (by Cross product)

If the product (multiplication) of the numerator of the first and the denominator of the second is equal to the product of the denominator of the first and the numerator of the Second than the given fractions are said to be equivalent condsider an example.

$$\frac{2}{7} \times \frac{6}{21}$$

$2 \times 21 = 42$ and $6 \times 7 = 42$

Both the cross products are same.

Thus two fractions are equivalent if their cross products are equal

Example 15: Are the following fractions equivalent?

(i) $\frac{3}{4}$ **and** $\frac{12}{16}$ (ii) $\frac{5}{6}$ **and** $\frac{25}{30}$ (iii) $\frac{4}{7}$ **and** $\frac{24}{27}$

Solution : (i) We have $\frac{3}{4} \times \frac{12}{16}$

By cross product, $3 \times 16 = 48$ and $12 \times 4 = 48$

Since two products are same.

So, the given fractions are equivalent.

(ii) We have $\frac{5}{6} \times \frac{25}{30}$

By cross product, $5 \times 30 = 150$ and $25 \times 6 = 150$

Since two products are same.

So, the given fractions are equivalent.

(iii) We have $\frac{4}{7} \times \frac{24}{27}$

By cross product, $4 \times 27 = 108$ and $24 \times 7 = 168$

Since two products are not same

So, the given fractions are not equivalent

Example 16: Replace □ in each of the following by the correct numbers:-

(i) $\frac{3}{4} = \frac{15}{\square}$ (ii) $\frac{2}{5} = \frac{\square}{30}$ (iii) $\frac{20}{28} = \frac{5}{\square}$

Solution : (i) Observe the numerators, we have $15 \div 3 = 5$

So we multiply both numerator and denominator of $\frac{3}{4}$ by 5.

i.e $\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$

Aliter :- We can solve it by cross product also.

$$\frac{3}{4} = \frac{15}{\square}$$

Let x be the correct number in $\square$.

$$\Rightarrow \quad \frac{3}{4} \times \frac{15}{x} \quad \Rightarrow 3 \times x = 15 \times 4$$

$$\Rightarrow x = \frac{15 \times 4}{3} = 20$$

(ii) Observe the denominators, we have $30 \div 5 = 6$

So, we multiply the numerator and denominator of $\frac{2}{5}$ by 6

i.e. $\frac{2}{5} = \frac{2 \times 6}{5 \times 6} = \frac{12}{30}$

(iii) Observe the numerators, we have $20 \div 5 = 4$

So, we divide the numerator and denominator of $\frac{20}{28}$ by 4

i.e. $\frac{20}{28} = \frac{20 \div 4}{28 \div 4} = \frac{5}{7}$

Example 17: (i) **Write a fraction equivalent to $\frac{2}{3}$ with numerator 16.**

(ii) Write a fraction equivalent to $\frac{5}{7}$ with denominator 28.

(iii) Write a fraction equivalent to $\frac{30}{45}$ with numerator 6.

Solution : (i) We have to find the equivalent fraction to $\frac{2}{3}$ with numerator 16.

or $\frac{2}{3} = \frac{16}{\square}$

Observe the numerators, we have $16 \div 2 = 8$

So we multiply the numerator and denominator of $\frac{2}{3}$ by 8

i.e $\frac{2}{3} = \frac{2 \times 8}{3 \times 8} = \frac{16}{24}$

Hence, equivalent fraction is $\frac{16}{24}$.

(ii) We have to find the equivalent fraction to $\frac{5}{7}$ with denominator 28.

or $\frac{5}{7} = \frac{\square}{28}$

Observe the denominators, we have $28 \div 7 = 4$

So, we multiply the numerator and denomiator of $\frac{5}{7}$ by 4.

i.e $\frac{5}{7} = \frac{5 \times 4}{7 \times 4} = \frac{20}{28}$

Hence equivalent fraction is $\frac{20}{28}$.

(iii) We have to find the equivalent fraction to $\frac{30}{45}$ with numerator 6.

or $\frac{30}{45} = \frac{6}{\square}$

Observe the numerators, we have $30 \div 6 = 5$

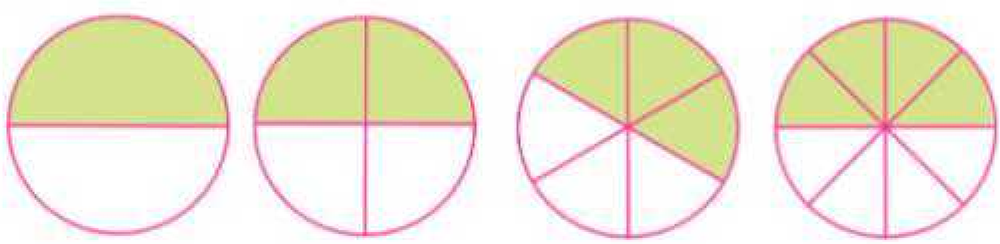
So we divide the numerator and denominator by $\frac{30}{45}$ by 5.

i.e $\frac{30}{45} = \frac{30 \div 5}{45 \div 5} = \frac{6}{9}$

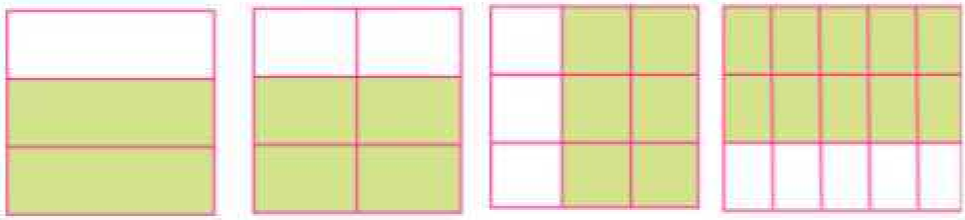
Hence, Equivalent fraction in $\frac{6}{9}$.

Exercise 5.3

1. Write the fraction for the shaded part and check whether these fractions are equivalent or not?

(i) 

(ii)



(iii)

2. Find four equivalent fractions of the followings:-

(i) $\frac{1}{4}$ (ii) $\frac{3}{5}$ (iii) $\frac{7}{9}$ (iv) $\frac{5}{11}$ (v) $\frac{2}{3}$

3. Write the lowest equivalent fraction (simplest form) of:-

(i) $\frac{10}{25}$ (ii) $\frac{27}{54}$ (iii) $\frac{48}{72}$ (iv) $\frac{150}{60}$ (v) $\frac{162}{90}$

4. Are the following fractions equivalent or not?

(i) $\frac{5}{12}, \frac{25}{60}$ (ii) $\frac{6}{7}, \frac{36}{42}$ (iii) $\frac{7}{9}, \frac{56}{72}$

5. Replace $\square$ in each of the following by the correct number.

(i) $\frac{2}{7} = \frac{12}{\square}$ (ii) $\frac{5}{8} = \frac{35}{\square}$ (iii) $\frac{24}{36} = \frac{6}{\square}$ (iv) $\frac{30}{48} = \frac{\square}{8}$ (v) $\frac{7}{4} = \frac{42}{\square}$

6. Find the equivalent fraction of $\frac{3}{5}$, having

(i) numerator 18 (ii) denominator 20 (iii) numerator 24

7. Find the equivalent fraction of $\frac{24}{40}$, having

(i) numerator 6 (ii) numerator 48 (iii) denominator 20

5.6 Like, Unlike Fractions and Unit Fractions

Like Fraction:- Fractions with the same denominators are called like fractions

e.g. $\frac{5}{7}, \frac{1}{7}, \frac{3}{7}$ are like fractions.

Unlike Fractions:- Fractions having different denominators are called unlike fractions

e.g. $\frac{2}{5}, \frac{3}{4}, \frac{7}{8}$ etc. are unlike fractions.

* **Unit Fractions:-** A fraction having 1 as its numerator is called unit fraction

e.g. $\frac{1}{3}, \frac{1}{2}, \frac{1}{7}, \frac{1}{11}, \frac{1}{8}$ etc.

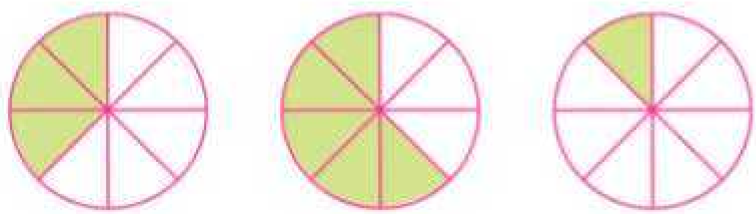
5.7 Comparing and Ordering of Fractions

We have already learnt how to compare natural numbers, whole number and integers. Here we will learn to compare the fractions, which are divided into three categories:-

5.7.1 Fractions with the Same Denominator :

Let us consider some fractions with the same denominator $\frac{3}{8}, \frac{5}{8}, \frac{1}{8}$

The pictorial representation of these fractions are given below:



$\frac{3}{8}$ $\frac{5}{8}$ $\frac{1}{8}$

A B C

By observing pictures,

we can conclude that shaded part of B > shaded part of A > shaded part of C

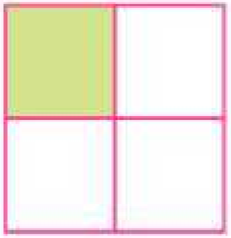
i.e $\frac{5}{8} > \frac{3}{8} > \frac{1}{8}$

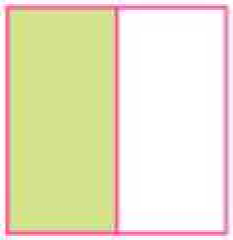
Thus, If two or more fractions having the same denominator then the fraction with greater numerator is greater fraction.

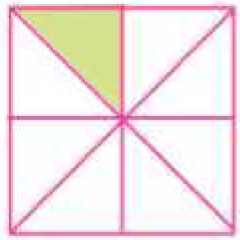
5.7.2 Fraction with the same Numerator

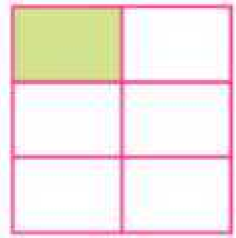
Let us consider some fractions with the same numerator $\frac{1}{4}, \frac{1}{2}, \frac{1}{8}, \frac{1}{6}$

The pictorial representation of these fractions are given below:


$\frac{1}{4}$
(A)


$\frac{1}{2}$
(B)


$\frac{1}{8}$
(C)

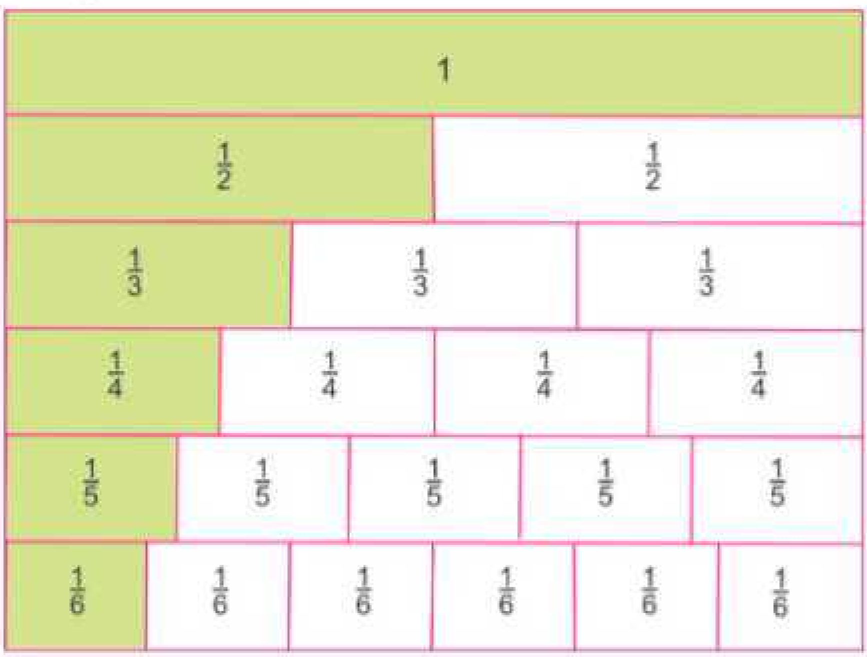

$\frac{1}{6}$
(D)

By observing pictures, we can conclude that shaded part of B> Shaded part of A> shaded part of D> shaded part of C

i.e $\frac{1}{2} > \frac{1}{4} > \frac{1}{6} > \frac{1}{8}$

Thus, If two or more fractions having the same numerator then the fraction with a small denominator is greater.

Representation of equivalent fractions.



In this way we can conclude that If two or more fractions have same numerator then the fractions with smaller denominator is greater.

i.e $1 > \frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \frac{1}{5} > \frac{1}{6}$

This activity can be performed by using chart and colours.

5.7.3 Fraction with different numerators and denominators

To compare the fractions with different numerators and denominators. First we convert that fraction into like fractions using the following steps:

Step 1: Find the LCM of the denominators of the fractions.

Step 2: Convert each fraction into its equivalent fraction with denominator equal to the LCM obtained in Step 1.

Step 3: Compare the numerators of the equivalent fractions whose denominator are same.

Example 18: Which is larger $\frac{2}{3}$ or $\frac{5}{6}$?

Solution : First find the LCM of 3 and 6,

Now we convert the given fraction into a fraction with denominator 6,

We have $\frac{2}{3}=\frac{2\times 2}{3\times 2}=\frac{4}{6}$ and

second fraction is $\frac{5}{6}$ clearly $\frac{4}{6}<\frac{5}{6} \Rightarrow \frac{2}{3}<\frac{5}{6}$

3	3, 6
2	1, 2
	1, 1

$\therefore$ LCM of 3 and 6 is $3 \times 2 = 6$

Aliter :-

If two fractions are there then we can compare the fractions with cross-product also

$$\frac{2}{3} \times \frac{5}{6}$$

$2 \times 6 = 12$ and $5 \times 3 = 15$

So $12 < 15 \Rightarrow \frac{2}{3}<\frac{5}{6}$

Example 19: Arrange the fractions in $\frac{3}{4}, \frac{7}{8}, \frac{15}{12}$ descending order.

Solution : First find the LCM of denominator 4, 8 and 12.

Now, we convert the given fractions into a fraction with denominator 24, we have

$$\frac{3}{4}=\frac{3\times 6}{4\times 6}=\frac{18}{24}$$

$$\frac{7}{8}=\frac{7\times 3}{8\times 3}=\frac{21}{24}$$

and $$\frac{5}{12}=\frac{5\times 2}{12\times 2}=\frac{10}{24}$$

2	4, 8, 12
2	2, 4, 6
2	1, 2, 3
3	1, 1, 3
	1, 1, 1

LCM $= 2 \times 2 \times 2 \times 3 = 24$

$\Rightarrow \quad \frac{21}{24} > \frac{18}{24} > \frac{10}{24} \qquad \Rightarrow \quad \frac{7}{8} > \frac{3}{4} > \frac{5}{12}$

Example 20: Arrange the following fractions in ascending order $\frac{2}{3}, \frac{5}{9}, \frac{3}{5}, \frac{7}{15}$

Solution : First find LCM of denominators 3, 9, 5 and 15

Now, we convert the given fractions into fractions with denominators 45, we have

3	3, 9, 5, 15
3	1, 3, 5, 5
5	1, 1, 5, 5
	1, 1, 1, 1

LCM = 3 × 3 × 5 = 45

$\frac{2}{3} = \frac{2 \times 15}{3 \times 15} = \frac{30}{45}$

$\frac{5}{9} = \frac{5 \times 5}{9 \times 5} = \frac{25}{45}$

$\frac{3}{5} = \frac{3 \times 9}{5 \times 9} = \frac{27}{45}$

$\frac{7}{15} = \frac{7 \times 3}{15 \times 3} = \frac{21}{45}$

∴ Ascending order is

$\Rightarrow \quad \frac{21}{45} < \frac{25}{45} < \frac{27}{45} < \frac{30}{45}$

$\Rightarrow \quad \frac{7}{15} < \frac{5}{9} < \frac{3}{5} < \frac{2}{3}$

Example 21: A boy reads $\frac{2}{5}$ of a book on the first day and $\frac{1}{4}$ of the same book on the second day on which day did he read the major part of the book?

Solution : Here, we have to compare both fractions $\frac{2}{5}$ and $\frac{1}{4}$ and find which is larger

By Cross Product $\frac{2}{5} \times \frac{1}{4}$

$2 \times 4 = 8$ and $1 \times 5 = 5$

So $8 > 5 \quad \Rightarrow \quad \frac{2}{5} > \frac{1}{4}$

Hence, he read the major part of the book on the first day.

Example 22: Arun exercised for $\frac{3}{4}$ of an hour while Jaspreet exercised for $\frac{3}{10}$ of an hour. Who exercised for longer time?

Solution : Arun exercised = $\frac{3}{4}$ of an hour

Jaspreet exercised = $\frac{3}{10}$ of an hour

Clearly, In both fractions, numerators are same.

So fraction with smaller denominator is larger.

i.e $\frac{3}{4} > \frac{3}{10}$

Hence, Arun exercised for a longer time.

Aliter :-

Arun exercised $= \frac{3}{4}$ of an hour $= \frac{3}{4} \times 60$ minutes [$\because$ 1 hours = 60 minutes]

$= 45$ minutes

Jaspreet exercised = $\frac{3}{10}$ of an hour $= \frac{3}{10} \times 60$ minutes

$= 18$ minutes

$\Rightarrow \quad 45 > 18$

Hence Arun exercised for a longer time.

Exercise 5.4

1. Find the different set of like fractions:

$\frac{3}{7}, \frac{5}{11}, \frac{2}{7}, \frac{6}{13}, \frac{3}{11}, \frac{1}{11}, \frac{2}{13}, \frac{5}{13}, \frac{6}{7}, \frac{10}{13}$

2. Write any three like fractions of:-

(i) $\frac{2}{5}$ (ii) $\frac{1}{4}$ (iii) $\frac{11}{6}$

3. Encircle unit fractions:-

$\frac{6}{11}, \frac{2}{3}, \frac{1}{8}, \frac{15}{7}, \frac{1}{9}, \frac{1}{7}, \frac{3}{3}$

4. Fill in the boxes with $>$, $<$ or $=$

(i) $\frac{4}{7} \square \frac{6}{7}$ (ii) $\frac{4}{5} \square \frac{3}{5}$ (iii) $\frac{7}{8} \square \frac{0}{8}$ (iv) $\frac{2}{3} \square \frac{5}{3}$ (v) $\frac{5}{13} \square \frac{7}{13}$

5. Compare using >, < or =

(i) $\frac{5}{7} \square \frac{5}{9}$ (ii) $\frac{1}{3} \square \frac{1}{2}$ (iii) $\frac{6}{11} \square \frac{6}{13}$ (iv) $\frac{11}{12} \square \frac{11}{17}$ (v) $\frac{7}{13} \square \frac{7}{10}$

6. Compare using > , < or =

(i) $\frac{5}{6} \square \frac{2}{5}$ (ii) $\frac{3}{4} \square \frac{1}{3}$ (iii) $\frac{3}{7} \square \frac{5}{9}$ (iv) $\frac{7}{10} \square \frac{4}{5}$ (v) $\frac{7}{7} \square 1$

7. Arrange the following fractions in ascending order:-

(i) $\frac{7}{10}, \frac{3}{10}, \frac{5}{10}$ (ii) $\frac{6}{7}, \frac{1}{7}, \frac{4}{7}$ (iii) $\frac{5}{8}, \frac{7}{8}, \frac{1}{8}, \frac{3}{8}$ (iv) $\frac{5}{7}, \frac{5}{9}, \frac{5}{3}$

(v) $\frac{3}{11}, \frac{3}{7}, \frac{3}{13}$ (vi) $\frac{1}{4}, \frac{1}{6}, \frac{5}{12}$ (vii) $\frac{2}{7}, \frac{11}{35}, \frac{9}{14}, \frac{13}{28}$ (viii) $\frac{1}{3}, \frac{4}{9}, \frac{5}{12}, \frac{4}{15}$

8. Arrange the following fractions in descending order:-

(i) $\frac{5}{9}, \frac{7}{9}, \frac{1}{9}$ (ii) $\frac{3}{11}, \frac{5}{11}, \frac{2}{11}, \frac{7}{11}$ (iii) $\frac{2}{7}, \frac{2}{13}, \frac{2}{9}$

(iv) $\frac{1}{5}, \frac{1}{3}, \frac{1}{8}, \frac{1}{2}$ (v) $\frac{1}{6}, \frac{5}{12}, \frac{5}{18}, \frac{2}{3}$ (vi) $\frac{3}{4}, \frac{9}{20}, \frac{11}{15}, \frac{17}{30}$

9. Kasvi covered $\frac{1}{3}$ of her journey by car, $\frac{1}{5}$ by rickshaw and $\frac{2}{15}$ on foot. Find by which means, she covered the major part of her journey.

10. Father distributed his property among his three sons. The eldest one got $\frac{3}{10}$, the middle got $\frac{1}{6}$ and the youngest got $\frac{1}{5}$ part of the property. State how the property was distribued in ascending order.

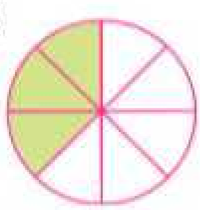
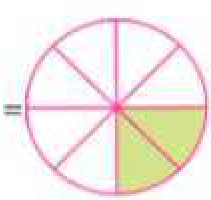
5.8 Operations on Fractions

In the previous section, we have learnt the like, unlike fractions and their comparison. In this section, we shall learn about addition and subtraction of the fractions. We follow certain methods for doing these operations:

5.8.1 Addition and Subtraction of like fractions

- **Addition:-** Look at the adjoining figure, there are 8 parts in a circle Let us represent

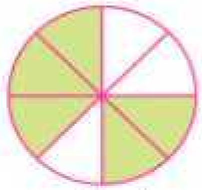
$\frac{3}{8} + \frac{2}{8}$

$\frac{3}{8} =$  $\frac{2}{8} =$ 

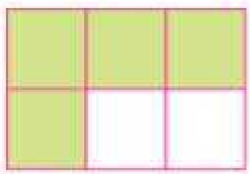
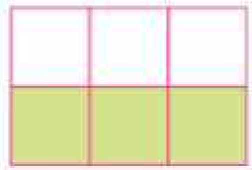
Shade 1:- Out of 8, 3 parts are shaded green.

Shade 2:- Out of 8, 2 parts are shaded green.

Count the total number of shaded parts there are 5 out of 8 i.e. $\frac{5}{8}$

i.e $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$ 

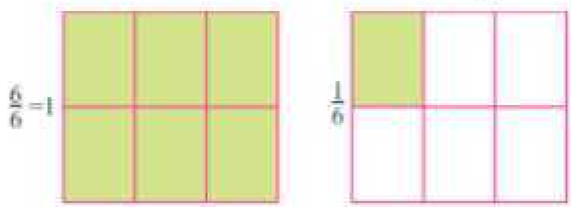
- Now Let us represent $\frac{4}{6} + \frac{3}{6}$

$\frac{4}{6} =$  , $\frac{3}{6} =$ 

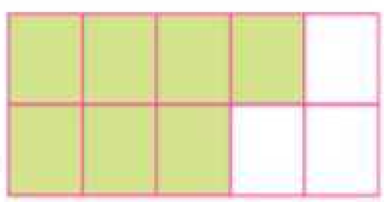
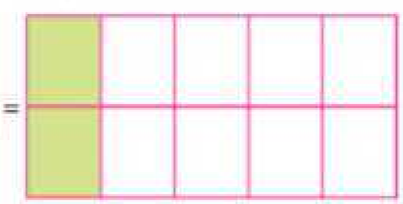
Shade 1:- Out of 6, 4 boxes are shaded with green colour.

Shade 2:- Here, we are to shade 3 parts out of 6, but we have 2 boxes left and we cannot shade more than 2. So we make one more same size figure with 6 parts and shade the remaining1 in those parts.

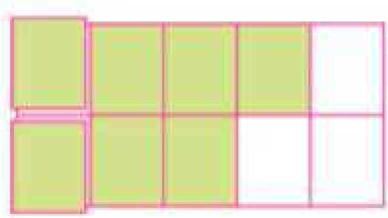
ie. $\frac{4}{6} + \frac{3}{6} = \frac{7}{6} = 1\frac{1}{6}$



- **Subtraction of Like fractions:-** Let us subtract $\frac{2}{10}$ from $\frac{7}{10}$

i.e $\frac{7}{10} - \frac{2}{10}$ $\frac{7}{10} =$  $\frac{2}{10} =$ 

- Shade 7 parts out of 10 and 2 out of 10 as shown
- Take away 2 shaded parts out of 10. Now, the remainder shaded parts are 5 thus $\frac{7}{10} - \frac{2}{10} = \frac{5}{10}$



Thus the sum and difference of two or more like fractions can be obtained as follows:-

Step 1:- Add/Subtract the numerators of all fractions.

Step 2:- Retain the common denominator of all fractions.

Step 3:- Write the fraction as $\frac{\text{Sum / difference of numerators}}{\text{Common denominator}}$

Example 23: Simplity:-

(i) $\frac{3}{10}+\frac{4}{10}$ (ii) $\frac{5}{11}+\frac{2}{11}+\frac{1}{11}$ (iii) $\frac{5}{14}+\frac{8}{14}+\frac{2}{14}$

(iv) $\frac{5}{8}-\frac{3}{8}$ (v) $\frac{4}{7}+\frac{5}{7}-\frac{6}{7}$ (vi) $\frac{8}{9}-\frac{2}{9}-\frac{3}{9}$

Solution : (i) $\frac{3}{10}+\frac{4}{10}=\frac{3+4}{10}=\frac{7}{10}$

(ii) $\frac{5}{11}+\frac{2}{11}+\frac{1}{11}=\frac{5+2+1}{11}=\frac{8}{11}$

(iii) $\frac{5}{14}+\frac{8}{14}+\frac{2}{14}=\frac{5+8+2}{14}=\frac{15}{14}$

(iv) $\frac{5}{8}-\frac{3}{8}=\frac{5-3}{8}=\frac{2}{8}=\frac{1}{4}$ $\left[\frac{2}{8}=\frac{2\div 2}{8\div 2}=\frac{1}{4}\right]$

(v) $\frac{4}{7}+\frac{5}{7}-\frac{6}{7}=\frac{4+5-6}{7}=\frac{3}{7}$

(vi) $\frac{8}{9}-\frac{2}{9}-\frac{3}{9}=\frac{8-2-3}{9}=\frac{3}{9}$

5.8.2 Addition / Subtraction of Unlike Fractions :

In last section, we have learnt about addition and subtraction of like decimals. In this section we shall learn about addition and subtraction of unlike fraction. First we convert unlike fractions to equivalent like fractions and then we add or subtract, We follow these steps:-

Step 1:- Find the L.C.M of the denominators.

Step 2:- Convert each fraction into equivalent fraction with denominator equal to the L.C.M. obtained in step 1.

Step 3:- Add or subtract like fractions as required.

Let us illustrate this with following examples:-

Example 24: Add $\frac{2}{3}$ and $\frac{3}{10}$

Solution : To add $\frac{2}{3}$ and $\frac{3}{10}$, we take LCM of denominators 3 and 10 = 30

New, we convert the given fractions into equivalent fractions with denominator 30:

$\frac{2}{3} = \frac{2 \times 10}{3 \times 10} = \frac{20}{30}$ and $\frac{3}{10} = \frac{3 \times 3}{10 \times 3} = \frac{9}{30}$

Thus, $\frac{2}{3} + \frac{3}{10} = \frac{20}{30} + \frac{9}{30} = \frac{20+9}{30} = \frac{29}{30}$

Aliter :- $\frac{(\text{Ist fraction}) \times \text{LCM} + (\text{2nd fraction}) \times \text{LCM}}{\text{LCM of denominators}}$

i.e $\frac{\frac{2}{3} \times 30 + \frac{3}{10} \times 30}{30} = \frac{2 \times 10 + 3 \times 3}{30} = \frac{20+9}{30} = \frac{29}{30}$

Aliter :- Cross Product Method.

$\frac{a}{b} \times \frac{c}{d} = \frac{a \times d + b \times c}{b \times d}$

Thus $\frac{2}{3} + \frac{3}{10} = \frac{2 \times 10 + 3 \times 3}{3 \times 10} = \frac{20+9}{30} = \frac{29}{30}$

This is applicable when denominators has no common factor

Example 25: **Add $\frac{5}{6} + \frac{1}{4}$**

Solution : To add $\frac{5}{6}$ and $\frac{1}{4}$, we take LCM of denominators 6 and 4 = 12

Now, convert the given fractions into equivalent fractions with denominator 12.

$$\begin{array}{c|l} 2 & 6, 4 \\ \hline & 3, 2 \end{array}$$

LCM = 2 × 3 × 2 = 12

$\frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$ and $\frac{1}{4} = \frac{1 \times 3}{4 \times 3} = \frac{3}{12}$

Thus $\frac{5}{6} + \frac{1}{4} = \frac{10}{12} + \frac{3}{12} = \frac{10+3}{12} = \frac{13}{12}$

Aliter :-

$\frac{(\text{Ist fraction}) \times \text{LCM} + (\text{2nd fraction}) \times \text{LCM}}{\text{LCM of denominators}}$

i.e $\frac{\left[\frac{5}{6} \times 12\right] + \left[\frac{1}{4} \times 12\right]}{12} = \frac{10+3}{12} = \frac{13}{12}$

Example 26:- **Add $3\frac{1}{4}+2\frac{4}{5}$**

Solution : Convert mixed fractions to Improper fraction

ie. $3\frac{1}{4}+2\frac{4}{5} = \frac{13}{4}+\frac{14}{5}$

$= \frac{13\times5}{4\times5}+\frac{14\times4}{5\times4}=\frac{65}{20}+\frac{56}{20}$

$= \frac{65+56}{20}=\frac{121}{20}=6\frac{1}{20}$

[∴ LCM of 4 and 5 is 20, So convert each fraction into an equivalent fraction with denominator 20]

∵ 20) 121 (6
−120
1

Aliter :-

$3\frac{1}{4}+2\frac{4}{5}$

$= 3+\frac{1}{4}+2+\frac{4}{5} = 5+\left[\frac{1}{4}+\frac{4}{5}\right]$

$= 5 + \left(\frac{1\times5}{4\times5}+\frac{4\times4}{5\times4}\right)$

Convert into equivalent fractions with denominator 20

$= 5 + \left(\frac{5}{20}+\frac{16}{20}\right)$

$= 5 + \left(\frac{21}{20}\right) = 5 + \left(1\frac{1}{20}\right)$

∵ 20) 21 (1
− 20
1

$= 5 + 1 + \frac{1}{20} = 6 + \frac{1}{20} = 6\frac{1}{20}$

Exmaple 27:- **Simplify:-**

(i) $\frac{3}{4}-\frac{5}{8}$ (ii) $\frac{2}{3}-\frac{1}{4}$

Solution : (i) $\frac{3}{4}-\frac{5}{8}$

To subtract $\frac{3}{4}$ and $\frac{5}{8}$, we take LCM of denominators 4 and 8 = 8

Now Equivalent fractions with denominators 8.

$\frac{3}{4}=\frac{3\times2}{4\times2}=\frac{6}{8}$ and $\frac{5}{8}$

2	4, 8
2	2, 4
	1, 2

LCM = 2 × 2 × 2 = 8

Thus, $\frac{3}{4}-\frac{5}{8}=\frac{6}{8}-\frac{5}{8}=\frac{6-5}{8}=\frac{1}{8}$

(ii) $\frac{2}{3}-\frac{1}{4}$

To subtract $\frac{2}{3}$ and $\frac{1}{2}$, we take LCM of denominator 3 and 4 = 12

So $\frac{2}{3}-\frac{1}{4}=\frac{2\times4}{3\times4}-\frac{1\times3}{4\times3}$

$=\frac{8}{12}-\frac{3}{12}=\frac{8-3}{12}=\frac{5}{12}$

Example 28: **Subtract $1\frac{1}{2}$ from $4\frac{3}{5}$**

Solution : (i) $4\frac{3}{5}-1\frac{1}{2}$

First convert mixed fraction into improper fraction.

i.e $4\frac{3}{5}-1\frac{1}{2}=\frac{23}{5}-\frac{3}{2}$

$=\frac{23\times2}{5\times2}-\frac{3\times5}{2\times5}$

$=\frac{46}{10}-\frac{15}{10}=\frac{46-15}{10}$

$=\frac{31}{10}=3\frac{1}{10}$

[∵ LCM of 5 and 2 is 10, So convert each fraction into an equivalent fraction of denominator 10]

∵ 10) 31 (3
 −30
 1

Aliter :-

$4\frac{3}{5}-1\frac{1}{2}=(4-1)+\left(\frac{3}{5}-\frac{1}{2}\right)$

$=3+\left(\frac{3\times2}{5\times2}-\frac{1\times5}{2\times5}\right)=3+\left(\frac{6}{10}-\frac{5}{10}\right)$

$=3+\frac{1}{10}=3\frac{1}{10}$

Example 29: **Simplify:-**

(i) $\frac{2}{3}+\frac{3}{8}-\frac{5}{6}$ (ii) $\frac{3}{5}+\frac{7}{10}-\frac{1}{4}$ (iii) $\frac{5}{12}-\frac{1}{6}+\frac{4}{15}$

Solution : (i) $\frac{2}{3}+\frac{3}{8}-\frac{5}{6}$

First, take LCM of 3, 8 and 6 = 24 then Convert each fraction into equivalent fraction with denominator 24.

$$\therefore \frac{2}{3}+\frac{3}{8}-\frac{5}{6}=\frac{2\times 8}{3\times 8}+\frac{3\times 3}{8\times 3}-\frac{5\times 4}{6\times 4}$$

$$=\frac{16}{24}+\frac{9}{24}-\frac{20}{24}=\frac{16+9-20}{24}=\frac{5}{24}$$

3	3, 8, 6
2	1, 8, 2
2	1, 4, 1
	1, 2, 1

LCM = 3 × 2 × 2 × 2 = 24

(ii) $\frac{3}{5}+\frac{7}{10}-\frac{1}{4}$

First, take LCM of 5, 10 and 4 = 20 then convert each fraction into equivalent fraction with denominator 20.

$$\therefore \frac{3}{5}+\frac{7}{10}-\frac{1}{4}=\frac{3\times 4}{5\times 4}+\frac{7\times 2}{10\times 2}-\frac{1\times 5}{4\times 5}$$

$$=\frac{12}{20}+\frac{14}{20}-\frac{5}{20}=\frac{12+14-5}{20}=\frac{21}{20}$$

2	5, 10, 4
5	5, 5, 2
	1, 1, 2

LCM = 3 × 5 × 2 = 20

(iii) $\frac{5}{12}-\frac{1}{6}+\frac{4}{15}$

LCM of denominators = 60

$$=\frac{5\times 5}{12\times 5}-\frac{1\times 10}{6\times 10}+\frac{4\times 4}{15\times 4}$$

$$=\frac{25}{60}-\frac{10}{60}+\frac{16}{60}=\frac{25-10+16}{60}=\frac{31}{60}$$

2	12, 6, 15
3	6, 3, 15
	2, 1, 5

LCM = 2 × 3 × 2 × 5 = 60

5.8.3 Adding or Subtracting a fraction with a whole number

In this section, we shall learn to add or subtract a fraction with a whole number. We express the whole number in a fraction by writing 1 in the denominator.

Then we add or subtract as we have discussed in the previous section.

Example 30: **Simplify**

(i) $4+\frac{2}{3}$ **(ii)** $2-\frac{5}{6}$

Solution : (i) $4+\frac{2}{3}=\frac{4}{1}+\frac{2}{3}=\frac{4\times 3}{1\times 3}+\frac{2}{3}$

(∵ LCM of 1 and 3 is 3)

$$=\frac{4\times 3+2\times 1}{3}=\frac{12+2}{3}=\frac{14}{3}=4\frac{2}{3}$$

Aliter :-

In additon, we can write it direct also.

i.e. $4+\frac{2}{3}=4\frac{2}{3}$

(ii) $2-\frac{5}{6}=\frac{2}{1}-\frac{5}{6}$

(∵ LCM of 1 and 6 is 6)

$=\frac{2\times6}{1\times6}-\frac{5}{6}=\frac{12}{6}-\frac{5}{6}$

$=\frac{12-5}{6}=\frac{7}{6}=1\frac{1}{6}$

6) 7 (1
−6
1

Example 31: **Sophia ran $\frac{3}{8}$ km in the morning and $2\frac{7}{10}$ km in the evening. How much did she run in that day?**

Solution : Total distance Sophia ran $=\frac{3}{8}+2\frac{7}{10}$

$=\frac{3}{8}+\frac{27}{10}$

$=\frac{3\times5}{8\times5}+\frac{27\times4}{10\times4}$

$=\frac{15}{40}+\frac{108}{40}=\frac{15+108}{40}=\frac{123}{40}=3\frac{3}{40}$

∴ Sophia ran $3\frac{3}{40}$ km on that day.

2 | 8, 10
| 4, 5

LCM of 8 and 10 = $2\times4\times5=40$

∵ 40) 123 (3
−120
3

Example 32:- **A piece of wire $3\frac{3}{4}$ metres long broke into two pieces. One piece was $2\frac{5}{6}$ metre long. How long is the other piece?**

Solution : Length of Second piece $=3\frac{3}{4}-2\frac{5}{6}$

$=\frac{15}{4}-\frac{17}{6}$

∵ LCM of 4 and 6 = 12

$=\frac{15\times3}{4\times3}-\frac{17\times2}{6\times2}=\frac{45}{12}-\frac{34}{12}=\frac{45-34}{12}=\frac{11}{12}$

So, length of other piece is $\frac{11}{12}$ m.

Example 33: **Pankaj bought a notebook for ₹ $11\frac{1}{2}$, pencil for ₹ $2\frac{3}{4}$ and a colour box for ₹ $6\frac{2}{5}$. How much money did he spend?**

Solution : Total money he spend = $11\frac{1}{2} + 2\frac{3}{4} + 6\frac{2}{5}$

$= \frac{23}{2} + \frac{11}{4} + \frac{32}{5}$ (∵ LCM of 2, 4, 5 = 20)

$= \frac{23 \times 10}{2 \times 10} + \frac{11 \times 5}{4 \times 5} + \frac{32 \times 4}{5 \times 4}$

$= \frac{230}{20} + \frac{55}{20} + \frac{128}{20} = \frac{230 + 55 + 128}{20}$

$= \frac{413}{20} = 20\frac{13}{20}$

$$\begin{array}{r} 20\,)\;413\,(\,20 \\ -400 \\ \hline 13 \\ \hline \end{array}$$

He spent ₹ $20\frac{13}{20}$ in all.

Exercise 5.5

1. Add the following:-

(i) $\frac{3}{7} + \frac{2}{7}$ (ii) $\frac{2}{11} + \frac{4}{11}$ (iii) $\frac{6}{13} + \frac{5}{13}$ (iv) $\frac{5}{14} + \frac{9}{14} + \frac{3}{14}$

(v) $\frac{1}{4} + \frac{2}{3}$ (vi) $\frac{1}{6} + \frac{5}{12}$ (vii) $\frac{3}{10} + \frac{4}{15}$ (viii) $\frac{3}{8} + \frac{1}{4}$

(ix) $\frac{5}{9} + 4$ (x) $\frac{4}{7} + \frac{2}{3} + \frac{5}{21}$ (ix) $\frac{3}{4} + \frac{7}{12} + \frac{2}{3}$ (xii) $\frac{3}{5} + \frac{1}{3}$

2. Subtract the following:-

(i) $\frac{5}{9} - \frac{2}{9}$ (ii) $\frac{6}{17} - \frac{3}{17}$ (iii) $\frac{7}{10} - \frac{3}{10}$ (iv) $\frac{11}{13} - \frac{6}{13} - \frac{2}{13}$

(v) $\frac{5}{12} - \frac{1}{4}$ (vi) $\frac{3}{5} - \frac{2}{10}$ (vii) $\frac{6}{7} - \frac{2}{3}$ (viii) $\frac{5}{6} - \frac{1}{4}$

(ix) $\frac{8}{3} - \frac{5}{9}$ (x) $2 - \frac{1}{7}$ (xi) $\frac{13}{7} - \frac{3}{4} - \frac{1}{14}$ (xii) $\frac{17}{24} - \frac{5}{16} - \frac{1}{3}$

3. Simplify the following:-

(i) $4\frac{2}{5} + 2\frac{1}{5}$ (ii) $5\frac{3}{4} + 2\frac{1}{6}$ (iii) $6\frac{1}{2} + 2\frac{2}{3}$ (iv) $4\frac{3}{4} - 1\frac{5}{6}$

(v) $2\frac{7}{10} - 1\frac{2}{15}$ (vi) $5 - 3\frac{1}{2}$ (vii) $7 + \frac{7}{4} + 5\frac{1}{6}$ (viii) $2\frac{1}{8} + 1\frac{1}{2} - \frac{7}{16}$

(ix) $5\frac{2}{3} + 6 - 3\frac{1}{4}$ (x) $2 - \frac{7}{16}$ (xi) $6 + 1\frac{1}{2}$ (xii) $2\frac{5}{6} - 3\frac{5}{8} + 2$

4. An iron pipe of length $6\frac{2}{3}$ metres long was cut into two pieces. One piece is $4\frac{3}{7}$ metre long. What is the length of other piece?

5. Ashok bought $\frac{7}{10}$ kg of mangoes and Tarun $\frac{11}{15}$ kg of apples. How much fruit did he buy in all?

6. Avi did $\frac{3}{5}$ of his homework on Saturday and $\frac{1}{10}$ of the same homework on Sunday. How much of the homework did he do over the weekend?

7. Charan spent $\frac{1}{4}$ of his pocket money on a movie and $\frac{3}{8}$ on a new pen and $\frac{1}{8}$ on a pencil. What fraction of his pocket money did he spend?

8. Simar lives at a distance of 4km from the school. Prabhjot lives at a distance of $\frac{2}{3}$ km less than simar's distance from the school. How far does prabhjot live from the school?

5.9 Multiplication of a fraction by a Whole Number

We have learnt the addition and subtraction of different type of fractions. Here we shall learn the multiplication of a fraction with a whole number in a brief view.

We know that in case of whole numbers, **multiplication means repeated addition.**

e.g. $4 \times 6 = 6 + 6 + 6 + 6 = 24$

Similarly, using the same rule, we multiply a fraction by a whole number.

Let us consider an example.

$$\frac{1}{3}\times 5 = 5\times\frac{1}{3} = \frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3} \qquad = \frac{1+1+1+1+1}{3} = \frac{5}{3}$$

or, we can say that $\frac{1}{3}\times 5 = 5\times\frac{1}{3} = \frac{5\times 1}{3} = \frac{5}{3}$

Thus, To multiply a fraction by a whole number, we multiply the numerator of the fraction by the whole number and the denominator of the fraction remains same.

Example 34:- Multiply :-

(i) $\mathbf{\frac{1}{8}\times 3}$ **(ii)** $\mathbf{\frac{5}{12}\times 4}$ **(iii)** $\mathbf{6\times\frac{7}{10}}$

Solution : (i) $\frac{1}{8}\times 3 = \frac{1}{8}\times\frac{3}{1} = \frac{1\times 3}{8} = \frac{3}{8}$

(ii) $\frac{5}{12}\times 4 = \frac{5}{12}\times\frac{4}{1} = \frac{5\times 4}{12} = \frac{20}{12} = \frac{20\div 4}{12\div 4} = \frac{5}{3}$

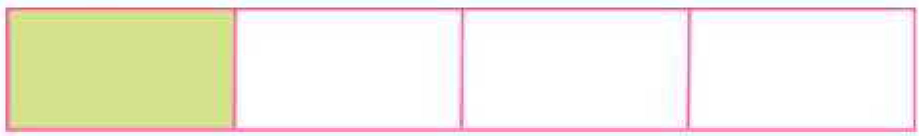
($\because$ HCF of 12 and 20 is 4. Convert in Simplest form)

(iii) $6\times\frac{7}{10}=\frac{6}{1}\times\frac{7}{10}=\frac{6\times7}{10}=\frac{42}{10}=\frac{42\div2}{10\div2}=\frac{21}{5}$

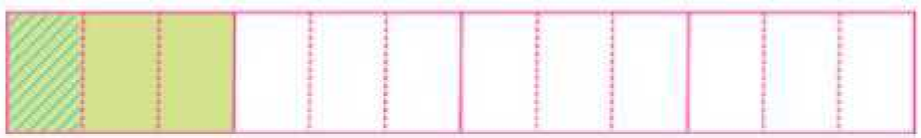
(∵ HCF of 42 and 10 is 2. Convert in Simplest form)

5.10 Division of a Fraction by a natural number

Let us divide $\frac{1}{4}$ by 3



Divide the whole figure in 4 equal parts. Now, we divide each part into 3 equal parts. Such that the whole figure is divided into 12 equal parts.



In figure, double shaded part represents $\frac{1}{12}$ of the whole figure.

i.e. $\frac{1}{4}\div3=\frac{1}{12}$

Also $\frac{1}{4}\div\frac{3}{1}=\frac{1}{4}\times\frac{1}{3}=\frac{1\times1}{4\times3}=\frac{1}{12}$

Thus, to divide a fraction by a natural number, we multiply the fraction by the reciprocal of the natural number.

Example 35: **Divide:-**

(i) $\frac{1}{6}$ by 2 (ii) $\frac{2}{3}$ by 5 (iii) $\frac{2}{5}$ by 4

Solution : (i) $\frac{1}{6}\div2=\frac{1}{6}\div\frac{2}{1}=\frac{1}{6}\times\frac{1}{2}=\frac{1\times1}{6\times2}=\frac{1}{12}$

(ii) $\frac{2}{3}\div5=\frac{2}{3}\div\frac{5}{1}=\frac{2}{3}\times\frac{1}{5}=\frac{2\times1}{3\times5}=\frac{2}{15}$

(iii) $\frac{2}{5}\div4=\frac{2}{5}\div\frac{4}{1}=\frac{2}{5}\times\frac{1}{4}=\frac{2\times1}{5\times4}=\frac{2}{20}=\frac{1}{10}$

Exercise 5.6

1. Multiply:-

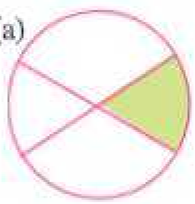
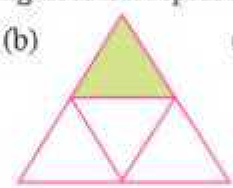
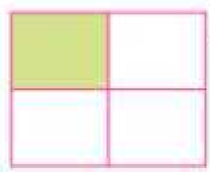
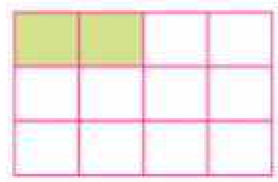
(i) $\frac{1}{5}\times 4$ (ii) $\frac{2}{7}\times 3$ (iii) $\frac{5}{8}\times 2$ (iv) $\frac{7}{12}\times 4$ (v) $10\times\frac{4}{5}$

2. Divide:-

(i) $\frac{1}{4}\div 5$ (ii) $\frac{3}{5}\div 3$ (iii) $\frac{5}{8}\div 3$ (iv) $\frac{6}{7}\div 2$ (v) $\frac{12}{15}\div 6$

Multiple Choice Questions

1. Which of the following does not represent any fraction ?

(a)

(b)

(c)

(d)


2. Which of the following is a proper fraction?

(a) $\frac{5}{5}$ (b) $\frac{12}{11}$ (c) $\frac{7}{9}$ (d) 7

3. Which of the following is an improper fraction?.

(a) $\frac{5}{8}$ (b) $2\frac{3}{4}$ (c) $\frac{7}{11}$ (d) $\frac{15}{16}$

4. The fractions having 1 as numerator are called .... fractions.

(a) Like (b) Unlike (c) Unit (d) Proper

5. The fractions having same denominators are called .... fractions.

(a) Proper (b) Unit (c) Improper (d) Like

6. The fraction having different denominators are called .... fractions.

(a) Unlike (b) Like (c) Improper (d) Unit

7. Express 8 hours as a fraction of 1 day.

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{8}{1}$ (d) $\frac{1}{8}$

8. Find : $\frac{2}{5}$ of ₹ 20

(a) ₹ 8 (b) ₹ 10 (c) ₹ 12 (d) ₹ 40

9. Write $\frac{19}{4}$ as mixed fraction

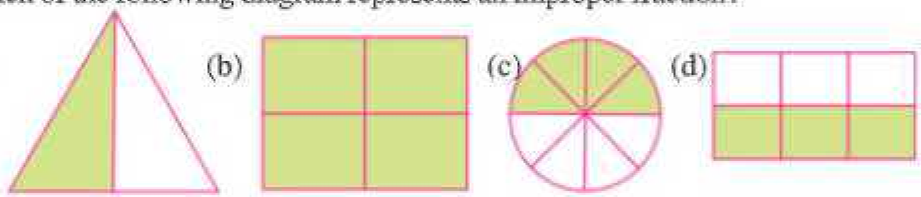
(a) $3\frac{4}{5}$ (b) $4\frac{4}{3}$ (c) $4\frac{3}{4}$ (d) $5\frac{1}{4}$

10. $7\frac{2}{3}$ = ...

(a) $\frac{17}{3}$ (b) $\frac{23}{3}$ (c) $\frac{13}{3}$ (d) $\frac{42}{3}$

11. Which of the following diagram represents an improper fraction?

(a) (b) (c) (d)



12. Which of the following fraction is an equivalent of $\frac{5}{7}$?

(a) $\frac{25}{49}$ (b) $\frac{20}{35}$ (c) $\frac{35}{49}$ (d) $\frac{35}{28}$

13. Replace □ by the correct number in $\frac{5}{8} = \frac{20}{\square}$

(a) 32 (b) 24 (c) 40 (d) 16

14. Which of the following are in ascending order?

(a) $\frac{2}{3}, \frac{2}{7}, \frac{2}{5}$ (b) $\frac{2}{3}, \frac{2}{5}, \frac{2}{7}$ (c) $\frac{2}{7}, \frac{2}{3}, \frac{2}{5}$ (d) $\frac{2}{7}, \frac{2}{5}, \frac{2}{3}$

15. Which of the following are in descending order?

(a) $\frac{1}{8}, \frac{1}{3}, \frac{1}{9}$ (b) $\frac{1}{3}, \frac{1}{8}, \frac{1}{9}$ (c) $\frac{1}{8}, \frac{1}{9}, \frac{1}{3}$ (d) $\frac{1}{3}, \frac{1}{9}, \frac{1}{8}$

16. $\frac{4}{6} + \frac{3}{6}$ = ...

(a) $\frac{7}{12}$ (b) $\frac{7}{8}$ (c) $1\frac{1}{6}$ (d) $1\frac{1}{12}$

17. $\frac{4}{9} + \frac{5}{9} - \frac{2}{9}$ = ...

(a) $\frac{7}{9}$ (b) $\frac{7}{18}$ (c) $\frac{11}{9}$ (d) $\frac{5}{9}$

18. $\frac{2}{3}+\frac{1}{6}=$ ....

(a) $\frac{3}{9}$ (b) $\frac{5}{6}$ (c) $\frac{7}{6}$ (d) $\frac{5}{9}$

19. $4-\frac{1}{3}=$ ....

(a) $4\frac{1}{3}$ (b) $3\frac{1}{3}$ (c) $4\frac{2}{3}$ (d) $3\frac{2}{3}$

20. Divide $\frac{1}{6}$ by 2

(a) $\frac{1}{3}$ (b) $\frac{1}{12}$ (c) $\frac{1}{18}$ (d) 12

Learning Outcomes

After completion of this chapter the students are now able to

- Recognise different types of fractions and their diagrammical representation
- Use fractions in daily life.
- Use fractions in different units i.e. money, length and temperature.

ANSWER KEY

Exercise 5.1

1. (i) $\frac{1}{4}$ (ii) $\frac{5}{8}$ (iii) $\frac{4}{9}$ (iv) $\frac{5}{8}$ (v) $\frac{7}{16}$ (vi) $\frac{3}{3}$ (vii) $\frac{2}{5}$ (viii) $\frac{4}{7}$

3. (i) $\frac{3}{4}$ (ii) $\frac{7}{10}$ (iii) $\frac{1}{4}$ (iv) $\frac{5}{8}$ (v) $\frac{3}{12}$

4. (i) $\frac{5}{9}$ (ii) $\frac{2}{11}$ (iii) $\frac{6}{7}$

5. (i) numerator = 2, denominator = 3 (ii) numerator = 1, denominator = 4
(iii) numerator = 5, denominator = 11 (iv) numerator = 9, denominator = 13
(v) numerator = 17, denominator = 16

6. (i) $\frac{1}{7}$ (ii) $\frac{40}{60}$ or $\frac{2}{3}$ (iii) $\frac{15}{24}$ or $\frac{5}{8}$ (iv) $\frac{2}{12}$ or $\frac{1}{6}$ (v) $\frac{45}{100}$ or $\frac{9}{20}$

7. (i) $\frac{12}{25}$ (ii) $\frac{9}{25}$ (iii) $\frac{8}{25}$

8. $\frac{24}{42}$ or $\frac{4}{7}$, $\frac{18}{42}$ or $\frac{3}{7}$

9. $\frac{6}{13}$, $\frac{7}{13}$

10. (i) $\frac{2}{10}$ or $\frac{1}{5}$ (ii) $\frac{3}{10}$ (iii) $\frac{4}{10}$ or $\frac{2}{5}$ (iv) Sidharth

11. Apples = $\frac{12}{24}$ or $\frac{1}{2}$, Oranges = $\frac{7}{24}$, Guava = $\frac{5}{24}$ **12.** Dishmeet = 15, Balkirat = 5

14. (i) 12 books (ii) 20 pens (iii) 6 copies (iv) 12 apples (v) 21 pencils

15. (i) 18 (ii) 8 (iii) 10

16. (i) False (ii) True (iii) True (iv) True

Exercise 5.2

1. Proper fractions :- $\frac{9}{13}, \frac{6}{11}, \frac{7}{9}, \frac{2}{15}, \frac{4}{17}, \frac{7}{8}$ Improper fractions:- $\frac{5}{4}, \frac{3}{2}, \frac{5}{2}, \frac{6}{6}$

2. (i) $5\frac{2}{5}$ (ii) $3\frac{1}{4}$ (iii) $5\frac{3}{8}$ (iv) $7\frac{2}{7}$ (v) $6\frac{2}{3}$

3. (i) $\frac{7}{3}$ (ii) $\frac{37}{7}$ (iii) $\frac{23}{5}$ (iv) $\frac{15}{4}$ (v) $\frac{77}{8}$

4. (i) $\frac{7}{3}, 2\frac{1}{3}$ (ii) $\frac{13}{4}, 3\frac{1}{4}$ (iii) $\frac{19}{5}, 3\frac{4}{5}$ (iv) $\frac{21}{8}, 2\frac{5}{8}$ (v) $\frac{25}{6}, 4\frac{1}{6}$

Exercise 5.3

1. (i) $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$; yes (ii) $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{10}{15}$; yes (iii) $\frac{4}{12}, \frac{2}{6}, \frac{3}{9}, \frac{1}{3}$; yes

2. (i) $\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20}$ (ii) $\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20} = \frac{15}{25}$

(ii) $\frac{7}{9} = \frac{14}{18} = \frac{21}{27} = \frac{28}{36} = \frac{35}{45}$ (iv) $\frac{5}{11} = \frac{10}{22} = \frac{15}{33} = \frac{20}{44} = \frac{25}{55}$

(v) $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15}$

3. (i) $\frac{2}{5}$ (ii) $\frac{1}{2}$ (iii) $\frac{2}{3}$ (iv) $\frac{5}{2}$ (v) $\frac{9}{5}$ **4.** (i) yes (ii) yes (iii) yes

5. (i) 42 (ii) 56 (iii) 9 (iv) 5 (v) 24

6. (i) $\frac{18}{30}$ (ii) $\frac{12}{20}$ (iii) $\frac{24}{40}$ **7.** (i) $\frac{6}{10}$ (ii) $\frac{48}{80}$ (iii) $\frac{12}{20}$

Exercise 5.4

1. $\frac{3}{7}, \frac{2}{7}, \frac{6}{7}$; $\frac{5}{11}, \frac{3}{11}, \frac{1}{11}$; $\frac{6}{13}, \frac{2}{13}, \frac{5}{13}, \frac{10}{13}$ **3.** $\frac{1}{8}, \frac{1}{9}, \frac{1}{7}$

4. (i) < (ii) > (iii) > (iv) < (v) <

5. (i) > (ii) < (iii) > (iv) > (v) <

6. (i) > (ii) > (iii) < (iv) < (v) =

7. (i) $\frac{3}{10}, \frac{5}{10}, \frac{7}{10}$ (ii) $\frac{1}{7}, \frac{4}{7}, \frac{6}{7}$ (iii) $\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}$ (iv) $\frac{5}{9}, \frac{5}{7}, \frac{5}{3}$

(v) $\frac{3}{13}, \frac{3}{11}, \frac{3}{7}$ (vi) $\frac{1}{6}, \frac{1}{4}, \frac{5}{12}$ (vii) $\frac{2}{7}, \frac{11}{35}, \frac{13}{28}, \frac{8}{14}$ (viii) $\frac{4}{15}, \frac{1}{3}, \frac{5}{12}, \frac{4}{9}$

(ix) $\frac{3}{16}, \frac{3}{8}, \frac{7}{12}, \frac{2}{3}$ (x) $\frac{2}{9}, \frac{7}{12}, \frac{11}{18}, \frac{5}{6}$

8. (i) $\frac{7}{9}, \frac{5}{9}, \frac{1}{9}$ (ii) $\frac{7}{11}, \frac{5}{11}, \frac{3}{11}, \frac{2}{11}$ (iii) $\frac{2}{7}, \frac{2}{9}, \frac{2}{13}$ (iv) $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{8}$

(v) $\frac{2}{3}, \frac{5}{12}, \frac{5}{18}, \frac{1}{6}$ (vi) $\frac{3}{4}, \frac{11}{15}, \frac{17}{30}, \frac{9}{20}$

9. Car

10. $\frac{1}{6}, \frac{1}{5}, \frac{3}{10}$

Exercise 5.5

1. (i) $\frac{5}{7}$ (ii) $\frac{6}{11}$ (iii) $\frac{11}{13}$ (iv) $\frac{17}{14}$ (v) $\frac{11}{12}$ (vi) $\frac{7}{12}$ (vii) $\frac{17}{30}$

(viii) $\frac{5}{8}$ (ix) $\frac{41}{9}$ (x) $\frac{31}{21}$ (xi) 2 (xii) $\frac{14}{15}$

2. (i) $\frac{1}{3}$ (ii) $\frac{3}{17}$ (iii) $\frac{2}{5}$ (iv) $\frac{3}{13}$ (v) $\frac{1}{6}$ (vi) $\frac{2}{5}$ (vii) $\frac{4}{21}$

(viii) $\frac{7}{12}$ (ix) $\frac{19}{9}$ (x) $\frac{13}{7}$ (xi) $\frac{29}{28}$ (xii) $\frac{1}{16}$

3. (i) $6\frac{3}{5}$ (ii) $7\frac{11}{12}$ (iii) $9\frac{1}{6}$ (iv) $2\frac{11}{12}$ (v) $1\frac{17}{30}$ (vi) $1\frac{1}{2}$ (vii) $13\frac{11}{12}$

(viii) $3\frac{3}{16}$ (ix) $8\frac{5}{12}$ (x) $1\frac{9}{16}$ (xi) $7\frac{1}{2}$ (xii) $1\frac{5}{24}$

4. $2\frac{5}{21}$ m 5. $1\frac{13}{30}$ kg 6. $\frac{7}{10}$ 7. $\frac{3}{4}$ 8. $3\frac{1}{3}$

Exercise 5.6

1. (i) $\frac{4}{5}$ (ii) $\frac{6}{7}$ (iii) $\frac{5}{4}$ (iv) $\frac{7}{3}$ (v) 8 2. (i) $\frac{1}{20}$ (ii) $\frac{1}{5}$ (iii) $\frac{5}{24}$ (iv) $\frac{3}{7}$ (v) $\frac{2}{15}$

Multiple Choice Questions

1. a 2. c 3. b 4. c 5. d 6. a 7. b 8. a 9. c 10. b
11. b 12. c 13. a 14. d 15. b 16. c 17. a 18. b 19. d 20. b

L7Y6Y4

6

DECIMALS

Objectives

In this chapter, you will learn

1. To understand about decimals places.
2. To provide knowledge of addition and subtraction of decimals.
3. To use decimals in length, capacity and weight etc.
4. To use decimals in daily life problems.

6.1 Introduction

The word decimal comes from Latin word **"Decem"** meaning 10. We have learnt about decimals in earlier classes. In this class, we shall study decimals as an extension of place value table as fractions and addition and subtraction of decimals.

6.2 Decimal Number

Consider the number 2145. the number 2145 in terms of place value can be written as

$2145 = 2000 + 100 + 40 + 5 = 2 \times 1000 + 1 \times 100 + 4 \times 10 + 5 \times 1$

It is observed that in the place value table each place value is 10 times of the next place value on its right. For example, place value of tens is 10 times the place value of ones. Place value of hundreds is 10 times of the place value of tens and place value of thousands is 10 times of the place value of hundreds and it continuous in the same way.

We can notice that as we move from left to right, the place value is divided by 10. If we extend this system to the right of units place digit 5 (as in above example), the place value of the digit to the right of 5 (unit digit) will be **tenths** i.e. $\frac{1}{10}$, the place value of next digit will be **hundredths** i.e. $\frac{1}{100}$ and next will be **thousandths** i.e. $\frac{1}{1000}$ and so on.

In such a situation, the place value table will take the following shape:

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
(1000)	(100)	(10)	(1)	$\left(\frac{1}{10}\right)$	$\left(\frac{1}{100}\right)$	$\left(\frac{1}{1000}\right)$

Consider a number 5432.167

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
(1000)	(100)	(10)	(1)	$\left(\frac{1}{10}\right)$	$\left(\frac{1}{100}\right)$	$\left(\frac{1}{1000}\right)$
5	4	3	2	1	6	7

The number 5432.167 in terms of place value can be written as 5432 + . 167

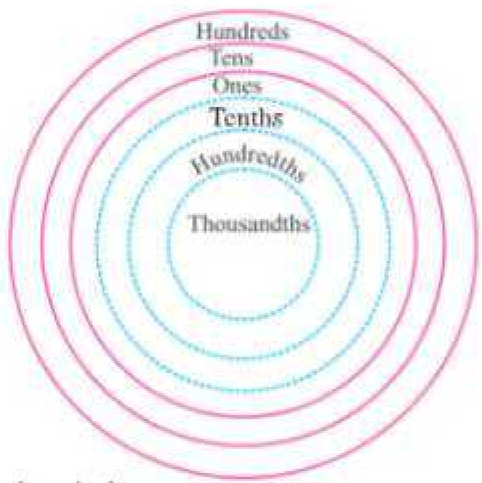
$$\underbrace{5\times1000+4\times100+3\times10+2\times1}_{\text{whole number part}}+\underbrace{1\times\frac{1}{10}+6\times\frac{1}{100}+7\times\frac{1}{1000}}_{\text{fractional part}}$$

To separate whole number and fractional part of the number, we put small dot in between which is called **decimal**.

The number is read as *"Five thousand four hundred thirty two point one six seven."*

Or *Five thousand four hundred thirty two and one hundred sixty seven thousandths."*

ACTIVITY



Students, Let's play an activity with numbers. Draw concentric circles on a card board (as shown). Write ones, tens, hundreds and tenths, hundredths, thousandths so on.

Take some marbles and throw these marbles gently on the this cardboard. Let us suppose that the marbles settle themselves in the place value circles as shown in following figure.

Observe the number of marbles in different place value circles.

Number of marbles in hundreds circle = 6

So place value = 6 ×100 = 600

Number of marbles in tens circle = 2

Place value = 2 × 10 = 20

Number of marbles in ones circle = 4

Place value = 4 × 1 = 4

Number of marbles in tenths circle = 1

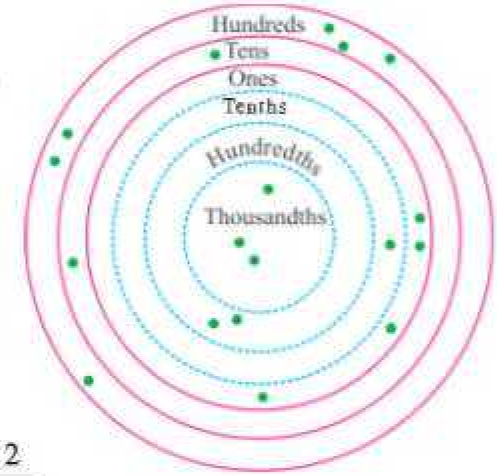
Place value = $1 \times \frac{1}{10} = \frac{1}{10}$

Number of marbles in hundredths circle = 2

Place value = $2 \times \frac{1}{100} = \frac{2}{100}$

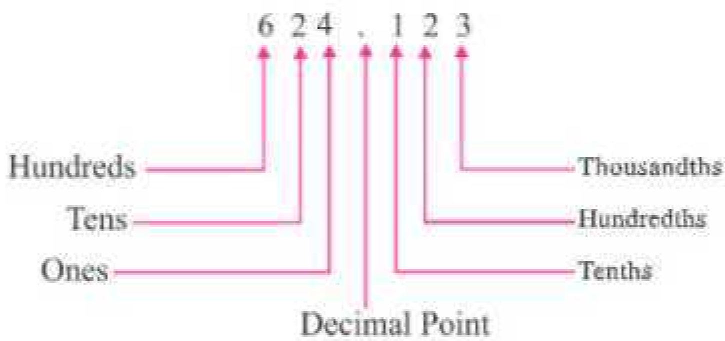
Number of marbles in thousandths circle = 3

Place value = $3 \times \frac{1}{1000} = \frac{3}{1000}$



Hence Decimal number = $600 + 20 + 4 + \frac{1}{10} + \frac{2}{100} + \frac{3}{1000}$

= 624.123



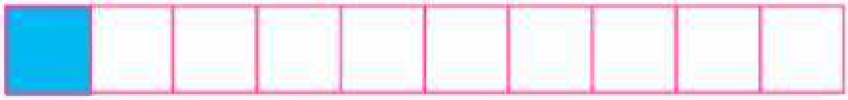
Now we shall discuss decimal fractions : tenths, hundredths, thousandths etc. in detail.

6.3 Decimal Fractions

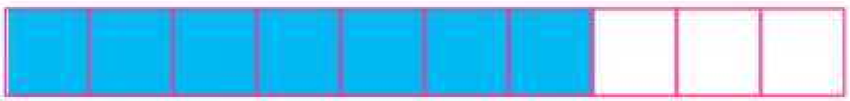
In the previous section, we have learnt that decimals are an extension of our number system. In this section, we shall see that decimals are another name of fractions whose denominators are 10,100, 1000 etc. Let us first define tenths, hundredths, thousandths etc. as fractions.

6.3.1 Tenths

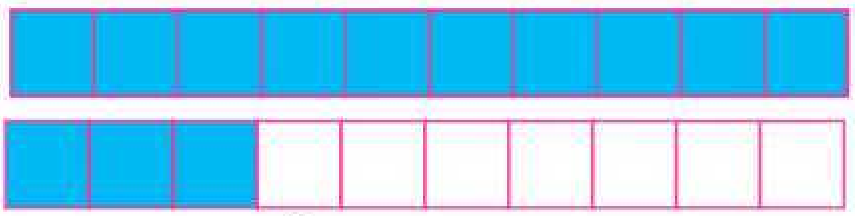
⇒ Consider a rectangle divided into ten equal parts and shade one part. The shaded part represents one-tenths of the whole figure. It is written as $\frac{1}{10}$ or .1, which is read as 'one tenth' or 'point one' or 'decimal one'.



⇒ The following figure is divided into ten equal parts and seven parts are shaded. The shaded part represents seven-tenths of the whole figure. It is written as $\frac{7}{10}$ or .7 and is read as 'point seven' or 'decimal seven'.



⇒ Similarly, In the following figure, one whole rectangle is shaded and 3 parts of another same rectangle are shaded.



The shaded part is written as $1\frac{3}{10}$ = 1.3 and read as 'one point 3' or 'one and three tenths'.

So, $.1 = \frac{1}{10} = 1$ tenths

Let's illustrate some examples:

Example 1. **Write each of the following as decimals:**

(i) Seven and three tenths **(ii) Two tenths**

(iii) Twenty four and one tenth

Solution : (i) Seven and three tenths $= 7 + \frac{3}{10}$

$= 7\frac{3}{10} = 7.3$

(ii) Two tenths $= \frac{2}{10} = .2$

(iii) Twenty four and one tenth $= 24\frac{1}{10} = 24.1$

Example 2. **Write each of the following as decimals:**

(i) $10 + 3 + \frac{2}{10}$ **(ii)** $200 + 7 + \frac{5}{10}$ **(iii)** $\frac{9}{10}$

Solution : (i) $10 + 3 + \frac{2}{10}$

There are 1 tens, 3 ones and 2 tenths.

$\therefore \quad 10 + 3 + \frac{2}{10} = 13 + \frac{2}{10} = 13.2$

(ii) $200 + 7 + \frac{5}{10}$

There are 2 hundreds, 7 ones and 5 tenths

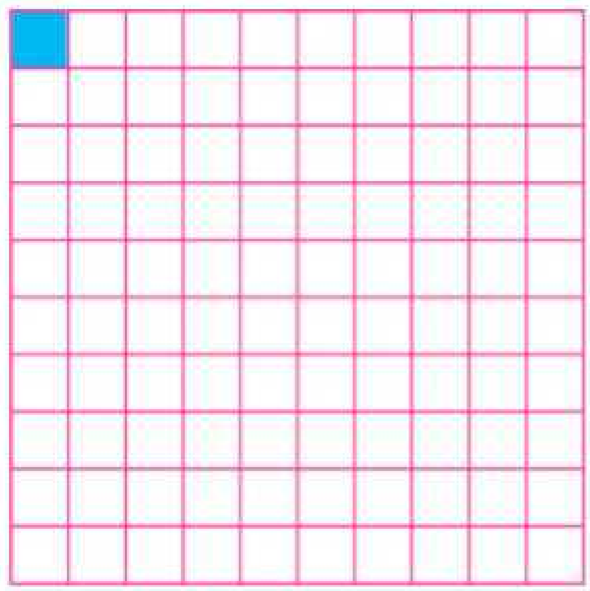
$\therefore \quad 200 + 7 + \frac{5}{10} = 207 + \frac{5}{10} = 207.5$

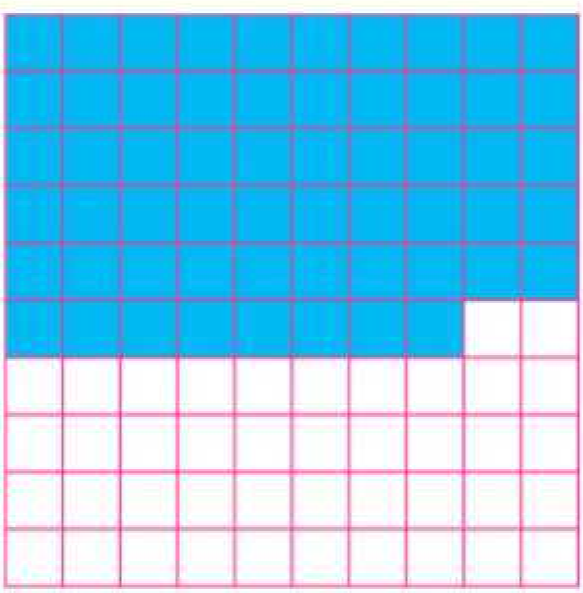
(iii) $\frac{9}{10}$, There are only 9 tenths

$\therefore \quad \frac{9}{10} = .9$

6.3.2 Hundredths

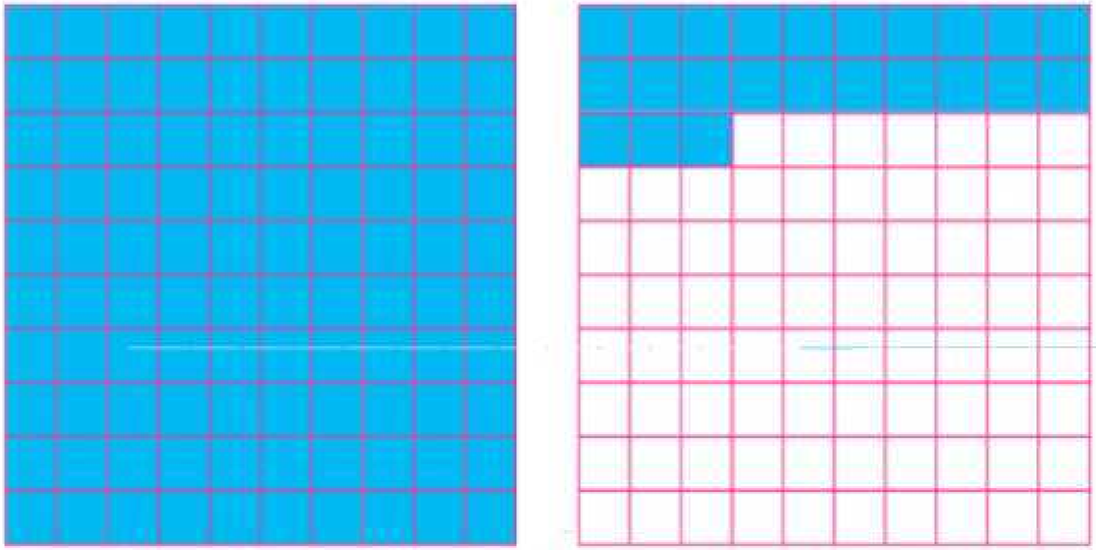
⇒ In the following figure, a square is divided into 100 equal parts and 1 part is shaded. Thus, the shaded part represents one-hundredths of the whole figure and is written as $\frac{1}{100}$ or .01 and read as 'one-hundredth' or 'point zero one' or 'decimal zero one'.





⇒ In the figure, a square is divided into 100 equal parts out of which 58 are shaded. Then the shaded part represents fifty-eight hundredths of the whole and is written as $\frac{58}{100}$ or .58 and read as 'point five eight' or 'decimal five eight'.

⇒ Similarly, In the figure, one whole square is shaded and 23 parts of other similar squares are shaded.



The shaded part is written as $1\frac{23}{100}$ or 1.23 and read as 'one point two three' or 'one and twenty three hundredths.

6.3.3 Thousandths

If an object is divided into 1000 equal parts then each part is one-thousandths of the whole. It is written as $\frac{1}{1000}$ or .001 and read as 'point zero zero one.'

- If we take 19 parts out of 1000 equal parts than 19 parts make $\frac{19}{1000}$ of the whole and written as .019 and read as 'point zero one nine'.

  Similarly we have,

$$\frac{102}{1000} = .102, \ \frac{519}{1000} = .519, \ \frac{1439}{1000} = 1.439$$

$$\frac{12508}{1000} = 12.508 \text{ etc.}$$

How to mark decimal ?

⇒ A fraction of the form $\frac{\text{Number}}{10}$ is written as a decimal obtained by putting decimal point by leaving **one right most digit**.

e.g. $\frac{34}{10} = 3.4$

$\Rightarrow$ A fraction of the form $\frac{\text{Number}}{100}$ is written as a decimal obtained by putting decimal point by leaving **two right most digits**. If the number is short of digits, insert zeroes at the left of the number.

e.g. (i) $\frac{1258}{100} = 12.58$ (ii) $\frac{6}{100} = .06$

$\Rightarrow$ A fraction of the form $\frac{\text{Number}}{1000}$ is written as a decimal obtaind by putting decimal point by leaving three right most digits. If the number is short of digits, insert zeros at the left of the number.

e.g. (i) $\frac{2345}{1000} = 2.345$ (ii) $\frac{16}{1000} = .016$

In decimals, we have some distinctions which are as follows:

(i) A decimal number may contain a whole number and a decimal part .4, .23, 6.25 etc.

(ii) If the decimal number consists only decimal part then zero can be written in the whole part

i.e. .3 = 0.3

.05 = 0.05 etc.

(iii) If the decimal numbers consists only whole part then zero can be written in the decimal part.

i.e 2 = 2.0

40 = 40.0 etc.

Let us consider some examples to understand the basic idea of decimals:

Example 3. **Write each of the following in numbers.**

(i) Twenty four point five

(ii) Sixty nine point three eight

(iii) One thousand fifty two point zero seven

(iv) Zero point six zero nine.

(v) Two point zero zero one.

Solution : (i) Twenty four point five = 24.5

(ii) Sixty nine point three eight= 69.38

(iii) One thousand fifty two point zero seven = 1052.07

(iv) Zero point six zero nine = 0.609

(v) Two point zero zero one = 2.001

Example 4. **Write each of the following in figures.**

(i) Five and seven tenths

(ii) Twenty nine and sixty one hundredths

(iii) Eighty two and one hundred fifty two thousandths

(iv) Four hundredths

(v) Seventy and two thousandths

Solution : (i) Five and seven tenths = $5+\frac{7}{10}=5.7$

(ii) Twenty nine and sixty one hundredths = $29+\frac{61}{100}=29.61$

(iii) Eighty two and one hundred fifty two thousandths = $82+\frac{152}{1000}=82.152$

(iv) Four hundredths $=\frac{4}{100}=0.04$

(v) Seventy and two thousandths = $70+\frac{2}{1000}=70.002$

Example 5. **Write the following decimals in the place value table:**

(i) 125.67 **(ii) 5.3** **(iii) 0.56**

(iv) 3.148 **(v) 10.007**

Solution :

Number	Thousands	Hundreds	Tens	ones	Tenths	Hundredths	Thousandths
125.67	–	1	2	5	6	7	–
5.3	–	–	–	5	3	–	–
0.56	–	–	–	0	5	6	–
3.148	–	–	–	3	1	4	8
10.007	–	–	1	0	0	0	7

Example 6. **Write the following decimal numbers in words:**

(i) 64.58 **(ii) 0.63** **(iii) 7.006** **(iv) 712.05**

(v) 0.725

Solution : (i) 64.58 = Sixty four point five eight

Or Sixty four and fifty eight hundredths

(ii) 0.63 = Zero point six three

Or Sixty three hundredths

(iii) 7.006 = Seven point zero zero six

Or Seven and six thousandths

(iv) 712.05 = Seven hundred twelve point zero five

Or Seven hundred twelve and five hundredths

(v) 0.725 = Zero point seven two five

Or Seven hundred twenty five thousandths

Example 7. **Write the decimals shown in the following place value table:**

	Thousands (1000)	Hundreds (100)	Tens (10)	Ones (1)	Tenth $\left(\frac{1}{10}\right)$	Hundredths $\left(\frac{1}{100}\right)$	Thousandth $\left(\frac{1}{1000}\right)$
(i)	–	5	6	0	3	4	–
(ii)	1	0	2	3	0	5	2
(iii)	2	1	5	0	0	0	6
(iv)	–	–	2	1	1	2	–
(v)	–	–	–	5	0	0	4

Solution : (i) We have $5 \times 100 + 6 \times 10 + 0 \times 1 + 3 \times \frac{1}{10} + 4 \times \frac{1}{100}$

$= 500 + 60 + 0 + \frac{3}{10} + \frac{4}{100} = 560.34$

(ii) We have $1 \times 1000 + 0 \times 100 + 2 \times 10 + 3\times 1 + 0 \times\frac{1}{10} + 5 \times\frac{1}{100} + 2\times\frac{1}{1000}$

$= 1000 + 0 + 20 + 3 + 0 + \frac{5}{100} + \frac{2}{1000}$

$= 1023.052$

(iii) We have $2\times 1000 + 1 \times 100 + 5 \times 10 + 0\times 1 + 0\times\frac{1}{10} + 0 \times\frac{1}{100} + 6 \times\frac{1}{1000}$

$= 2000 + 100 + 50 + 0 + 0 + 0 + \frac{6}{1000} = 2150.006$

(iv) We have $2\times 10 + 1 \times 1 + 1 \times \frac{1}{10} + 2 \times\frac{1}{100}$

$= 20 + 1 + \frac{1}{10} + \frac{2}{100} = 21.12$

(v) We have $5 \times 1 + 0 \times \frac{1}{10} + 0 \times \frac{1}{100} + 4\times \frac{1}{1000}$

$= 5 + 0 + 0 + \frac{4}{1000} = 5.004$

Example 8. **Write the following decimals in expanded form:**

(i) 5.6 (ii) 2.12 (iii) 14.89 (iv) 45.067 (v) 130.008

Solution : (i) 5.6 $= 5+.6 = 5 + \frac{6}{10}$

(ii) 2.12 $= 2 + .12 = 2 + .1 + .02$

$$= 2 + \frac{1}{10} + \frac{2}{100}$$

(iii) 14.89 $= 14 + .89$

$$= 10 + 4 + .8 + .09$$

$$= 10 + 4 + \frac{8}{10} + \frac{9}{100}$$

(iv) 45.067 $= 45 + .067$

$$= 40 + 5 + .06 + .007$$

$$= 40 + 5 + \frac{6}{100} + \frac{7}{1000}$$

(v) 130.008 $= 130 + .008$

$$= 100 + 30 + \frac{8}{1000}$$

Exercise 6.1

1. Write each of the following in figures:
   - (i) Seventy two point one four.
   - (ii) Two hundred fifty seven point zero eight
   - (iii) Eight point two five six.
   - (iv) Forty five and twenty three hundredths.
   - (v) Six hundred twenty one and two hundred fifty three thousandths
   - (vi) Twelve and eight thousandths.
2. Write the following decimal numbers in words:

   (i) 12.52 (ii) 7.148 (iii) 0.24 (iv) 5.018 (v) .009
3. Write the following decimals in the place value table:

   (i) 21.569 (ii) 0.64 (iii) 3.51 (iv) 14.087 (v) 3.002
4. Write the following as decimals:

   (i) $40 + \frac{2}{10}$ (ii) $700 + 5 + \frac{3}{10} + \frac{4}{100}$

   (iii) $10 + \frac{5}{100} + \frac{3}{1000}$ (iv) $\frac{7}{10} + \frac{4}{1000}$ (v) $\frac{5}{1000}$

5. Write the decimals shown in the following place value table:

	Thousands	Hundreds	Tens	Ones	Tenth	Hundredths	Thousandths
(i)	–	5	2	4	1	2	–
(ii)	2	0	3	4	2	1	–
(iii)	–	–	6	1	0	2	3
(iv)	–	–	–	4	0	0	1
(v)	–	1	0	0	0	3	

6. Expand the following decimals.

(i) 2.5 (ii) 18.43 (iii) 4.05 (iv) 13.123 (v) 245.456 (vi) 20.057

6.4 Conversion of decimals and fractions

In last section, we have learnt about reading and writing of decimals and their expanded form. Now we shall learn the conversion of decimals into fractions and vice versa.

6.4.1 Decimals into fractions

Consider a decimal say 2.3 which can be written as:

$$2.3 = 2 + .3 = 2 + \frac{3}{10}$$

$$= \frac{20}{10} + \frac{3}{10} = \frac{20+3}{10} = \frac{23}{10}$$

i.e. $2.3 = \frac{2.3}{1} = \frac{23}{10}$

$$2.3 = \frac{23}{10} = \frac{\text{Number without decimal}}{\text{1 in place of decimal followed by as many zeroes as the number of digits after decimal}}$$

Example 9. Convert the following decimals into fraction and reduce it to its lowest terms.

(i) 2.5 (ii) 1.52 (iii) .006 (iv) 24.6 (v) 4.32

Solution : (i) 2.5

$$2.5 = \frac{25}{10} = \frac{25 \div 5}{10 \div 5} = \frac{5}{2} \quad \text{(HCF of 25 and 10 = 5)}$$

Here, numerator of the fraction is the given number without decimal, i..e 25. since the number of digits after decimals in 2.5 is 1, So the denominator of the fraction is 1 followed by one zero.

(ii) 1.52

$$1.52 = \frac{152}{100} = \frac{152 \div 4}{100 \div 4} \text{ (HCF of 152 and 100 is 4)}$$

$$= \frac{38}{25}$$

Here, numerator of the fraction is the given number without decimal i.e. 152 since the number of digits after decimals in 1.52 are 2, So the denominator of the fraction is 1 followed by two zeros.

(iii) $.006 = \frac{6}{1000} = \frac{6 \div 2}{1000 \div 2}$ (HCF of 6 and 1000 is 2)

$$= \frac{3}{500}$$

(iv) $24.6 = \frac{246}{10} = \frac{246 \div 2}{10 \div 2}$ (HCF of 246 and 10 is 2)

$$= \frac{123}{5}$$

(v) $4.32 = \frac{432}{100} = \frac{432 \div 4}{100 \div 4}$ (HCF of 432 and 100 is 4)

$$= \frac{108}{25}$$

6.4.2 Fractions into decimals

As we have learnt in last section that the fractions with denominators 10, 100 or 1000 can easily be converted into decimals.

e.g. $\frac{43}{10} = 4.3$; $\frac{125}{100} = 1.25$; $\frac{65}{1000} = 0.065$

$\frac{2143}{1000} = 2.143$; $\frac{619}{100} = 6.19$ etc

> To convert a fraction with denominator 10,100 or 1000 into decimal, place the decimal point (from right to left) in the numerator after as many digits as there are zeroes (after) in the denominator.

But some fractions have denominators other than 10,100 or 1000, those can be changed into fractions with denominators 10,100 or 1000 by finding their equivalent fractions or by division method.

In this class, we shall learn only that fractions whose denominators are multiples of 2 or 5 or both.

Equivalent Fractions Method

⇒ Consider an example, say $\frac{3}{5}$

Here denominator is 5. So we need to convert it into 10,100 or 1000. We know that after multiplying 5 by 2, we get 10.

$$\therefore \quad \frac{3}{5} = \frac{3\times 2}{5\times 2} = \frac{6}{10} = 0.6$$

Now consider $\frac{5}{4}$ Here denominatior is 4, so we need to convert it into 10, 100 or 1000. We know that after multiplying 4 by 25, we get 100

$$\therefore \quad \frac{5}{4} = \frac{5\times 25}{4\times 25} = \frac{125}{100} = 1.25$$

Division Method

As above method is quite difficult in many cases when we have denominator 8, 16 or 40 etc. So we have another method, Division method which is quite easy. Consider an example say, $\frac{14}{5}$

We know, Here dividend = 14

And divisor = 5

```
     2
5 ) 14 (
  - 10
  ----
     4
```

Step 1. Divide 14 by 5 we know quotient and remainder will be 2 and 4 respectively.

```
     2
5 ) 14.0 (
  - 10
  ----
     4
```

Step 2. In dividend, insert decimal point and put zero right to it i.e. 14 = 14.0

Step 3: Take 0 of 14.0 down to the remainder 4 which becomes 40.

Step 4 : In quotient, put decimal point after 2 i.e. 2.

Step 5. Now divide 40 by 5, we have quotient 8.

```
     2.8
5 ) 14.0 (
  - 10 ↓
  ------
     4.0
    -4 0
  ------
       0
```

i.e. $\frac{14}{5} = 2.8$

Let's consider some examples.

Example 10. Convert the following fractions into decimals by equivalent fraction method.

(i) $\frac{5}{10}$ **(ii)** $\frac{423}{100}$ **(iii)** $\frac{9}{1000}$ **(iv)** $\frac{15}{2}$ **(v)** $\frac{12}{25}$

(vi) $\frac{23}{20}$

Solution : (i) $\frac{5}{10}$ = .5 or 0.5 (Here denominator is 10)

(ii) $\frac{423}{100}$ = 4.23 (Here denominator is 100)

(iii) $\frac{9}{1000}$ = .009 or 0.009 (Here denominator is 1000)

(iv) $\frac{15}{2}$

Here denominator is 2, convert into equivalent fraction with denominator 10 by multiplying it by 5.

$$\therefore \quad \frac{15}{2}=\frac{15\times 5}{2\times 5}=\frac{75}{10}=7.5$$

(v) $\frac{12}{25}$

Here denominator is 25.

Convert into equivalent fraction with denominator 100 by multiplying it by 4.

$$\therefore \quad \frac{12}{25}=\frac{12\times 4}{25\times 4}=\frac{48}{100}=.48 \text{ or } 0.48$$

(vi) $\frac{23}{20}$

Here denominator is 20.

Convert into equivalent fraction with denominator 100 by multiplyig it by 5.

$$\therefore \quad \frac{23}{20}=\frac{23\times 5}{20\times 5}=\frac{115}{100}=1.15$$

Example 11. **Convert the following fraction into decimal by division method:**

(i) $\frac{13}{2}$ **(ii)** $\frac{34}{5}$ **(iii)** $\frac{47}{4}$ **(iv)** $\frac{21}{8}$ **(v)** $\frac{18}{25}$

Solution : (i) $\frac{13}{2}=6.5$

$$\begin{array}{r} 6.5 \\ 2\,\overline{)\,13.0} \\ -12 \\ \hline 1.0 \\ -\,1.0 \\ \hline 0 \end{array}$$

(ii) $\frac{34}{5}=6.8$

$$\begin{array}{r} 6.8 \\ 5\,\overline{)\,34.0} \\ -30 \\ \hline 4.0 \\ -\,4.0 \\ \hline 0 \end{array}$$

(iii) $\frac{47}{4} = 11.75$

```
       11.75
   4 ) 47.00
      -4
       7
      -4
       30
      -28
        20
       -20
         0
```

(iv) $\frac{21}{8} = 2.625$

```
       2.625
   8 ) 21.000
      -16
        50
       -48
         20
        -16
          40
         -40
           0
```

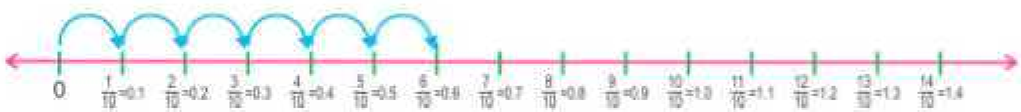
(v) $\frac{18}{25} = 0.72$

```
        0.72
   25 ) 18.00
        -0
        180
       -175
          50
         -50
           0
```

6.5 Representation of Decimals on Number line

We have learnt to represent natural numbers, whole numbers, integers and fractions on number line. Here we shall represent decimals on a number line. Representation of decimal is similar like of fractions.

$\Rightarrow$ Let's represent .6 or $\frac{6}{10}$ on number line. The fraction $\frac{6}{10}$ is smaller than 1 but greater than 0. So we divide the distance from 0 to 1 on the number line into 10 equal parts and count 6 steps starting from 0 towards the right.



Since $\frac{6}{10} = 0.6$, 0.6 represents the same point on the number line as $\frac{6}{10}$.

⇒ Now we represent 1.3 on the number line. We know that 1.3 = 1 + .3 i.e. 1 + 3 tenths is greather than 1 but smaller than 2. So we start from 1 and count 3 steps towards the right.

Example 12. **Represent the following decimals on the number line:**

(i) 0.4 (ii) 2.8 (iii) 4.5

Solution : (i) 0.4 lies between 0 and 1.

(ii) 2.8 lies between 2 and 3.

(iii) 4.5 lies between 4 and 5.

6.6 Like and Unlike decimals

As we know the number of digits contained in the decimal part of a decimal number gives the number of decimal places.

e.g. ⇒ 5.34 has two decimal places.

⇒ 4.156 has three decimal places.

⇒ 42.01 has two decimal places.

Like Decimals : The decimals with the same number of decimal places are called like decimals. e.g. 2.56, 42.01, 1.68,2.30 are like decimals, each having two places of decimals.

Unlike Decimals: The decimals having different number of decimal places are called unlike decimals e.g. 2.1, 3.14, 42.356 are unlike decimal as they contain one, two, three decimal places respectively.

Now, convert all unlike decimals into like decimals by putting zeroes at the end of the decimal number so that all of them have same number of decimal places.

i.e. 2.1 = 2.100 ; 3.14 = 3.140; 42.356 are all like decimals.

Adding extra zeroes to the right of a decimal does not change its value

i.e $\quad 2.5 = 2.50 = 2.500$

6.7 Comparing Decimals

To compare decimals following steps are followed:

Step 1 : Convert unlike decimals to like decimals.

Step 2 : Compare the whole number part. Number with greater whole number part will be the greater decimal number.

e.g. 6.8 and 5.9

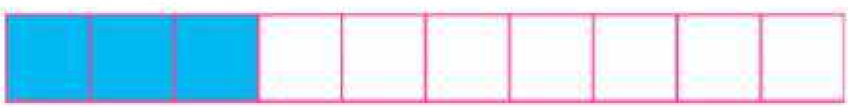
6>5

$\therefore \quad 6.8 > 5.9$

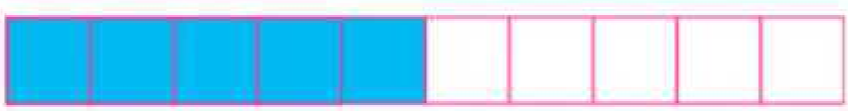
Step 3: If the whole number part is equal then compare the digits in the tenths place, the decimal number having greater number at the tenths place will be greater.

e.g. (i) Compare 0.3 and 0.5

$0.3 = \frac{3}{10}$ i.e. 3 out of 10 parts are shaded



and $\quad 0.5 = \frac{5}{10}$ i.e. 5 out of 10 parts are shaded.



$\therefore 0.5 > 0.3$

As

same

0.3 $\quad$ 0.5

3<5

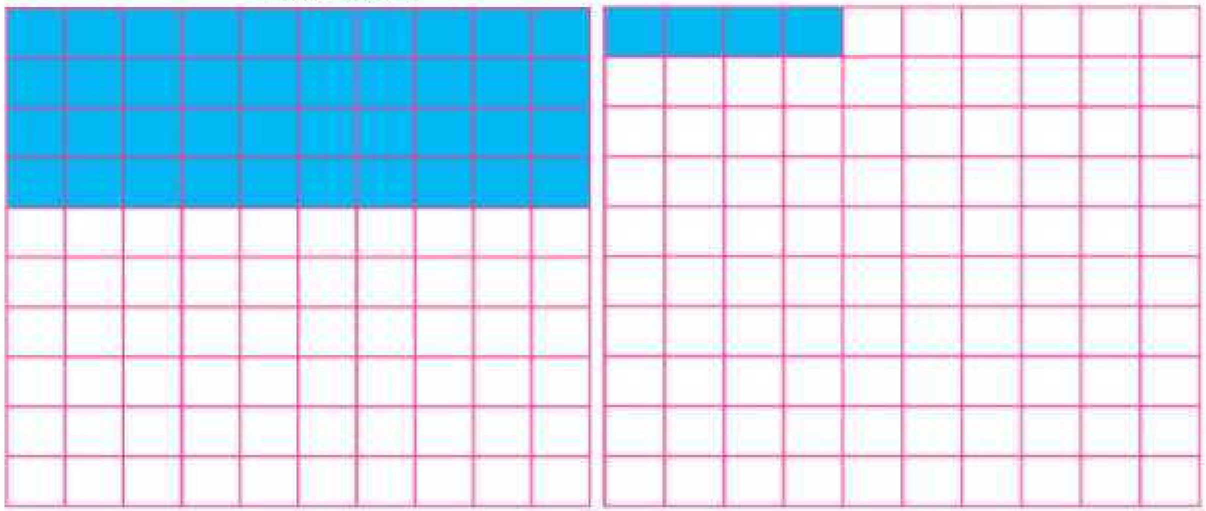
$\therefore \quad 0.3 < 0.5$ or $0.5 > 0.3$

(ii) Now compare 0.4 and 0.04

$0.4 = \frac{4}{10} = \frac{40}{100}$ i.e. 40 out of 100 parts are shaded.

and $0.04 = \frac{4}{100}$ i.e. 4 out of 100 parts are shaded.

Now $40 > 4$



Thus $\frac{40}{100} > \frac{4}{100}$ or $.4 > .04$

Step 4 : If the digits in tenths place are also equal then compare the digits in the hundredths place and so on.

Example 13. Which is greater?

(i) 1.4 and 0.5 **(ii) 3.18 and 13.28**

(iii) 4.3 and 4.03 **(iv) 5.168 and 5.169** **(v) 24.3 and 24.31**

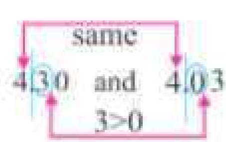
Solution : (i)

Since 1>0, so $1.4 > 0.5$

(ii)

Since 3<13, so $3.18 < 13.28$

(iii)

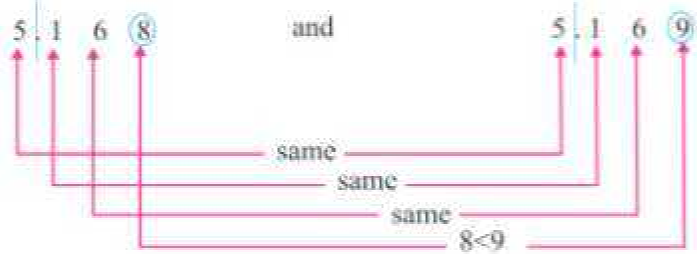


(Convert 4.3 into 4.30)

Since whole part is same then comparing tenths digit, $3 > 0$

$\Rightarrow \quad 4.3 > 4.03$

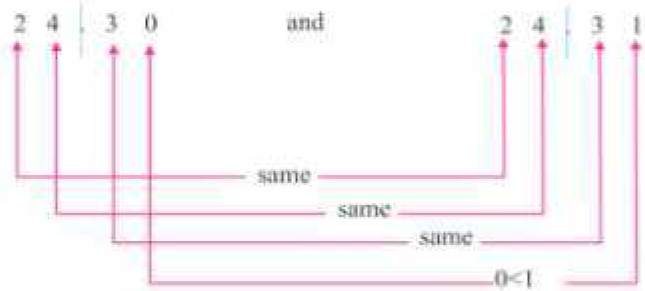
(iv)



Comparing the thousandths digit $8<9$.

$\Rightarrow \quad 5.169 > 5.168$

(v)



Comparing hundredths digit, $0<1$

$\Rightarrow \quad 24.31 > 24.3$

Exercise 6.2

1. Convert the following decimal numbers into fractions and reduce it to lowest form.

(i) 1.4 (ii) 2.25 (iii) 18.6 (iv) 4.04 (v) 21.6

2. Convert the following fractions into decimal numbers :

(i) $\frac{7}{100}$ (ii) $\frac{12}{10}$ (iii) $\frac{215}{100}$ (iv) $\frac{18}{1000}$ (v) $\frac{245}{10}$

3. Convert the following fractions into decimal numbers by equivalent fraction method:

(i) $\frac{5}{2}$ (ii) $\frac{3}{4}$ (iii) $\frac{28}{5}$ (iv) $\frac{135}{20}$ (v) $\frac{17}{4}$

4. Convert the following fractions into decimals by long division method:

(i) $\frac{17}{2}$ (ii) $\frac{33}{4}$ (iii) $\frac{76}{5}$ (iv) $\frac{24}{25}$ (v) $\frac{5}{8}$

5. Represent the following decimals on number line:

(i) 0.7 (ii) 1.6 (iii) 3.7 (iv) 6.3 (v) 5.4

6. Write three decimal numbers between:

(i) 1.2 and 1.6 (ii) 2.8 and 3.2 (iii) 5 and 5.5

7. Which number is greater:

(i) 0.4 or 0.7 (ii) 2.6 or 2.5 (iii) 1.23 or 1.32

(iv) 12.3 or 12.4 (v) 18.35 or 18.3 (vi) 12 or 1.2

(vii) 5.06 or 5.061 (viii) 2.34 or 23.3 (ix) 13.08 or 13.078

(x) 2.3 or 2.03

8. Arrange the decimal numbers in ascending order:

(i) 2.5, 2, 1.8, 1.9 (ii) 3.4, 4.3, 3.1, 1.3

(iii) 1.24, 1.2, 1.42, 1.8

9. Arrange the deecimal numbers in descending order:

(i) 4.1, 4.01, 4.12, 4.2 (ii) 1.3, 1.03, 1.003, 13

(iii) 8.02, 8.2, 8.1, 8.002

6.8 Use of decimals in daily life

Decimals are very useful in our daily life in form of money, weight, capacity etc. In this section, we shall learn about use of decimals in different fields of our life.

6.8.1 CURRENCY (Money)

Conversion of paise into rupees :

In India, money is expressed in rupees and paise.

i.e. 100 paise = ₹1

So 1 paisa is one hundredth of a rupee.

i.e. 1 paisa = ₹ $\frac{1}{100}$ = ₹ 0.01

Similarly 2 paise = ₹ $\frac{2}{100}$ = ₹0.02

5 paise = ₹ $\frac{5}{100}$ = ₹0.05

45 paise = ₹ $\frac{45}{100}$ = ₹0.45

Let's consider some examples:

Example 14: **Write the following money in rupees using decimals:**

(i) 60 paise **(ii) 125 paise** **(iii) 5 rupees 50 paise**

(iv) 18 rupees 99 paise **(v) 25 rupees 5 paise**

Solution : (i) 60 paise $= ₹\frac{60}{100} = ₹0.60$ $(\because 1 \text{ paisa} = ₹\frac{1}{100})$

(ii) 125 paise $= ₹\frac{125}{100} = ₹1.25$ $(\because 1 \text{ paisa} = ₹\frac{1}{100})$

(iii) 5 rupees 50 paise

$= (5 \text{ rupees}) + (50 \text{ paise})$

$= ₹5 + ₹\frac{50}{100} = ₹5 + ₹0.50 = ₹5.50$ $(\because 1 \text{ paisa} = ₹\frac{1}{100})$

(iv) 18 rupees 99 paise

$= (18 \text{ rupees}) + (99 \text{ paise})$

$= ₹18 + ₹\frac{99}{100}$ $(\because 1 \text{ paisa} = ₹\frac{1}{100})$

$= ₹18 + ₹0.99 = ₹18.99$

(v) 25 rupees 5 paise

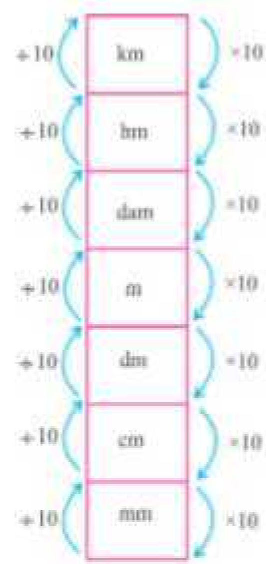
$= (25 \text{ rupees}) + (5 \text{ paise})$

$= ₹25 + ₹\frac{5}{100}$ $(\because 1 \text{ paisa} = ₹\frac{1}{100})$

$= ₹25 + ₹0.05 = ₹25.05$

6.8.2 Length or Distance

Conversion of centimetre into metre :



We know that 100cm = 1 metre

$1\text{cm} = \frac{1}{100}\text{m} = 0.01\text{m}$

Similarly $2\text{cm} = \frac{2}{100}\text{m} = 0.02\text{m}$

$7\text{cm} = \frac{7}{100}\text{m} = 0.07\text{m}$

$35\text{cm} = \frac{35}{100}\text{m} = 0.35\text{m}$

Example 15. **Express as metre using decimal.**

(i) 4cm **(ii) 185cm** **(iii) 3m 32cm**

Solution : (i) $4\text{cm} = \frac{4}{100}\text{m} = 0.04\text{m}$ $(\because 1\text{cm} = \frac{1}{100}\text{m})$

(ii) $185\text{cm} = \frac{185}{100}\text{m} = 1.85\text{m}$ ($\because$ $1\text{cm} = \frac{1}{100}\text{m}$)

(iii) $3\text{m } 32\text{cm} = 3\text{m} + 32\text{cm}$

$= 3\text{m} + \frac{32}{100}\text{m}$ ($\because$ $1\text{cm} = \frac{1}{100}\text{m}$)

$= 3\text{m} + 0.32\text{m} = 3.32\text{m}$

Conversion of millimere into centimetre

We know that $10\text{mm} = 1\text{ cm}$

$\Rightarrow$ $1\text{mm} = \frac{1}{10}\text{cm} = 0.1\text{ cm}$

Similarly $3\text{mm} = \frac{3}{10}\text{cm} = 0.3\text{cm}$

$8\text{mm} = \frac{8}{10}\text{ cm} = 0.8\text{cm}$

Example 16. Express as centimetre using decimals:

(i) 5mm (ii) 28mm (iii) 5cm 3mm

Solution : (i) $5\text{mm} = \frac{5}{10}\text{ cm} = 0.5\text{cm}$ ($1\text{mm} = \frac{1}{10}\text{cm}$)

(ii) $28\text{mm} = \frac{28}{10}\text{cm} = 2.8\text{cm}$ ($1\text{mm} = \frac{1}{10}\text{cm}$)

(iii) $5\text{cm } 3\text{mm} = 5\text{cm} + 3\text{mm}$

$= 5\text{cm} + \frac{3}{10}\text{ cm} = 5\text{ cm} + 0.3\text{ cm}$

$= 5.3\text{cm}$

Conversion of metre into kilometre

- We know that $1000\text{m} = 1\text{km}$

$\Rightarrow$ $1\text{m} = \frac{1}{1000}\text{km} = 0.001\text{km}$

Similarly $42\text{m} = \frac{42}{1000}\text{km} = 0.042\text{km}$

$180\text{m} = \frac{180}{1000}\text{km} = 0.180\text{km}$

Example 17. Express as kilometre using decimals:

(i) 35m (ii) 1250m (iii) 5km 45m

Solution : (i) $35\text{m} = \frac{35}{1000}\text{km} = 0.035\text{km}$ ($\because$ $1\text{cm} = \frac{1}{1000}\text{km}$)

(ii) $1250\text{m} = \frac{1250}{1000}\text{km} = 1.250\text{km}$ $\quad (\because \; 1\text{m} = \frac{1}{1000}\text{km})$

(iii) $5\text{km } 45\text{m} = 5\text{km} + 45\text{m}$

$= 5\text{km} + \frac{45}{1000}\text{km}$ $\quad (\because \; 1\text{m} = \frac{1}{1000}\text{km})$

$= 5\text{km} + 0.045\text{ km} = 5.045\text{km}$

6.8.3 Weight

Conversion of grams into kilograms :

We know that $1000\text{g} = 1\text{kg}$

$1\text{g} = \frac{1}{1000}\text{kg} = 0.001\text{kg}$

Similarly $8\text{g} = \frac{8}{1000}\textbf{kg} = 0.008\text{ kg}$

$72\text{g} = \frac{72}{1000}\text{kg} = 0.072\text{kg}$

$430\text{g} = \frac{430}{1000}\text{kg} = 0.430\text{kg}$

Example 18. Express as kilogram using decimals:

(i) 3g (ii) 765g (iii) 4kg 80g

Solution : (i) $3\text{g} = \frac{3}{1000}\text{kg} = 0.003\text{kg}$ $\quad (\because \; 1\text{g} = \frac{1}{1000}\text{kg})$

(ii) $765\text{g} = \frac{765}{1000}\text{kg} = 0.765\text{kg}$ $\quad (\because \; 1\text{g} = \frac{1}{1000}\text{kg})$

(iii) $4\text{kg } 80\text{g} = 4\text{kg} + 80\text{g}$

$= 4\text{kg} + \frac{80}{1000}\text{kg} = 4\text{kg} + 0.080\text{ kg}$

$= 4.080\text{kg}$

6.8.4 Capacity

Conversion of millilitre into litre

We know that $1000\text{ m}\ell = 1\text{litre}$

$1\text{ m}\ell = \frac{1}{1000}\ell = 0.001\ell$

Similarly $9\text{ m}\ell = \frac{9}{1000}\ell = 0.009\ell$

$65\text{ m}\ell = \frac{65}{1000}\ell = 0.065\ell$

$325\text{ m}\ell = \frac{325}{1000}\ell = 0.325\ell$

Example 19. **Express as litre using decimals :**

(i) **50 mℓ** (ii) **665mℓ** (iii) **2ℓ25mℓ**

Solution: (i) $50m\ell = \frac{50}{1000}\ \ell = 0.050\ell$ $\quad (1m\ell = \frac{1}{1000}\ell)$

(ii) $665m\ell = \frac{665}{1000}\ell = 0.665\ell$ $\quad (1m\ell = \frac{1}{1000}\ell)$

(iii) $2\ell\ 25m\ell = 2\ell + 25m\ell$

$$= 2\ell + \frac{25}{1000}\ell = 2\ell + 0.025\ell$$

$$= 2.025\ell$$

Exercise 6.3

1. Express as rupee using decimals:
   (i) 35 paise (ii) 4 paise (iii) 240 paise
   (iv) 12 rupees 25 paise (v) 24 rupees 5 paise
2. Express as metre using decimals:
   (i) 5cm (ii) 62cm (iii) 135cm (iv) 5m 20cm (v) 12m 8cm
3. Express as centimetre using decimals:
   (i) 2mm (ii) 28mm (iii) 8cm 4mm
4. Express as kilometre using decimals:
   (i) 7m (ii) 50m (iii) 425m (iv) 2475m (v) 3km 225m
5. Express as kilogram using decimals:
   (i) 5g (ii) 75g (iii) 423g (iv) 1265g (v) 5kg 418g
6. Express as litre using decimals:
   (i) 2mℓ (ii) 80mℓ (iii) 725mℓ (iv) 3ℓ 423mℓ (v) 8ℓ 20mℓ

6.9 Addition of Decimals

Addition of decimals is same as addition of whole numbers. The only difference is that we ensure that all decimal points will be in same column before addition. We use the following steps to add the decimals.

Step 1. Draw a dotted line to represents decimal point.

Step 2. Write the decimals in column so that tenths place digit comes under tenths place digits, hundredths comes under hundredths and so on.

Step 3. Convert the given decimals into like decimals.

Step 4. Add as we add whole numbers.

The following examples will make this concept more clear.

Example 20. **Add the followings**

(i) **4.23 + 5.69** (ii) **3.15 + 4.234** (iii) **1.2 + 18.67**

(iv) **2.4 + 1.35 + 24.567** (v) **13.25 + 2.4 + 18**

Solution : (i) We have 4.23 + 5.69

- Draw a dotted line for representation of decimal point.
- Ensure that all decimal points of given decimals must be on this line below each other as shown.
- Add the numbers.

Hence the required answer is 9.92

$$\begin{array}{r} 4.23 \\ +\ 5.69 \\ \hline 9.92 \\ \hline \end{array}$$

(ii) We have 3.15 + 4.234

- Draw a dotted line for representation of decimal point.
- Convert the given decimals into like decimals and add numbers.

Hence, the required answer is 7.384

$$\begin{array}{r} 3.150 \\ +\ 4.234 \\ \hline 7.384 \\ \hline \end{array}$$

(iii) We have 1.2 + 18.67

- Draw a dotted line for representation of decimal point.
- Convert the given decimals into like decimals and add.

Hence, the required answer is 19.87

$$\begin{array}{r} 01.20 \\ +\ 18.67 \\ \hline 19.87 \\ \hline \end{array}$$

Common Error:- This is common error that students can add in this way

- It is not correct as decimals points are not in a column.

So to avoid this mistake ensure that decimal points lie on a vertical line.

$$\begin{array}{r} .12 \\ +\ 18.67 \\ \hline 18.79 \\ \hline \end{array}$$

(iii) We have 2.4 + 1.35 + 24.567

- Draw a dotted line for representation of decimal point.
- Convert into like decimals and add.

Hence, the required answer is 28.317

$$\begin{array}{r} 02.400 \\ 01.350 \\ +\ 24.567 \\ \hline 28.317 \\ \hline \end{array}$$

(iv) We have 13.25 + 2.4 + 18

- Draw a dotted line for representation of decimal point.
- Convert into like decimals as 18 can be written as 18.00 and then add

Hence, the required answer is 33.65

$$\begin{array}{r} 13.25 \\ 02.40 \\ +\ 18.00 \\ \hline 33.65 \\ \hline \end{array}$$

6.10 Subtraction of Decimals

Subtraction of decimals is same as subtraction of whole numbers. The only difference is to ensure that all decimal points will be in same column before subtraction. We use the following steps for subtraction of decimals.

Step 1. Draw a dotted line to represent decimal point.

Step 2. Write the decimals in column so that tenths place digit comes under tenths place digit, hundredths comes under hundredths and so on.

Step 3. Convert the given decimals into like decimals.

Step 4. Subtract as we subtract in whole numbers.

Let's consider some examples:

Example 21. **Subtract the decimals:**

(i) **14.82 – 5.97** (ii) **25.18 – 18.07** (iii) **42.3 – 15.78**

(iv) **47.39 – 13.412** (v) **40 – 4.156**

Solution : (i) We have 14.82 – 5.97

- Draw a dotted line for representation of decimal point.
- Ensure that all decimal points of given decimals must be on this line.
- Subtract the numbers.

$$\begin{array}{r} 14.82 \\ -\ 05.97 \\ \hline 8.85 \\ \hline \end{array}$$

Hence, the required answer is 8.85

(ii) We have 25.18 – 18.07

- Draw a dotted line for representation of decimal point.
- Ensure that all decimal points of given decimals must be on this line.
- Subtract the numbers.

$$\begin{array}{r} 25.18 \\ -\ 18.07 \\ \hline 7.11 \\ \hline \end{array}$$

Hence, the required answer is 7.11

(iii) We have 42.3 – 15.78

- Draw a dotted line for representation of decimal point.
- Convert the given decimals into like decimals and then subtract

$$\begin{array}{r} 42.30 \\ -\ 15.78 \\ \hline 26.52 \\ \hline \end{array}$$

Hence, the required answer is 26.52

(iv) We have 47.39 – 13.412

- Draw a dotted line for representation of decimal point.
- Convert the given decimals into like decimals and then subtract

$$\begin{array}{r} 47.390 \\ -\ 13.412 \\ \hline 33.978 \\ \hline \end{array}$$

Hence, the required answer is 33.978

(v) We have 40 – 4.156

- Draw a dotted line for representation of decimals point.
- Convert the given decimals into like decimals and then subtract

$$\begin{array}{r} 40.000 \\ -\ 04.156 \\ \hline 35.844 \\ \hline \end{array}$$

Hence, the required answer is 35.844

Example 22 : (i) **Subtract 12.83 from 19.672.**

(ii) **Subtract 24.67 from 32.**

Solution: (i)

$$\begin{array}{r} 19.672 \\ -\ 12.830 \\ \hline 6.842 \\ \hline \end{array}$$

(ii)

$$\begin{array}{r} 32.00 \\ -\ 24.67 \\ \hline 7.33 \\ \hline \end{array}$$

6.11 Word Problems

In this section, we shall deal with daily life problems of decimals in addition and subtraction.

Example 23. **Three bags contain 45kg, 38.16kg and 47.258kg of rice respectively. What is the total weight of the rice in the bags?**

Solution : Total weight of the rice in the bags = Sum of weight of all three bags

$$\begin{array}{r} 45.000 \\ 38.160 \\ +\ 47.258 \\ \hline 130.418 \\ \hline \end{array}$$

Hence, total weight of rice is 130.418 kg.

Example 24. **Mandeep buys books worth ₹ 86.75, pencils for ₹ 28.2 and geometry box for ₹ 54.25. How much she has to pay?**

Solution : Price of books = ₹ 86.75

Price of pencils = ₹ 28.20

Price of geometry box = ₹ 54.25

Total amount she has to pay = ₹ 86.75 + ₹28.20 + ₹54.50

= ₹169.20

Example 25. **The height of Raman and Aashish are 1.64 m and 0.98 m respectively. How much Aashish is shorter than Raman?**

Solution : To solve sum, we subtract heights of boths

Height of Raman = 1.64 m

Height of Aashish = 0.98 m

∴ Differnce of heights = 1.64 m – 0.98 m = 0.66 m

Hence, Aashish is 0.66m shorter than Raman.

Example 26. **From a ribbon of length 25m, two pieces of 8.2m and 5.65 m were cut. Find the length of the remaining part.**

Solution : Given, Total length of the ribbon = 25 m

Length of first piece = 8.2 m

and length of 2nd piece = 5.65 m

Now length of both pieces = 8.2 m + 5.65 m = 13.85 m

∴ Length of Remaining part = (Total length) – (Sum of length of both pieces)

25.00 m – 13.85m = 11.15 m

Hence the required length is 11.15m.

Exercise 6.4

1. Solve the following:

(i) 12.15 + 4.87	(ii) 23.5 + 13.47	(iii) 12.56 + 6.234
(iv) 24.25 –13.12	(v) 18.8 – 4.26	(vi) 42.34 – 5.256

(vii) 45.4 + 13.25 + 28.68 (viii) 52.9 + 26.893 + 13.62

(ix) 42 – 27.563 (x) 64.26 – 43.589 + 13.42

(xi) 18.3 + 2.56 – 11.643 (xii) 66.5 – 13.49 – 29.712

2. (i) Subtract 21.92 from 32.683

   (ii) Subtract 14.812 from 23.

3. What should be added to 3.412 to get 7?

4. Khan spent ₹63.25 for Maths book and ₹48.99 for English book. Find the total amount spent by Khan.

5. Samar walked 3km 450m in morning and 2km 585m in evening. How much distance did he walk in all?

6. Sheetal has ₹190.50 in her pocket. She buys a school bag for ₹123.99. How much money is left with her now?

7. A piece of 18.56m long ribbon is cut into three pieces. If the length of two pieces are 8.75m and 3.125m respectively. Find the length of the third piece.

8. Veerpal bought vegetables weighing 20kg. Out of this 6kg 750kg are onions, 5kg 25g are potatoes and rest are tomatoes. What is the weight of the tomatoes?

9. Ashish's school is 28km far from his house. He covers 14km 250m by bus, 12km 650m by car and the remaining distance by foot. How much distance does he cover on foot?

Multiple Choice Questions

1. $3+\frac{2}{10}$ = ...

   (a) 302 (b) 3.2 (c) 3.02 (d) 30.2

2. $200 + 4 + \frac{5}{10}$ = ...

   (a) 24.5 (b) 204.05 (c) 204.5 (d) 24.05

3. $\frac{7}{100}$ = ...

   (a) .07 (b) 700 (c) .007 (d) 7

4. $50+\frac{3}{1000}$ = ...

   (a) 50.3 (b) 503000 (c) 50.0003 (d) 50.003

5. Seventy and four thousandths = ...

   (a) 74000 (b) 70.004 (c) .00074 (d) .074

6. 2.03 in expanded form = ...

(a) $2+\frac{3}{10}$ (b) $20+\frac{3}{10}$ (c) $2+\frac{3}{100}$ (d) $20+\frac{3}{100}$

7. 2.5 = ...

(a) $\frac{5}{2}$ (b) $\frac{25}{2}$ (c) $\frac{5}{10}$ (d) $\frac{1}{4}$

8. $\frac{13}{2}$ = ...

(a) 6 (b) 6.1 (c) 1.3 (d) 6.5

9. Which of the following decimals is largest?

(a) 0.5 (b) 0.05 (c) 0.51 (d) 0.005

10. Which of the following decimals is smallest?

(a) 2.13 (b) .213 (c) 21.3 (d) 213

11. 75g = ...kg

(a) .075kg (b) .75kg (c) 7.5kg (d) 75kg

12. 27mm = ...cm

(a) .27cm (b) 27cm (c) 2.7cm (d) .027cm

13. 2.5 +4.23 = ...

(a) 4.48 (b) 6.73 (c) 4.73 (d) 6.48

14. 15 + 3.84 = ...

(a) 3.99 (b) 18.99 (c) 3.84 (d) 18.84

15. 13.5 – 4.23 = ...

(a) 2.87 (b) 7.29 (c) 9.27 (d) 9.37

16. 20 – 12.56 = ...

(a) 7.44 (b) 8.44 (c) 9.44 (d) 6.44

17. 14.8 + 2.62 – 8.4 = ...

(a) 8.02 (b) 9.12 (c) 9.02 (d) 6.44

18. 5ℓ 7mℓ = ... ℓ

(a) 5.07ℓ (b) 5.7ℓ (c) 5.70ℓ (d) 5.007ℓ

19. 12kg 85g = ... kg

(a) 12.085kg (b) 12.85kg (c) 128.5kg (d) 12.0085kg

20. 235 paise = ...

(a) ₹235 (b) ₹23.5 (c) ₹2.35 (d) ₹.235

Learning Outcomes

After completion of this chapter students are now able to

- Know about decimal's place.
- Perform additions and substraction of decimals.
- Use decimals in practical life.
- Use decimals in lengths, capacity and weight.

ANSWER KEY

Exercise 6.1

1. (i) 72.14 (ii) 257.08 (iii) 8.256 (iv) 45.23 (v) 621.253
(vi) 12.008

2. (i) Twelve point five two. *or* Twelve and fifty two hundredths.
(ii) Seven point one four eight. *or* Seven and one hundred forty eight thousandths
(iii) Point two four. *or* Twenty four hundredths
(iv) Five point zero one eight. *or* Five and eighteen thousandths.
(v) Point zero zero nine. *or* Nine thousandths

4. (i) 40.2 (ii) 705.34 (iii) 10.053 (iv) .704 (v) .005

5. (i) 524.12 (ii) 2034.21 (iii) 61.023 (iv) 4.001 (v) 100.03

6. (i) $2+\frac{5}{10}$ (ii) $10+8+\frac{4}{10}+\frac{3}{100}$

(iii) $4+\frac{5}{100}$ (iv) $10+3+\frac{1}{10}+\frac{2}{100}+\frac{3}{1000}$

(v) $200+40+5+\frac{4}{10}+\frac{5}{100}+\frac{6}{1000}$ (vi) $20+\frac{5}{100}+\frac{7}{1000}$

Exercise 6.2

1. (i) $\frac{7}{5}$ (ii) $\frac{9}{4}$ (iii) $\frac{93}{5}$ (iv) $\frac{101}{25}$ (v) $\frac{108}{5}$

2. (i) 0.07 (ii) 1.2 (iii) 2.15 (iv) 0.018 (v) 24.5

3. (i) 2.5 (ii) 0.75 (iii) 5.6 (iv) 6.75 (v) 4.25

4.	(i)	8.5	(ii)	8.25	(iii)	15.2	(iv)	0.96	(v)	0.625
6.	(i)	1.3, 1.4, 1.5			(ii)	2.9, 3, 3.1			(iii)	5.1, 5.2, 5.3, 5.4
7.	(i)	0.7	(ii)	2.6	(iii)	1.32	(iv)	12.4	(v)	18.35
	(vi)	12	(vii)	5.061	(viii)	23.3	(ix)	13.08	(x)	2.3
8.	(i)	1.8, 1.9, 2, 2.5			(ii)	1.3, 3.1, 3.4, 4.3			(iii)	1.2, 1.24, 1.42, 1.8
9.	(i)	4.2, 4.12, 4.1, 4.01			(ii)	13, 1.3, 1.03, 1.003			(iii)	8.2, 8.1, 8.02, 8.002

Exercise 6.3

1.	(i)	₹0.35	(ii)	₹0.04	(iii)	₹2.40	(iv)	₹12.25	(v)	₹24.05
2.	(i)	0.05m	(ii)	0.62m	(iii)	1.35m	(iv)	5.20m	(v)	12.08m
3.	(i)	0.2cm	(ii)	2.8cm	(iii)	8.4cm				
4.	(i)	0.007km	(ii)	0.050km	(iii)	0.425km	(iv)	2.475km	(v)	3.225km
5.	(i)	0.005kg	(ii)	0.075kg	(iii)	0.423kg	(iv)	1.265kg	(iv)	5.418kg
6.	(i)	0.002ℓ	(ii)	0.080ℓ	(iii)	0.725ℓ	(iv)	3.423ℓ	(v)	8.020ℓ

Exercise 6.4

(1)	(i)	17.02	(ii)	36.97	(iii)	18.794	(iv)	11.13	(v)	14.54
	(vi)	37.084	(vii)	87.33	(viii)	93.413	(ix)	14.437	(x)	34.091
	(xi)	9.217	(xii)	23.298						
(2)	(i)	10.763	(ii)	8.188						
(3)	(i)	3.588	(4)	₹ 112.24	(5)	6km 035m	(6)	₹ 66.51		
(7)	6.685m		(8)	8.225kg	(9)	1km 100m				

Multiple Choice Questions

(1)	b	(2)	c	(3)	a	(4)	d	(5)	b
(6)	c	(7)	a	(8)	d	(9)	c	(10)	b
(11)	a	(12)	c	(13)	b	(14)	d	(15)	c
(16)	a	(17)	c	(18)	d	(19)	a	(20)	c

J2TSN7

7

ALGEBRA

Objectives

In this chapter you will learn

- To impart knowledge of variables.
- To use variables in different situations.
- To find the value of the equation.
- The practical use of equation in daily life.
- To make algebraic expressions.

7.1 Introduction

We have been dealing with numerals 0, 1, 2, 3, ... so far and the four operations addition, subtraction, multiplication and division. This branch of mathematics is called **Arithmetic**. We have also learnt about the two dimensional and three dimensional figures, study of which is called **Geometry**. Here we shall study a new branch of mathematics, called **Algebra**. It is an arabic word which means **"bringing together broken parts"**. Algebra evolved from the rules and operations of arithmetic.

7.2 Main features of Algebra

* The main feature of Algebra is the use of letters, which allow us to write rules and formulas in a general way. By using letters, one can talk about any number not just a particular number.

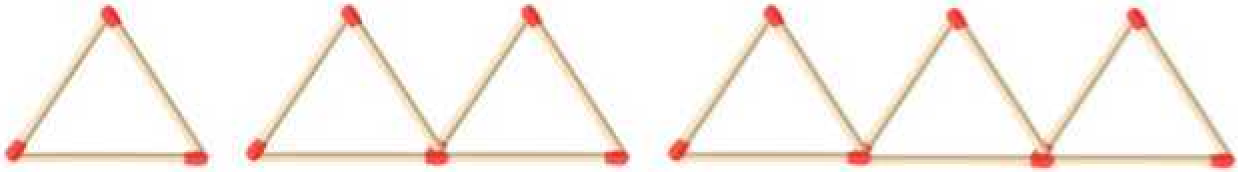
* These letters in algebra represent unknown numbers and are called **literals** or **variables**. The letters like a, b, c, ... p, q, r, ... x, y, z are called literals or variables. By learning methods of determining variables, we develop powerful rules for solving puzzles and many other daily life problems.

* Since letters stand for numbers, operations can be performed on them as on numbers. This leads to the study of algebraic expressions and their properties.

You will find that Algebra is quite interesting and useful in solving problems. Let us begin our study with following examples.

7.3 Identiying Patterns

Look at the match stick pattern



The table shows the number of matchsticks required to make 1, 2, 3, ... triangles.

Number of Triangles	1	2	3	4	5	–	–	8	–	–
Number of Matchsticks	3	6	9	12	15	–	–	24	–	–

It is observed that

1. One triangle is formed by using $3 \times 1 = 3$ matchsticks.
2. Two triangles are formed by using $3 \times 2 = 6$ matchsticks
3. Three triangles are formed by using $3 \times 3 = 9$ matchsticks.

Thus, the number of matchsticks required

$= 3 \times$ Number of triangles.

Let's represent the number of triangles by the letter 'n'

Thus, the general rule for the number of matchsticks required for n number of triangles $= 3 \times n$

This rule is very powerful and useful. By using this rule we do need to draw the pattern or make a rule, we can find number of matchsticks even for 100, 500, ... triangles.

* n is an example of a variable. Its value is not fixed. It can take any value 1, 2, 3, 4, ...

7.4 Variables and Constants

In the previous section, we have seen that the value of a letter is not fixed. It can take any numerical value such as 1, 2, 3, ... so on. The letters which stand for unknown numbers and can take any numerical value are called **variables**.

In other words, a letter whose value varies is called a variable.

Generally, we use small letters to denote a variable. The numbers 0, 1, 2, 3, ... have fixed values. They have a fixed numerical value and are called **constants**.

As numbers are the foundation stones of Arithmetic, variables are the foundation stones of Algebra

Example 1 : **If there are 10 pencils in a box. How will you write the total number of pencils in terms of number of boxes? Use 'n' for number of boxes.**

Solution : Let us make a table for number of pencils and boxes.

Number of boxes	1	2	3	-	-	10	-	-	n
Number of pencils	$10\times1=10$	$10\times2=20$	$10\times3=30$	-	-	$10\times10=100$	-	-	10n

It is observed that In a box there are 10 pencils,

In 2 boxes, there are $10 \times 2 = 20$ pencils

In 3 boxes, there are $10 \times 3 = 30$ pencils.

Thus, number of pencils in n boxes = (Number of pencils) × (Number of boxes) = $10 \times n = 10n$

Example 2 : **During a prayer for a school, 15 students stand in a row, If there are 'x' number of rows, give the rule to find the total number of students.**

Solution : Let us make a table for the number of students in rows.

Number of Rows	1	2	3	-	-	8	-	-	x
Number of Students	15	30	45	-	-	120	-	-	15x

It is observed from the table that

Total number of students in number of rows

= (Number of students) × (Number of Rows)

= $15 \times x = 15x$

Example 3: **There are 16 keys on a telephone set. Give the rule to find the total number of keys in terms of the number of telephone sets, if 't' represents the number of sets.**

Solution : We know

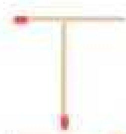
Total number of telephone keys in number of telephone sets

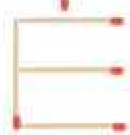
= (Number of keys in a telephone set) × (Number of telephone sets)

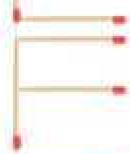
= $16 \times t = 16t$

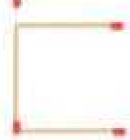
Exercise 7.1

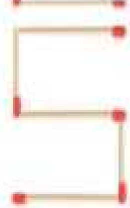
1. Find the rule which gives the number of matchsticks required to make the following 'n' matchstick patterns. Use a variables to write the rule:-

(i) A pattern of letter T as 

(ii) A pattern of letter E as 

(iii) A pattern of letter F as 

(iv) A pattern of letter C as 

(v) A pattern of letter S as 

2. Students are sitting in rows. There are 12 students in a row. What is the rule which gives the number of students in 'n' rows? (Represent by table)

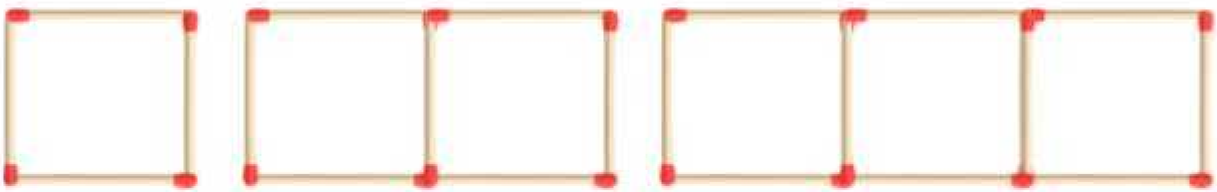
3. The teacher distributes 3 pencils to a student. What is the rule which gives the number of pencils, if there are 'a' number of students?

4. There are 8 pens in a pen stand. What is the rule that gives the total cost of the pens. If the cost of each pen is represented by a variable 'c'?

5. Gurleen is drawing pictures by joining dots. To make one picture, she has to join 5 dots. Find the rule that gives the number of dots, if the number of pictures is represented by the symbol 'p'.

6. The cost of a dozen bananas is ₹ 50. Find the rule of total cost of bananas if there are 'd' dozens bananas.

7. Look at the following matchsticks patterns of squares given below. The squares are not separate as there are two adjoined adjacent squares have a common match stick. Observe the patterns and find the rule that gives the number of matchsticks in terms of the number of squares.



(Hint: If you remove the vertical stick at the end you will get a patterns of C)

7.4.1 Operations on Literal Numbers or Variables

Since literal numbers or variables are used to represent numbers, they follow all the rules for four fundamental operations of numbers.

1. **Addition:-** Let x and y be two literals then the sum of x and y is written as $x + y$.
2. **Subtraction:-** Let x and y be two literals then the difference of these two literals is written as $x - y$ or $y - x$.
3. **Multiplication:-** Let x and y be two literals then the product of x and y is written as $x \times y$. Generally, we write xy. (As there may be confusion between x and '×').
4. **Division :-** Let x and y be two literals then 'x is divided by y' is written as $x \div y$ or $\frac{x}{y}$

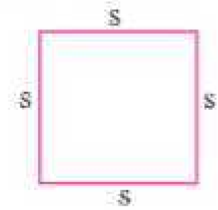
Let us consider the use of variables in some real life situations.

7.5 Algebra As Generalisation

Algebra is often referred to as generalised form of arithmetic. In mathematics, any rule or formula is generalised by expressing it through variables. Let's discuss its wider use in geometry and arithmetic.

7.5.1 Geometry

In earlier classes, we have studied the terms perimeter and area. Let us work about the general rules for them in terms of variables.



1. **Square:-** We know that a square has 4 sides of equal length. Let the length of each side is 's'.

Perimeter:-

$\therefore$ Perimeter of square = Sum of the lengths of the sides of the square

$= s + s + s + s = 4 \times s = 4s$

Perimeter also can be represented by another variable 'P'. Then the general rule for the perimeter of square is expressed as $P = 4s$

Area:- Let Area is represented as 'A'. We know, Area of Square = (Side) × (Side)

Thus, General rule for the area of square is expressed as $A = s \times s$

Thus, we get the rule for the Area of a square

2. **Rectangle:-** A rectangle is a closed figure having four sides. Its oppsite sides are equal in measurement.

Perimeter:- Let ℓ and b be length and breadth of rectangle and P be perimeter.

$\therefore$ Perimeter of rectange = Length + Breadth + Length + Breadth

Thus, we get the general rule for the perimeter of rectangle.

$P = \ell + b + \ell + b$

$= \ell + \ell + b + b$

$= 2\ell + 2b = 2(\ell + b)$

Area:- Let A be the area of rectangle

$\therefore$ Area of rectangle = (length) × (breadth)

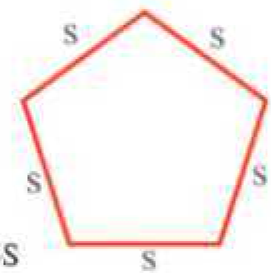
i.e $A = \ell \times b$

Thus, we get the general rule for the area of rectangle

Let us consider some examples of geometrical shapes.

Example 4: **Each side of regular pentagon is denoted by S. Express the perimeter of the regular pentagon using S.**

Solution : Each side of regular pentagon = S.



Perimeter of regular Pentagon

= Sum of all sides

$= S + S + S + S + S = 5 \times S = 5S$

Example 5: The diameter of a circle is twice its radius. If d is the diameter of the given circle and r is its radius. Express the diameter of the circle in terms of its radius.

Solution : Radius of circle = r

We know diameter of circle = twice of radius = $2 \times$ radius

$\therefore$ d = 2r

7.5.2. Arithmetic :

In the chapter of whole numbers, we have learnt some properties. Here we shall discuss those properties in form of algebra.

1. Commutative Property

- **Addition:-** If the order of numbers in addition is changed, it does not change their sum.

  e.g. $4 + 5 = 5 + 4 = 9$

  This property holds true for all set of numbers.

  Thus, we can form a general rule.

  Let a and b be two variables representing any two numbers. Then

  In this way, we $\mathbf{a + b = b + a}$. We can verify this general rule for every pair of numbers.

  e. g. a = 6, b = 7 then $6 + 7 = 7 + 6 = 13$

- **Multiplication:-** If the order of numbers in multiplication is changed, it does not change their product

  e.g. $3 \times 7 = 7 \times 3 = 21$

  This property holds true for any set of numbers, Thus we can form a general rule.

  Let a and b two variables representing any two numbers then $\mathbf{a \times b = b \times a}$

  We can verify this general rule for every pair of numbers.

  e.g. If a = 8, b = 5 then $8 \times 5 = 5 \times 8 = 40$

2. Associative Property

- **Addition:-** If three numbers can be added in any order, it does not change their sum.

  e. g $(3 + 4) + 5 = 7 + 5 = 12$

  $3 + (4 + 5) = 3 + 9 = 12$

  Thus, we can form a general rule.

  Let a, b and c be any three variables representing any three numbers. then

  $\mathbf{(a + b) + c = a + (b + c)}$

- **Multiplication:-** If three numbers can be multiplied in any order, it does not change their product.

  e.g. $(2 \times 3) \times 4 = 6 \times 4 = 24$

  $2 \times (3 \times 4) = 2 \times 12 = 24$

Thus, we can form a general rule.

Let a, b and c be any three variables representing any three numbers. then

$\mathbf{a \times (b \times c) = (a \times b) \times c}$

3. Distributive Property

- **Multiplication over Addition:-** In this property, while multiplying two numbers we split the larger number into sum of two numbers and then multiply these numbers with smaller number one by one and then add.

e.g. $5 \times 53 = 5 \times (50 + 3)$

$= 5 \times 50 + 5 \times 3 = 250 + 15 = 265$

In this multiplication, 5 is distributed over the addition of 50 and 3.

It is always true for any three numbers. Thus, we can form a general rule.

Let a, b and c be the variables representing any three numbers. then

$\mathbf{a \times (b + c) = a \times b + a \times c}$

- **Multiplication over subtraction:-** In this property while multiplying two numbers, we break the larger number into difference of two numbers and then multiply these numbers with smaller number one by one and then subtract.

e.g. $9 \times 48 = 9 \times (50 - 2)$

$= 9 \times 50 - 9 \times 2 = 450 - 18 = 432$

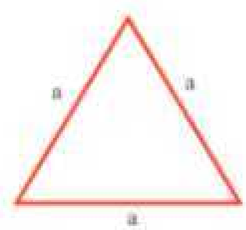
In this multiplication, 9 is distributed over the subtraction of 50 and 2. It is always true for any three numbers. Thus, we can form a general rule.

Let a, b, c be the variables representing three numbers. then

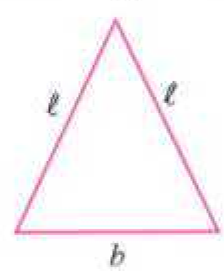
$\mathbf{a \times (b - c) = a \times b - a \times c}$

Exercise 7.2

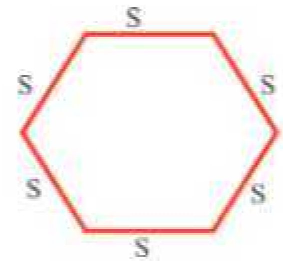
1. Each side of equilateral triangle is denoted by 'a' then express the perimeter of the triangle using 'a'.



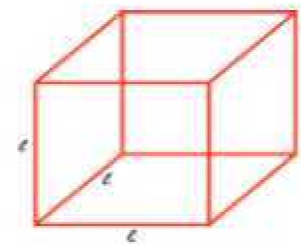
2. An isosceles triangle is shown. Express its perimeter in terms of 'ℓ' and 'b'



3. Each side of regular hexagon is denoted by 'S' then express the perimeter of the regular hexagon using 'S'.



4. The cube has 6 faces and all of them are identify squares. If ℓ is the length of an edge of a cube, find the total length of all edges of the cube in terms of 'ℓ'.



5. Write communtative property of addition using variables x and y.
6. Write associative property of multiplication using variables l, m and n.
7. Write distributive property of multiplication over addition in terms of variables p, q and r respectively.

7.6. Algebraic Expressions (Expressions with Variables)

In arithmetic, we come across many expressions such as $(6+5)\times 3$, $10+5\times 3-2$, $12\div 4\times 7-8$ etc. These expressions are formed by connecting the numbers with four operations i.e. addition, subtraction, multiplication and division. These are arithmetic expressions or expressions with numbers.

We can also make expression with variables. e.g 3a, x – 10, $l+4$. 5m + 3 etc. are expressions with variables and numbers connecting by operations i.e. additon, subtraction, multiplication and division.

"A collection of variables and numbers connecting by one or more signs of operations (+ , – × , ÷) is called an algebraic expression".

Some examples of algebraic expressions are $5l$, 6m– 2, $4l+3$, x + 12, $2l+3$m etc. Each part of the algebraic expression along with (+ or –) is known as its **term**.

Algebraic Expressions	Number of Terms	Terms
7a	1	7a
15m + 12	2	15m, 12
4a + 2b – 3c	3	4a, 2b, – 3c
x^2-4x+5	3	x^2, –4x, 5

Note:- One important point must always be kept in mind that expression with numbers can be solved very easily.

like 5 × 2 + 3 = 10 + 3 = 13

But expression with variables can not be solved. like 4x – 3 is in one variable x and value of x is unknown. So it is possible only if value of x is known.

e.g. If $x = 2$ then $4x - 3 = 4 \times 2 - 3 = 8 - 3 = 5$

Let us consider that how algebraic expressions are formed:

Algebraic Expressions	How these formed
$x + 5$	5 is added to x
$a - 8$	8 is subtracted from a
$3a$	a is multiplied by 3.
$\frac{\ell}{5}$	ℓ is divided by 5.
$2s + 3$	First s is multipled by 2 then 3 is added

7.6.1 Operations On Variables (Literals) and Numbers

We have learnt about operations on only variables or numbers. In this section, we shall learn about operations on variables and numbers taken together which is very useful in high classes algebra.

1. **Addtion of Variables and Numbers :-** Here, we shall discuss the addition of variables and numbers. Some expressions are as follows:

- 5 is added to x then sum is $x + 5$
- 8 is added to y then sum is $y + 8$
- a is added to b then sum is $b + a$

Note:- These above expressions can not be solved further, these have to be left as it is.

Example 6: **Write the following algebraic expressions:-**

(i) 9 is added to m (ii) 3 more than x (iii) 10 is added to p

Solution : (i) 9 is added to m = $m + 9$

(ii) 3 more than x = $x + 3$

(iii) 10 is added to p = $p + 10$

2. **Subtraction of Varibles and Numbers:-** Here, we shall discuss the subtraction of variables and numbers. Some expressions are as follows:-

- Subtract 3 from a = $a - 3$
- Subtract 6 from x = $x - 6$
- Subtract x from 4 = $4 - x$

Note:- Leave these expressions as it is, it cannot be solved further.

Example 7: **Write the following algebraic expressions:-**

(i) Subtract 1 from y (ii) Decrease *l* by 8 (iii) Subtract a from 5

Solution : (i) Subtract 1 from y = $y - 1$

(ii) Decrease l by 8 = $l - 8$

(iii) Subtract a from 5 = $5 - a$

3. **Multiplication of Variables and Numbers:-** Here, we shall discuss the multiplication or product of variables and numbers. Some expressions are as follows:

- Multiply a by 3 = $a \times 3 = 3a$ (In short form)
- The product of 5 and x = $5 \times x = 5x$
- l times m = $l \times m = lm$

Example 8: **Write the following expressions:-**

(i) Multiply 5 by p (ii) Product of 4 and z. (iii) Twice of l

Solution : (i) Multiply 5 by p = $5 \times p = 5p$

(ii) Product of 4 and z = $4 \times z = 4z$

(iii) Twice of l = Two times $l = 2 \times l = 2l$

4. **Division of literals and Numbers:-** Here, we shall discuss the division or quotient of variables and numbers. Some expressions are as follows:-

- b divided by 2 = $b \div 2 = \frac{b}{2}$
- y divided by 3 = $y \div 3 = \frac{y}{3}$
- Quotient of l by m = $l \div m = \frac{l}{m}$

Example 9: **Write the following expressions:-**

(i) x divided by 8 (ii) 3 divided by k. (iii) Quotient of a by 2

Solution : (i) x divided by 8 = $x \div 8 = \frac{x}{8}$

(ii) 3 divided by k = $3 \div k = \frac{3}{k}$

(iii) Quotient of a by 2 = $a \div 2 = \frac{a}{2}$

Now the applications of four basic operations are illustrated through following examples:

Example 10: **Study the following expressions and tell how are they formed?**

(i) $a - 8$ (ii) $l + 1$ (iii) $2m$ (iv) $\frac{a}{5}$ (v) $3z + 9$ (vi) $5p - 8$

Solution : (i) $a - 8$; 8 is Subtracted from a

(ii) $l + 1$; 1 is added to l

(iii) 2m; Twice of m

(iv) $\frac{a}{5}$; a is divided by 5

(v) $3z + 9$; First z is multipled by 3 then 9 is added to the product

(vi) $5p - 8$; First p is multipled by 5 then 8 subtracted from the product

Example 11: Give expressions for the following:-

(i) 12 is subtracted from x

(ii) 8 is added to y

(iii) p is multiplied by – 2

(iv) a is multiplied by –5 and 3 is added to the result

(v) ℓ is multiplied by 2 and then divided by 7.

Solution : (i) 12 is subtracted from x = $x - 12$

(ii) 8 is added to y = $8 + y$ or $y + 8$

(iii) p is multiplied by – 2 = $p \times (-2) = -2p$

(iv) a is multiplied by –5 and 3 is added to the result = $a \times (-5) + 3 = -5a + 3$

(v) ℓ is multiplied by 2 and then divided by 7= $(\ell \times 2) \div 7 = \frac{2\ell}{7}$

7.6.2.Use of Algebraic Expressions In Life:-

In last section, we have learnt the algebraic expressions using fundamental operatons. In this section we shall discuss the use of algebraic expressions in our daily life. Let us consider some examples.

Example 12: Find the number which is 12 more than a.

Solution : The required number = 12 more than a

= $12 + a$ or $a + 12$

Example 13: Write the number which is 8 less than x.

Solution : The required number = 8 less than x

= $x - 8$

Exampe 14: Vasu's present age is x years. Express the following in algebraic form.

(i) What will be Vasu's age after 6 years?

(ii) What will his age 3 years ago?

(iii) If Vasu's mother's age is 3 times his present age, what is the age of Vasu's mother?

(iv) If Vasu's elder brother Ankit is 10 years older than him. What is Ankit's age?

(v) Find his father's age, if he is 7 years more than twice of present age of Vasu?

Solution : Given Vasu's present age = x years

(i) After 6 years, Vasu's age = 6 years more than his present age (x)

= $(x + 6)$ years

(ii) 3 years ago, Vasu's age = 3 years less than his present age (x)

= $(x - 3)$ years

(iii)	Vasu's mother's age	=	3 times Vasu's present age
		=	$3 \times x = 3x$
(iv)	Ankit's age	=	10 years more than Vasu's present age
		=	$(10 + x)$ or $(x + 10)$ years
(v)	Father's age	=	Twice Vasu's present age + 7
		=	$(2x + 7)$ years

Example 15: The length of a room is 5 metres less than 3 times the breadth of the room. What is the length, if the breadth is b metres?

Solution : Given length of the room = 5 metres less than 3 times breadth

= 3 times breadth – 5 = 3 × breadth – 5

= $(3\,b - 5)$ metres

Exercise 7.3

1. Pick the algebraic expressions and the arithmetic expressions from the following:
   (i) $2l - 3$ (ii) $5 \times 3 + 8$ (iii) $6 - 3x$ (iv) $5l$
   (v) $2 \times (21 - 18) + 9$ (vi) $\frac{6a}{5} + 2$ (vii) $7 \times 20 \div 5 + 3$ (viii) 8
2. Write the terms for the following expressions:
   (i) $2y + 5z$ (ii) $6x - 3y + 8$ (iii) $7a$ (iv) $3l - 5m + 2n$ (v) $\frac{2\ell}{3} + x$
3. Tell how the following expressions are formed.
   (i) $a + 11$ (ii) $12 - x$ (iii) $3z + 8$ (iv) $6 - 5l$ (v) $\frac{5a}{4}$
4. Give expressions for the following:
   (i) 10 is added to p (ii) 5 is subtracted from y (iii) d is divided by 3
   (iv) l is multiplied by – 6 (v) m is subtracted from 1 (vi) 11 is added to 3x
   (vii) y is multiplied by – 2 and then 2 is added to the result
   (viii) c is divided by 5 and then 7 is multiplied to the result
   (ix) x is multiplied by 3 then subtracted this result from y
   (x) a is added to b then c is multiplied with this result
5. Write the number which is 15 less than y.
6. Write the number which is 3 more than a.
7. Find the number which is 1 more than twice of x.
8. Find the number which is 7 less than 5 times of y.
9. Somi's present age is 'a' years. Express the following in algebraic form:

(i) Her age after 15 years.

(ii) Her age 2 years ago.

(iii) If Somi's father's age is 5 more than twice of her present age, express her father's age.

(iv) If Somi's sister is 4 years younger to her. Express her sister's age.

(v) If Somi's mother is 3 less than 3 times her present age. Express her mother's age.

10. The length of a floor is 10 more than two times of breadth what is the length if breadth is l metres.

7.7 What Is An Equation ?

We have studied that algebraic expressions contain variables and constants. In the last section, We have learnt to change mathematical statements to algebraic expressions.

Let us consider a small puzzle:

Think of a number and add 5 to get 8. What is the number?

We can easily say that the number must be 3. If we use a variables (Literals) suppose 'x' in place of unknown number we can write this puzzle as follows : (Unknown number) + 5 = 8

ie. $x + 5 = 8$ This is an equation

Let us review some following statements:

(i) A number x increased by 7 is 12.

$\Rightarrow \quad x + 7 = 12$

(ii) A number x when decreased by 3 is 10

$\Rightarrow \quad x - 3 = 10$

(iii) Three times a number *l* gives 27.

$\Rightarrow \quad 3l = 27$

(iv) A number 'a' divided by 2 gives 6.

$$\Rightarrow \quad \frac{a}{2} = 6$$

(v) Sum of a number p and four times the number q is 18.

$\Rightarrow \quad p + 4q = 18$

Each of the above statement is a statement of equality. When the above statements written mathematically, contains one variables as in (i), (ii), (iii), (iv) or two variables as in (v) Each one of them is an equaltion.

"An equation is a mathematical statement equating two expressions. The expression on the left side of the equal sign is called Left Hand Side (LHS) and the expression on the right side of the equal sign is called Right Hand Side (RHS). The expressions on either side of equal sign are called members of the equation".

Those equations which have one or more variables (unknown values) and their highest power is 1 are called **linear equation**. Here, we shall study linear equations having only one variable.

Example 16: **Write the following mathematical statements as algebraic equations:-**

(i) The sum of a and 8 gives 13.

(ii) Twice of a number p gives 14.

(iii) One-fourth of a number is 16.

(iv) 5 more than 3 times of y gives 23.

(v) 2 less than from four times a number gives 26.

Solution : (i) The sum of a and 8 = a + 8

It gives 13

$\therefore$ The equation is $a + 8 = 13$

Aliter : Direct Method :

Sum of a and 8 = 13

$\Rightarrow \quad a + 8 = 13$

(ii) Twice of a number p = $2 \times p = 2p$

It is 14.

$\therefore$ The equation is $2p = 14$

Aliter

Twice of a number p = 14

ie. $2 \times p = 14 \Rightarrow 2p = 14$

(iii) One-fourth of a number = 16

$$\Rightarrow \quad \frac{1}{4} \times (\text{number}) = 16$$

Let the number be x

$\therefore$ The equation is $\frac{1}{4} \times x = 16$

$$\Rightarrow \quad \frac{x}{4} = 16$$

(iv) 3 times of y = $3 \times y = 3y$

5 more than 3 times of y = $3y + 5$

It is 23

$\therefore$ The equation is $3y + 5 = 23$

Aliter

5 more than 3 times of y = 23

i.e 3 times of y + 5 = 23

$3 \times y + 5 = 23 \Rightarrow 3y + 5 = 23$

(v) 2 less than from four times a number = 26

$\Rightarrow$ 4 times number – 2 = 26

$\Rightarrow$ $4 \times (\text{number}) - 2 = 26$

Let the number be a

$\therefore$ The equation is $4 \times a - 2 = 26$

$\Rightarrow \quad 4a - 2 = 26$

7.8 Solution of An Equation

To find the solution of an equation means to find a number which when substituted for the variable in the equation, makes its LHS equal to RHS. This number which satisfies the equation is called the **solution** or **root** of the equation.

To solve an equation or to find the solution (root) of an equation, we can follow these methods:

1. Trial and Error Method
2. Systematic Method
3. Transposition Method

7.8.1 Trial and Error Method

In this method, we try different values for the variable (unknown number) to make both sides of an equation equal. When we get a particular value of the variable which makes LHS equal to RHS. This particular value is said to be the root of the equation.

Example 17: **Solve $x + 6 = 9$**

Solution : We try different values of x to make LHS = RHS

Value of x	LHS = x + 6	RHS = 9	LHS = RHS
1	$1 + 6 = 7$	9	NO
2	$2 + 6 = 8$	9	NO
3	$3 + 6 = 9$	9	YES

From the above table, we find that LHS = RHS when $x = 3$

$\therefore$ Solution is $x = 3$

Example 18: **Solve $3x - 2 = 13$**

Solution : We try different values of x to make LHS = RHS

Value of x	LHS = 3x – 2	RHS = 13	LHS = RHS
1	$3 \times 1 - 2 = 3 - 2 = 1$	13	NO
2	$3 \times 2 - 2 = 6 - 2 = 4$	13	NO
3	$3 \times 3 - 2 = 9 - 2 = 7$	13	NO
4	$3 \times 4 - 2 = 12 - 2 = 10$	13	NO
5	$3 \times 5 - 2 = 15 - 2 = 13$	13	YES

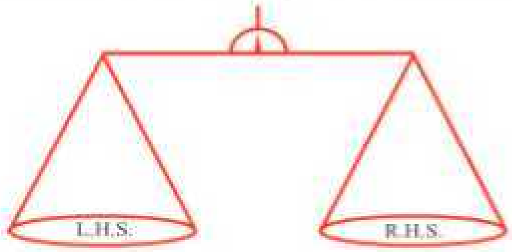
From the above table, we find that LHS = RHS when $x = 5$

$\therefore$ Solution is $x = 5$

7.8.2 Systematic method

The method of trial and error to solve linear equations can be time consuming. It is not a proper way to find the solution of an equation.

An equation behaves like a weighing balance. Both sides of equation are balanced in the same manner as the scales of a balance. When the weights in both sides are equal, the weighing balance is balanced.

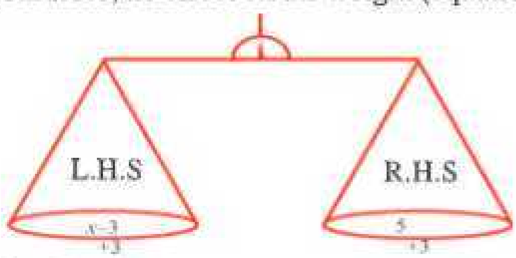


We can add equal weights to both sides or remove equal weigths from both sides then also the two sides will be in balance. Here we have four rules (axioms) for balancing linear equations.

Rule 1:- If we add the same number (quantity) on both sides of an equation, the equality holds true.

e.g. Take an equation $x - 3 = 5$

If we add on 3 both sides, no effect on the weight (equation)



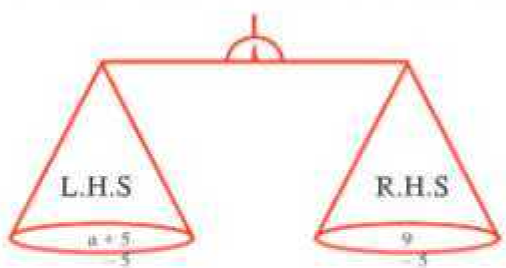
i.e. $(x - 3) + 3 = 5 + 3$

$\Rightarrow \quad x = 8$

Rule 2: If we subtract same number from both sides of an equation. the equality holds true

e.g. $a + 5 = 9$

If we subtract 5 from both sides, no effect on the weight (equation)



i.e $(a + 5) - 5 = 9 - 5$

$\Rightarrow \quad a = 4$

Rule 3:- If we multiply both sides of an equation by the same number, the equality holds true.

e.g. $\frac{x}{2} = 7$

If we multiply by 2 to both sides, no effect on the equation i.e $\frac{x}{2} \times 2 = 7 \times 2$

$\Rightarrow \quad x = 14$

Rule 4:- If we divide both sides of an equation by the same number, the equality holds true

e.g. $7l = 21$

If we divide both sides by 7, no effect on the equation

ie. $\frac{7l}{7} = \frac{21}{7}$

$\Rightarrow \quad l = 3$

Now, let us consider some examples using these rules.

Example 19: Solve:- $a - 8 = 4$

Solution : Given equation is $a - 8 = 4$

Adding 8 on both sides, we get

$a - 8 + 8 = 4 + 8$

$\Rightarrow \quad a = 12$ is the required solution.

Example 20: Solve:- $3x - 1 = 14$

Solution : Given equation is $3x - 1 = 14$

Adding 1 on both sides, we get

$3x - 1 + 1 = 14 + 1$

$\Rightarrow \quad 3x = 15$

Dividing both sides by 3, we get

$$\frac{3x}{3} = \frac{15}{3}$$

$\Rightarrow \quad x = 5$ is the required solution.

Example 21: Solve:- $2x + 5 = 21$

Solution : Given equation is $2x + 5 = 21$

Subtracting 5 from both sides, we get

$2x + 5 - 5 = 21 - 5$

$\Rightarrow \quad 2x = 16$

Dividing both sides by 2, we get

$$\frac{2x}{2} = \frac{16}{2}$$

$\Rightarrow \quad x = 8$ is the required solution.

7.8.3 Method of Transposition a number

We know that to solve a linear equation, we add, subtract, multiply or divide both sides of the equation by the same number.

Transposing a number (i.e. changing the side of the number) is the same as adding or subtracting the number, multiplying or dividing by the number to both sides of the equation ; we change the sign '+' into '–' and vice-versa, 'x' into '÷' and vice-versa.

Consider:- equation $x - 2 = 6$ .... (i)

Adding 2 to both sides, we get

$x - 2 + 2 = 6 + 2$

$x = 6 + 2$ ....(ii)

Comparing equation (i) and (ii), we observe that the number 2 is shifted from LHS to RHS of the equation but with sign changed i.e.**'–' Sign to '+' sign**. This process is known as **transposition**.

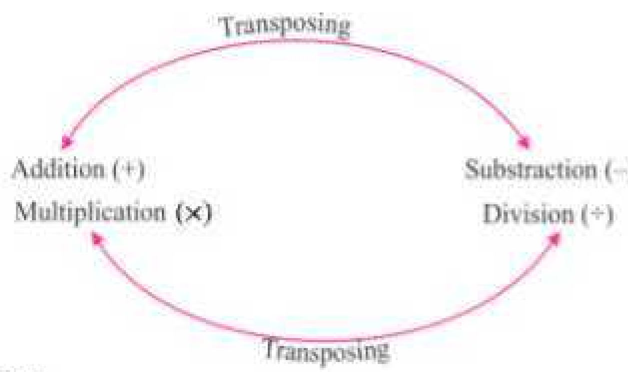
• Consider an equation $3a = 12$ .... (i)

Dividing both sides by 3, we get

$$\frac{3a}{3} = \frac{12}{3}$$

$$a = \frac{12}{3} \text{ or } 12 \div 3 \text{ .... (i)}$$

Comparing equation (i) and (ii), we observe that the number 3 is shifted from LHS to RHS of the equation but with operation changed i.e. **Multiply to Divide.** This process is known as transposition.



Example 22:- Solve the following equation:-

(i) $x + 2 = 11$ (ii) $y - 3 = 8$ (iii) $4x = 24$ (iv) $\frac{a}{3} = 6$ (v) $3b - 2 = 19$

Solution : (i) Given equation : $x + 2 = 11$

$\Rightarrow$ $x = 11 - 2$ (Transposing + 2 to other side, it becomes – 2)

$\therefore$ $x = 9$ is the required answer.

(ii) Given equation : $y - 3 = 8$

$\Rightarrow$ $y = 8 + 3$ (Transposing – 3 to other side, it becomes + 3)

$\therefore$ $y = 11$ is the required solution.

(iii) Given equation : $4x = 24$

$\Rightarrow$ $x = \frac{24}{4}$ (Transposing 'multiplication', it becomes 'division')

$\therefore$ $x = 6$ is the required solution

(iv) Given equation : $\frac{a}{3} = 6$

$\Rightarrow$ $a = 6 \times 3$ (Transposing 'division', it becomes 'multiplication')

$\therefore$ $a = 18$ is the required answer

(v) Give equation : $3b - 2 = 19$

$\Rightarrow$ $3b = 19 + 2$ (Transposing – 2, it becomes + 2)

$\Rightarrow \quad 3b = 21$

$\Rightarrow \quad b = \frac{21}{3}$ (Transposing 'Multiplication', it becomes 'division')

$\therefore$ b = 7 is the required solution.

Exercise 7.4

1. Write the following statements as algebraic equations:-
   (i) The sum of x and 3 gives 10.
   (ii) 5 less than a number 'a' is 12.
   (iii) 2 more than 5 times of p gives 32.
   (iv) Half of a number is 10.
   (v) Twice of a number added to 3 gives 17.
2. Write the LHS and RHS for the following equations:-
   (i) $l + 5 = 8$ (ii) $13 = 2m + 3$ (iii) $\frac{t}{4} = 6$ (vi) $2h - 5 = 13$ (v) $\frac{5x}{7} = 15$
3. Solve the following equations by trial and error method:
   (i) $x + 2 = 7$ (ii) $5p = 20$ (iii) $\frac{a}{5} = 2$ (iv) $2l - 4 = 8$ (v) $3x + 2 = 11$
4. Solve the following equations by systematic method.
   (i) $z - 4 = 10$ (ii) $a + 3 = 15$ (iii) $4m = 20$ (iv) $3x - 3 = 15$ (v) $4x + 5 = 13$
5. Solve the following equation by transposition:
   (i) $x - 5 = 6$ (ii) $y + 2 = 3$ (iii) $5x = 10$ (iv) $\frac{a}{6} = 4$ (v) $4y - 2 = 30$
6. Solve the following equations:
   (i) $x + 7 = 11$ (ii) $x - 3 = 15$ (iii) $x - 2 = 13$ (iv) $6x = 18$
   (v) $3x = 24$ (vi) $\frac{x}{4} = 7$ (vii) $\frac{x}{8} = 5$ (viii) $2x - 5 = 17$
   (ix) $4x + 5 = 21$ (x) $5x - 2 = 13$

Multiple Choice Questions

1. Each side of square is represented by 's' then perimeter of square is :
   (a) $4 + s$ (b) $s - 4$ (c) $4s$ (d) s
2. Write commutative property of multiplication using variables x and y
   (a) $xy = yx$ (b) $x + y = y + x$ (c) $x + y$ (d) xy

3. How many terms in expression $7l - 3$?

(a) 1 (b) 3 (c) 2 (d) 4

4. 5 is subtracted from m = ...

(a) $5 - m$ (b) $m + 5$ (c) $5 + m$ (d) $m - 5$

5. Multiply p by 3 then 2 is added = ...

(a) $2p + 3$ (b) $3p - 2$ (c) $3p + 2$ (d) $2p - 3$

6. If Armaan 's present age is x years then what will be his age after 4 years?

(a) $x - 4$ (b) $x + 4$ (c) $4x$ (d) $4 - x$

7. Write as algebraic equation : 7 more than 4 times of y gives 23.

(a) $4 + 7y = 23$ (b) $7 + y = 23$ (c) $4y - 7 = 23$ (d) $4y + 7 = 23$

8. Find x if $x - 3 = 2$

(a) 3 (b) 6 (c) 5 (d) 2

9. Solve; $4l - 3 = 5$

(a) 3 (b) 4 (c) 1 (d) 2

10. If $\frac{a}{4} = 5$ then a = ...

(a) 5 (b) 20 (c) 4 (d) 18

Learning Outcomes

After completion of this chapter the students are now able to

- Learn the concept of variables.
- Use variables in different situation.
- Know the meaning of equation.
- Find the value of the equation.
- Make algebraic expressions from the statement.

ANSWER KEY

Exercise 7.1

1. (i) 2n (ii) 4n (iii) 3n (iv) 3n (v) 5n 2. 12n 3. 3a

4. 8c 5. 5p 6. 50d 7. $1 + 3x$ or $3x + 1$

Exercise 7.2

1. 3a 2. $2l + b$ 3. 6S 4. $12l$ 5. $x + y = y + x$

6. $l \times (m \times n) = (l \times m) \times n$ 7. $p \times (q + r) = p \times q + p \times r$

Exercise 7.3

1. Algebraic expressions : (i), (iii), (iv), (vi) Arithmetic expressions : (ii), (v), (vii), (viii)

2. (i) 2y, 5z (ii) 6x, –3y, 8 (iii) 7a (iv) $3l, -5m, 2n$ (v) $\frac{2l}{3}, x$

3. (i) a is increased by 11 (ii) x is subtracted from 12
(iii) Three times of z is increased by 8 (iv) 5 times ℓ is subtracted from 6
(v) 5 times a is divided by 4.

4. (i) p + 10 (ii) y – 5 (iii) $\frac{d}{3}$ (iv) $-6l$ (v) 1 – m
(vi) 3x + 11 (vii) –2y + 2 (viii) $\frac{7c}{5}$ (ix) y – 3x (x) (a + b) c

5. y – 15 6. a + 3 7. 2x + 1 8. 5y – 7

9. (i) a + 15 (ii) a – 2 (iii) 2a + 5 (iv) a – 4 (v) 3a – 3 10. $2l + 10$

Exercise 7.4

1. (i) x + 3 = 10 (ii) a – 5 = 12 (iii) 5p + 2 = 32 (iv) $\frac{x}{2} = 10$ (v) 2x + 3 = 17

2. (i) LHS = $l + 5$, RHS = 8 (ii) LHS = 13, RHS = 2m + 3
(iii) LHS = $\frac{t}{4}$, RHS = 6 (iv) LHS = 2h – 5, RHS = 13 (v) LHS = $\frac{5x}{7}$, RHS = 15

3. (i) x = 5 (ii) p = 4 (iii) a = 10 (iv) $l = 6$ (v) x = 3

4. (i) z = 14 (ii) a = 12 (iii) m = 5 (iv) x = 6 (v) x = 2

5. (i) x = 11 (ii) y = 1 (iii) x = 2 (iv) a = 24 (v) y = 8

6. (i) x = 4 (ii) x = 18 (iii) x = 15 (iv) x = 3 (v) x = 8
(vi) x = 28 (vii) x = 40 (viii) x = 11 (ix) x = 4 (x) x = 3

Multiple Choice Questions

1. c 2. a 3. c 4. d 5. c 6. b 7. d 8. c 9. d 10. b


P7W2A1

8

BASIC GEOMETRICAL CONCEPTS

Objectives

In this chapter you will learn

- About point, line, line segment and ray.
- About curve i.e. simple, open and closed curve.
- About polygon i.e. triangle and quadrilateral and their parts.
- About circle and its parts.
- To correlate these geometrical concepts with the surroundings.

8.1 Introduction

Geometry, a branch of Mathematics, concerned with position, size and shape of figures.

The word 'Geometry' has been derived from Greek word **"Geo"** and **"metron"**. **Geo means earth** and **metron means measurement**. In earlier days, Geometry was used in various fields of our life e.g art, architecture, measurement etc, we learn about the construction of geometrical figures and study their basic properties.

Just as numbers are basic elements or foundation blocks of arithmetic and algebra has numbers, alphabets and the four fundamental operations as its basic elements; Geometry too has its own basic elements or foundation blocks. In this section, we shall learn about some of these. There are three basic elements **point, line and plane.** These terms cannot be precisely defined. However, we can give some ideas to illustrate the meaning of these terms.

8.2 Point

A small dot marked by a sharp pencil on a sheet of paper or a tiny prick made by a fine needle or pin on a paper, *Bindi* are examples of a point.


Sharped end of a pencil


Tips of Compasses


Pointed end of a needle

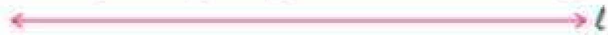
A point is just a location marker. It depicts the exact position of an object. A point has no length, breadth or height i.e. it does not have any size.

A point is represented by a single capital letter of the alphabet such as A, B, C. .... P, Q, R etc and read as 'point A, 'point B' (as shown)

8.3 Line

A line is a collection of points which can be extended infinitely on both the sides. It has only length neither breadth nor thickness. There are two ways of naming a line.

(i) A line can be named by writing a single small letter of the alphabet such as ℓ, m etc.

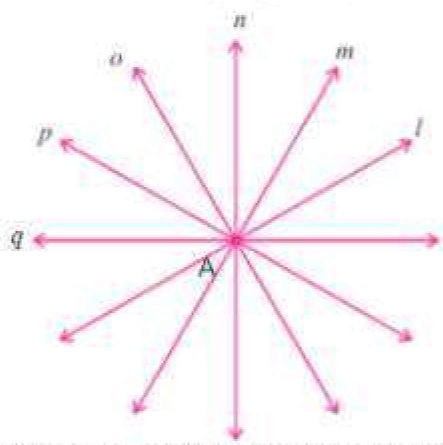


(ii) By taking two points say A and B on the line named as $\overleftrightarrow{AB}$

A line has some properties as follows:

- A line has no end points. The arrows show that the line goes an endlessly in all directions.
- A line has infinite many points on it.

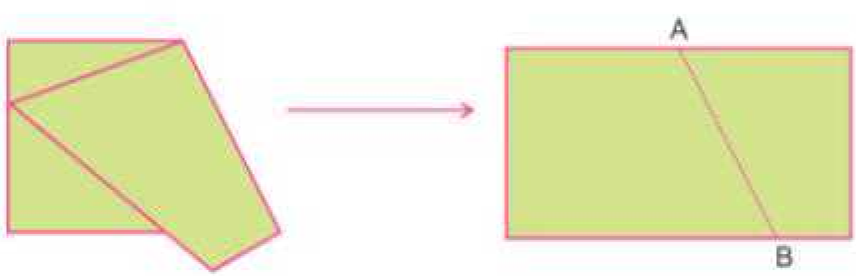
- Only one line can be drawn through two points.
- Line does not have definite length.
- Infinite lines can be drawn through a given point.



In the figure, lines ℓ, *m, n, o, p, q* all passes through a given point A.

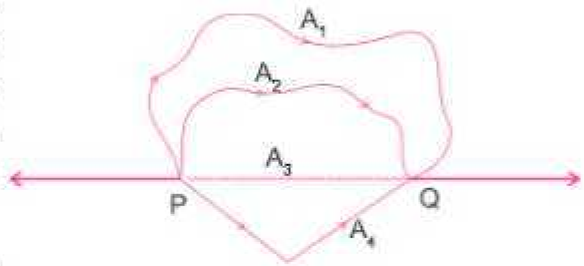
8.3.1 LINE SEGMENT

Fold a piece of paper and then unfold it. You get a crease on the paper.



The crease you get is the representation of a line segment. This line segment has two end points, A and B.

Now consider two points P and Q. There are several possible ways to reach from P to Q as shown as A_1, A_2, A_3 and A_4. The shortest path is represented by dotted line, (A_3) which is straight. So, the straight path from P to Q is a segment (portion) of line passing through P and Q.



Since, there is only one line passing through P and Q. It is obvious that there is only one line segment joining P and Q.

A line segment is a part of a line that is bounded by two distinct end points. It is the shortest distance between two end points.

A line segment from P to Q is represented by $\overline{PQ}$ or $\overline{QP}$. There are infinite points on a line segment.

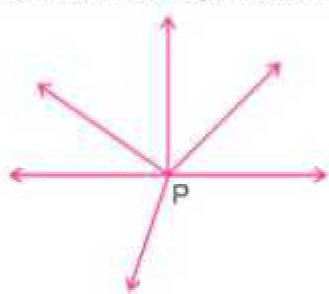
8.3.2 Ray

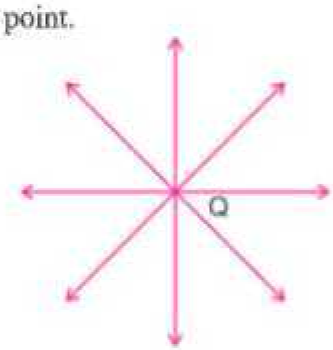
It is a part of a line which has only one end point and can be extended indefinitely in one direction. $\overrightarrow{AB}$ is a ray with initial point A and extended indefinitely from A to B.

$\overrightarrow{BA}$ is a ray with initial point B and extended indefinitely from B to A.

$\overrightarrow{AB}$ and $\overrightarrow{BA}$ are two different rays.

Infinite number of rays can be drawn from a point.



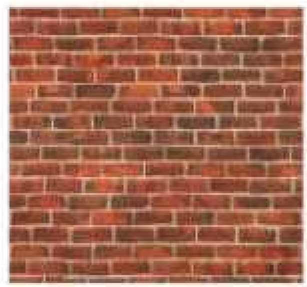


Infinite rays from a point P

Infinite rays from a point Q

8.4 Plane

We come across a lot of flat surfaces in our everyday life like top of a table, surface of a wall, surface of a blackboard.



So every solid has a surface which is flat or curved. In Geometry, we take totally flat or curved surfaces.

A plane is a flat surface which extends endlessly in all directions. It has no boundary. It has length and breadth but no height.

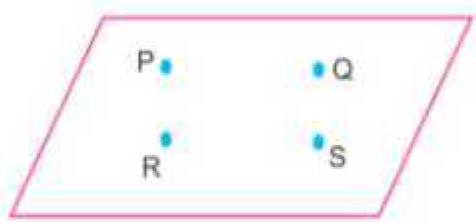
As plane extends indefinitely in all directions, we cannot draw it on a plain paper, only a portion of a plane is drawn.

A plane can be named in two ways:

(i) By writing a single small letter such as p or q. It is read as 'plane p' or 'plane q'.

(ii) By writing three or more capital letters say A, B and C but not on the same line. It is read as 'plane ABC'.

Plane A B C

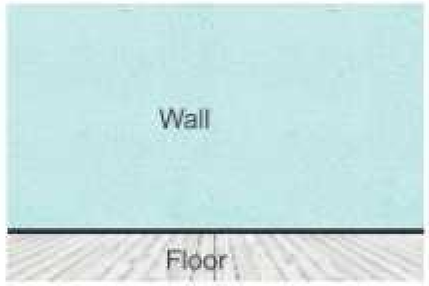


Plane P Q R S

8.4.1 Properties of Points and Lines in a Plane

1. Any two points on the same plane can be connected with one and only one line passing through them. This line wholly lies in the plane.

2. Two planes intersects in a line, e.g. wall and floor of a room intersect in a line called edge.

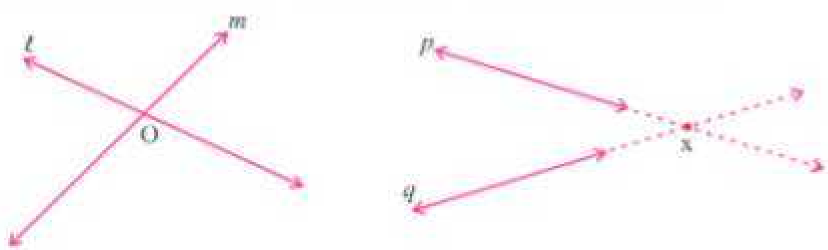


3. If we consider two lines in a plane, there can be two possibilities.
   (i) They may cut each other in the plane.
   (ii) They are parallel to each other i.e. they do not intersect each other.

8.5 Intersecting Lines

In a plane two lines that meet at a point are called **intersecting lines** and the point is called the **point of intersection**.

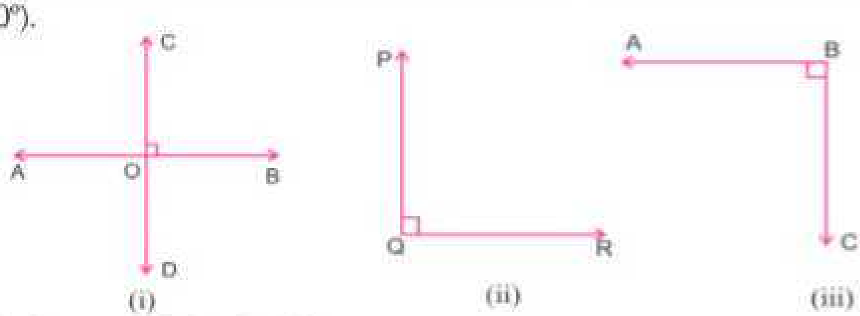
(i) ℓ and m are intersecting lines and O is the point of intersection.



(ii) After extending p and q lines, they intersect at x, point of intersection.

8.5.1 Perpendicular lines

In a plane two lines are said to be perpendicular to each other if the angle formed by them is a right angle (90°).



symbol of perpendicular is '⊥'

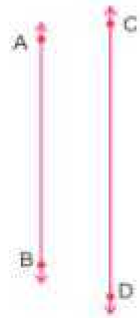
(i) Lines AOB and COD are perpendicular to each other as $\angle COB = 90^\circ$, we write COD⊥ AOB are read as 'CD is perpendicular to AB'. Or AOB ⊥ COD as AB is perpendicalar to CD.

(ii) PQ⊥QR as $\angle PQR = 90^\circ$ Or RQ ⊥ PQ as $\angle RQP = 90^\circ$

(iii) AB⊥BC as $\angle ABC = 90^\circ$ Or BC ⊥ AB as $\angle CBA = 90^\circ$

8.6. Parallel lines

In a plane, the lines which never meet even after extending are known as **parallel lines.** The distance between the set of parallel lines remain same.

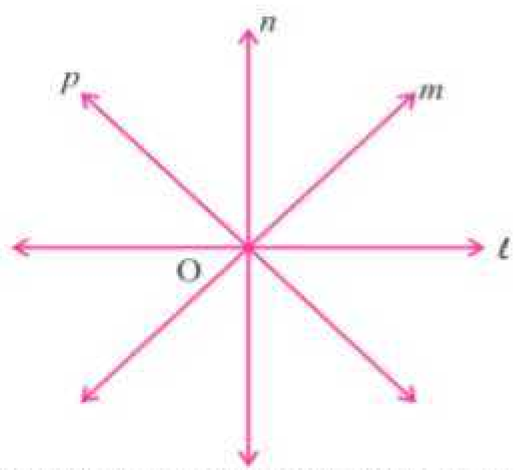


p||*q* AB||CD

The opposite edge of a ruler (scale) or blackboard, railway lines are best examples of parallel lines.

8.7 Concurrent lines

Three or more lines in a plane when pass through a same point are called concurrent lines and that point is called the **point of concurrence**.



Lines ℓ, *m*, *n*, *p* pass through O are called concurrent lines and O is called the point of concurrence.

8.8 Collinear Points

Three or more points in a plane are said to be collinear, if they all lie on the same line.

Collinear Points

Non-Collinear points

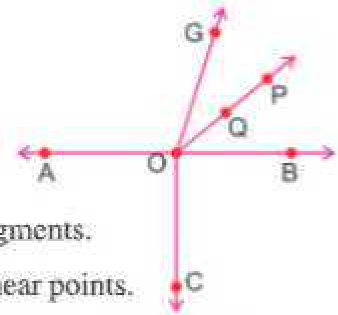
Let's illustrate some examples:

Example 1: **From the given figure, name**

(i) **Any four rays.**

(ii) **Any four line segments.**

(iii) **Set of collinear points.**

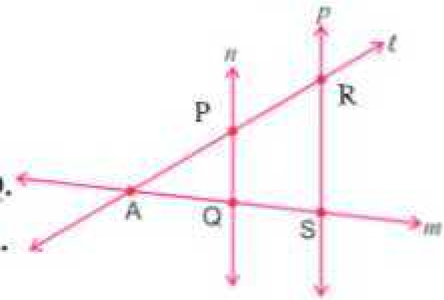


Solution : (i) $\overrightarrow{OQ}$, $\overrightarrow{OP}$, $\overrightarrow{OG}$, $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ $\overrightarrow{QP}$ are rays.

(ii) OQ, QP, OP, OG, OA, OB, OC, AB are line segments.

(iii) A, O, B are collinear points or O, Q, P are collinear points.

Example 2 : **From the given figure name.**



(i) **Pair of parallel lines.**

(ii) **Pairs of intersecting lines.**

(iii) **Lines whose point of intersection is Q.**

(iv) **Lines whose point of intersection is A.**

(v) **Collinear points**

Solution : (i) Pair of parallel lines : n and p

(ii) Pairs of intersecting lines : ℓ and m, n and ℓ , p and ℓ, n and m, p and m.

(iii) Q is the point of intersection of n and m.

(iv) A is the point of intersection of ℓ and m.

(v) Set of collinear points : A, P, R and A, Q, S.

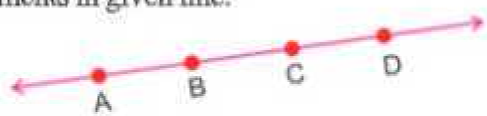
Exercise 8.1

1. Give the examples of :

   (i) A point (ii) A line segment (iii) Parallel lines

   (iv) Intersecting lines (v) Concurrent lines

2. Name the line segments in given line.



3. How many lines may pass through a point?

4. How many points may lie on a line?

5. How many lines pass through two points?

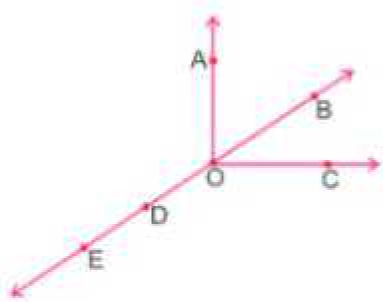
6. Use the figure to name.

   (i) Five points

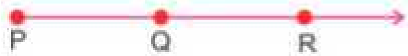
   (ii) A line

   (iii) Four rays

   (iv) Five line segments



7. Name the given ray in all possible ways.



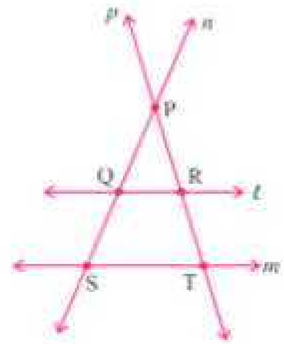
8. Use the figure to name :

(i) Pair of parallel lines.

(ii) All pairs of intersecting lines.

(iii) Lines whose point of intersection is S.

(iv) Collinear points.



9. Use the figure to name:

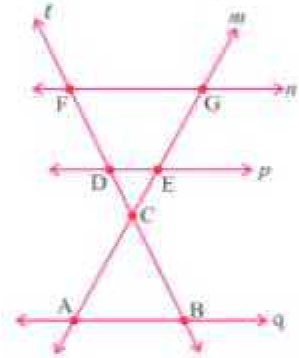
(i) All pairs of parallel lines.

(ii) All pairs of intersecting lines.

(iii) Lines whose point of intersection is D.

(iv) Point of intersection of lines m and p.

(v) All sets of collinear points.



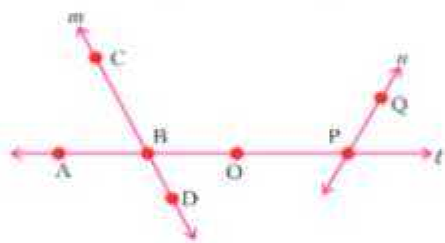
10. Use the figure to name:

(i) Line containing point P.

(ii) Lines whose point of intersection is B.

(iii) Point of intersection of lines m and ℓ.

(iv) All pairs of intersecting lines.



11. State which of the following statements are True (T) or False (F):

(i) Two lines in a plane, always intersect at a point.

(ii) If four lines intersect at a point, those are called concurrent lines.

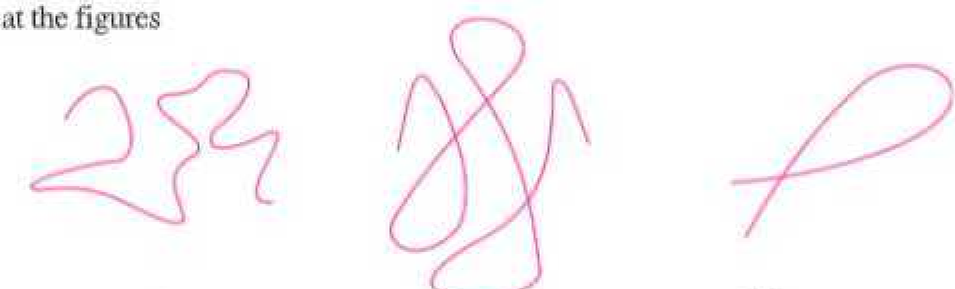
(iii) Point has a size because we can see it as a thick dot on the paper.

(iv) Through a given point, only one line can be drawn.

(v) Rectangle is a part of the plane.

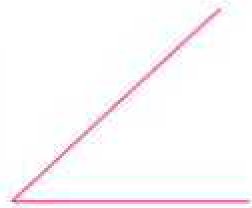
8.9 Curves

Look at the figures



(i) (ii) (iii)

(iv)



(v)

(vi)

You may have drawn several such drawings, names on sand, walls or mirror. All these drawings are curves.

Take a pencil and a paper. Put the sharp tip of the pencil on the paper and move it aimlessly from one point to other without lifting the pencil. The pictures obtained as a result are called curves.

Generally 'curve' means 'not straight'.

But in mathematics a curve can be straight figure (v), (vi).

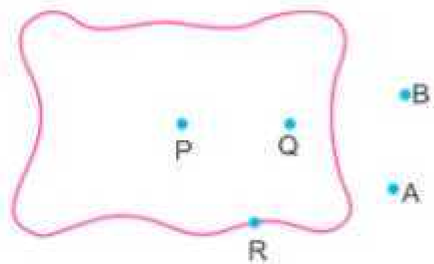
Simple Curve : If a curve does not cross itself, then it is called simple curve. figure (i), (iv), (v), (vi).

Open Curve : An open curve is a curve where the begining and end points are different.

Figure (i), (ii), (iii), (v)

Closed Curve : A curve whose initial and terminating point lies on same point is called a closed curve. (Figure (iv) is a closed curve), there are three parts.

- **Interior of the curve :** Part of curve made by all those points that are enclosed by the curve is called interior of the curve. Points P, Q are inside (interior) of the curve.



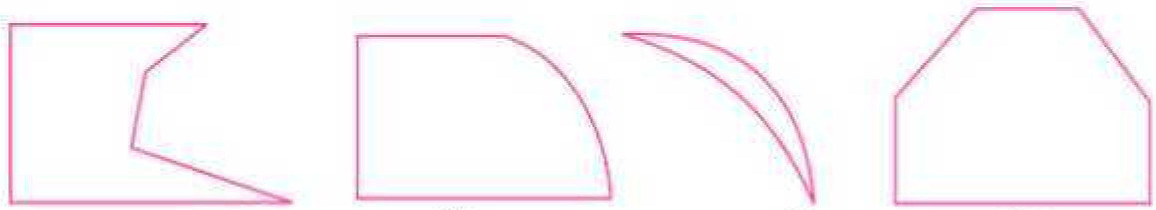
- **On the Boundary of the curve :** Part of the curve made by all those points that are on the curve is called the boundary of the curve. In figure point R is on the boundary of the curve.
- **Exterior of the curve :** Part of the curve made by all those points that are not enclosed by the curve is called the exterior of the curve. In figure, A and B are the points exterior to the curve.

  For Example : Your school has a boundary, your class rooms are inside the school boundary and your school gate is on the boundary. There is a road outside the boundary of your school.

- A curve divides the plane in three disjoint parts.

8.10 Polygons

Can you identify the difference between the following closed curves?



(i) (ii) (iii) (iv)

Figure (i) and (iv) are made up of line segments while figure (i) and (iii) are not.

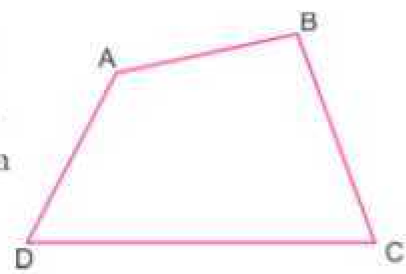
The figures which are entirely made up of line segments are known as polygons, thus figure (i) and (iv) are polygons.

Polygon means 'Poly' and 'gon'. 'Poly' means 'many' and 'gon' means 'sides'. So 'Polygon' means 'having many sides'.

"A polygon is simple closed curve having three or more line segments. such that."

- No two line segments intersect except at their end points.
- No two line segments with a common end point are coincident.

The line segments forming a polygon are called its**sides** and the end points of the line segments are called its **vertices.**



Sides : The line segments which form a polygon are called its sides. AB, BC, CD, DA are the sides of the polygon ABCD.

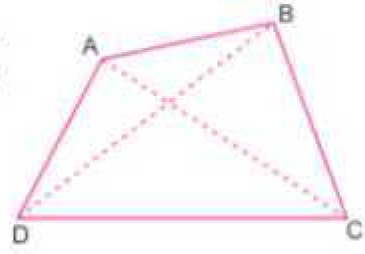
Vertices : The meeting point of a pair of sides of a polygon is called its vertex. In the polygon ABCD, sides AB and BC intersect at B, BC and CD intersect at C and so on. So A, B, C and D are the vertices of the polygon ABCD.

Adjacent Sides: Any two sides with a common end-point (vertex) are called the adjacent sides of the polygon. AB and BC have common vertex B. So AB and BC are adjacent sides.

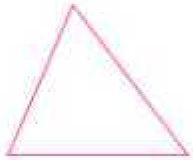
Similarly AB and AD; AD and DC; DC and CB are pairs of adjacent sides.

Adjacent vertices : The end-points of the same side of a polygon are known as adjacent vertices. Side AB has end points A and B. So A and B are adjacent vertices. Similarly A, D; D, C; C, B are pairs of adjacent vertices.

Diagonals : The line segments obtained by joining non-adjacent vertices are called the diagonals of the polygon. AC and BD are the diagonals of the polygon ABCD.



Polygons are further divided into various categories, depending upon the number of line segments they have

(i)  A three-sided polygon is called a **Triangle.**

(ii)  A four-sided polygon is called a **Quadrilateral.**

(iii)  A five-sided polygon is called a **Pentagon.**

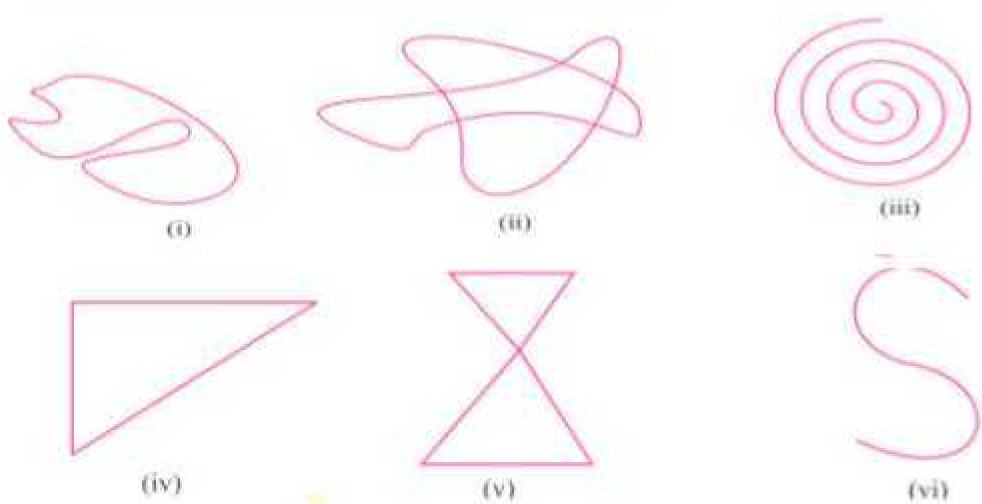
(iv)  A six-sided polygon is called a **Hexagon.**

Similarly seven, eight, nine and ten sided polygons are called heptagon, octagon, nonagon and decagon respectively.

Regular Polygon : If all sides of a polygon are equal and all angles are also equal, then it is called a regular polygon.

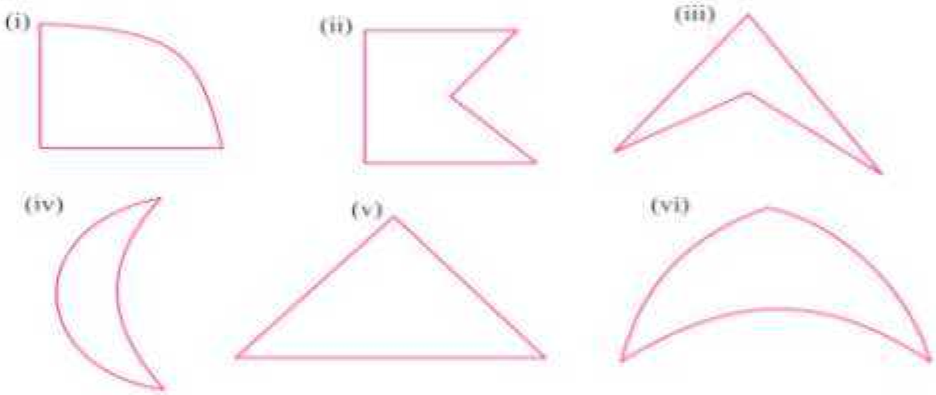
Exercise 8.2

1. (a) Which of the following are simple curves ?
(b) Classify the following as open or closed curve.



(vii) (viii)

2. Identify the polygons:



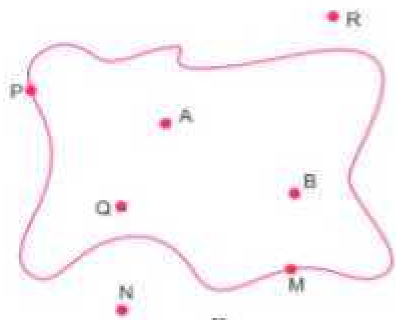
3. Draw any polygon and shade its interior.

4. Name the points which are:

   (i) In the interior of the closed figure.

   (ii) In the exterior of the closed figure.

   (iii) On the boundary of the closed figure.



5. In the given figure, name:

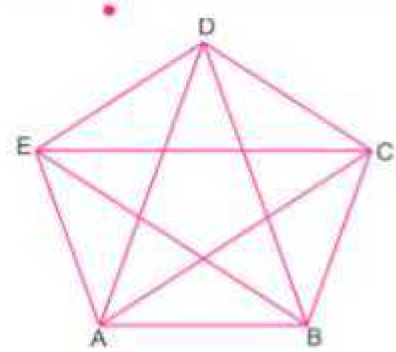
   (i) The vertices

   (ii) The sides

   (iii) The diagonals

   (iv) Adjacent sides of AB

   (v) Adjacent vertices of E.



8.11 Angle

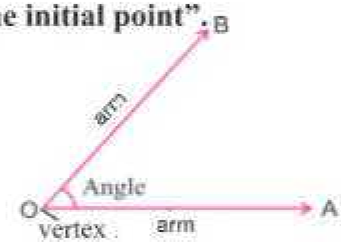
In our daily life, we come across many physical objects that have two edges (arms) joined together by a hinge. For example, two fingers of a hand, two hands of a clock, two sharp parts of scissors are inclined towards each other and have an opening (angle) between them.

Such objects give us the concept of an angle in Geometry.

"An angle is a figure formed by two rays with the same initial point".

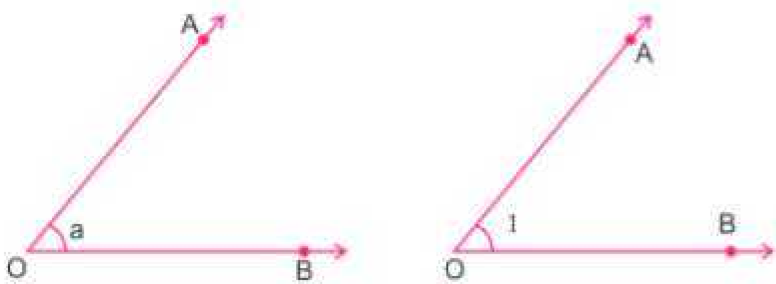
The common initial point is called the vertex of the angle and the rays forming the angle are called its arms.

In the figure, the common initial point O is the vertex and rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are the arms of the angle.



8.11.1 Naming an Angle

The symbol '∠' is used to denote an angle. There are several ways of naming an angle.



(i) The vertex is written in the middle and any two points on the arms of the angle are written as two extreme letters.

Thus, the given angle can be named as ∠AOB or ∠BOA.

(ii) Only the letter at the vertex of the angle alone can be written to name the angle.

Thus, the given angle can also be named as ∠O.

(iii) We can place a number 1, 2, 3......... etc. or a small letter a, b, c..... etc. near the small curve connecting the two arms of angle (as shown) and name the angle using that number or letter.

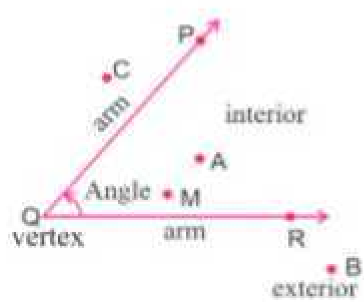
Thus, the given angle can also be named as ∠a or ∠1.

- **In naming an angle, the letter at the vertex should be in the middle.**

8.11.2 Interior and Exterior of an angle

An angle divides all the points in a plane into three parts.

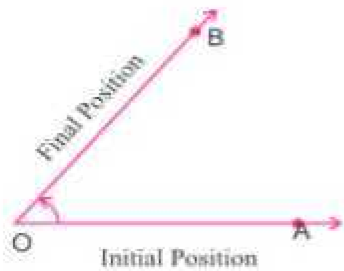
(i) The part of the plane which is within the arms of an angle produced indefinitely is called the **interior** of the angle. In the figure, points A and M are in the interior of ∠PQR.



(ii) The part of the plane which is outside the arms of an angle produced indefinitely is called the **exterior** of the angle. In the figure, points B and C are in the exterior of ∠PQR.

(iii) The part of the plane made by all those points that are on the angle is called the **boundary** of the angle. In the figure P, Q, R are on the boundary on ∠PQR.

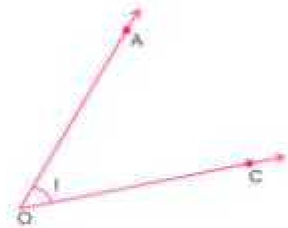
8.11.3 Angles as rotation of a ray



An angle can also be described by rotating a ray. Let there be a ray $\overrightarrow{OA}$ with initial point O. Suppose we rotate it and it occupies the final position $\overrightarrow{OB}$. We say that ∠AOB has been described by rotating a ray with O as vertex.

Its magnitude is the amount of rotation through which one of the arms must be rotated about the vertex to bring it to the position of the other arm.

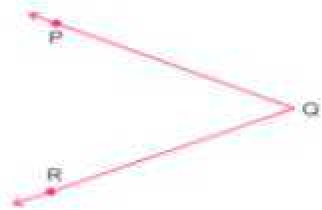
Let us consider some examples.



Example 3 : **Name the given angle in all ways.**

Solution : ∠AOC or ∠COA or ∠O.

Or ∠1

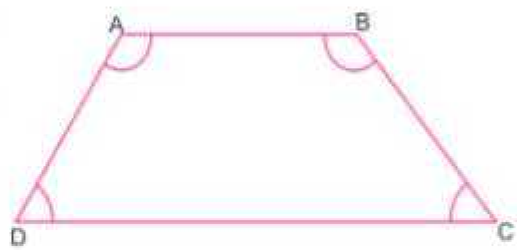


Example 4 : **Name the vertex and the arms of given ∠PQR.**

Solution : Vertex = Q

Arms of ∠PQR = $\overrightarrow{QP}$ and $\overrightarrow{QR}$

Example 5 : **Name all the angles of given figure.**



Solution : There are four angles in the given figure.

(i) ∠DAB or ∠BAD(A as vertex, $\overrightarrow{AB}$ and $\overrightarrow{AD}$ are arms)

(ii) ∠ABC or ∠CBA (B as vertex, BC and BA are arms)

(iii) ∠BCD or ∠DCB(C as vertex, CB and CD are arms)

(iv) ∠CDA or ∠ADC (D as vertex, DC and DA are arms)

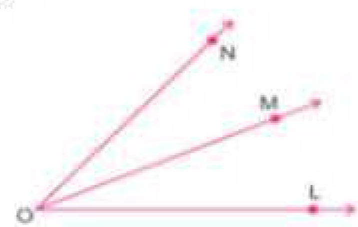
Example 6 : **Name all the angles in given figure.**

Solution : Clearly, there are three angles formed in the given figure

(i) ∠LOM or ∠MOL

(ii) ∠MON or ∠NOM

(iii) ∠NOL or ∠LON

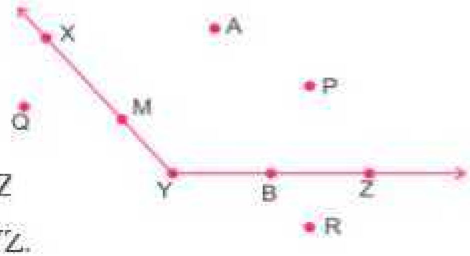


Example 7 : **In the given figure, name the points that lie :**

(i) In the interior of ∠XYZ

(ii) In the exterior of ∠XYZ

(iii) On ∠XYZ



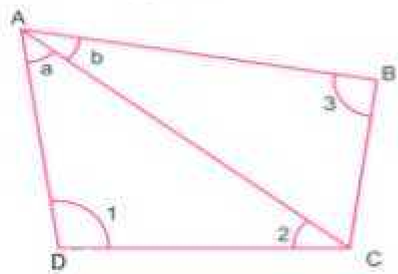
Solution : (i) Points A and P are in interior of ∠XYZ

(ii) Points Q are R in the exterior of ∠XYZ.

(iii) Points X, M, Y, B and Z are on ∠XYZ.

Example 8 : **In the given figure, write another name for the following angles.**

(i) ∠1 (ii) ∠2 (iii) ∠3 (iv) ∠a (v) ∠b



Solution : (i) ∠1 = ∠ADC or ∠CDA

(ii) ∠2 = ∠ACD or ∠DCA

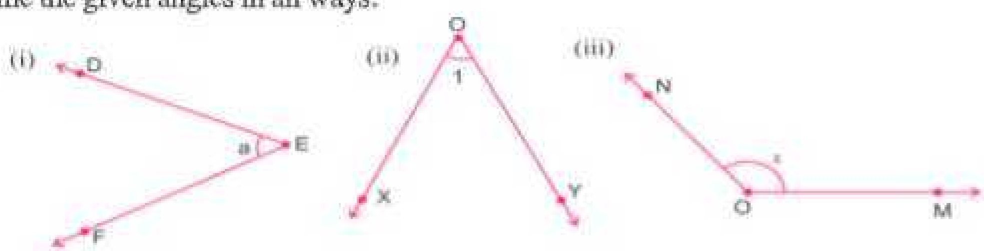
(iii) ∠3 = ∠CBA or ∠ABC

(iv) ∠a = ∠DAC or ∠CAD

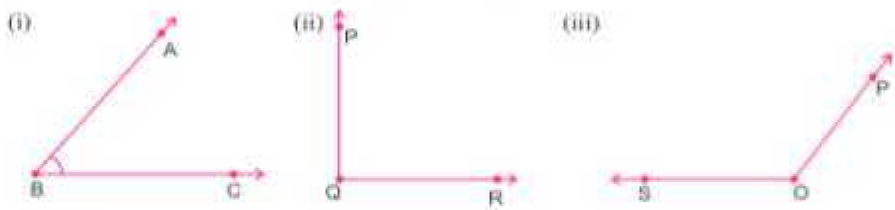
(v) ∠b = ∠BAC or ∠CAB

Exercise 8.3

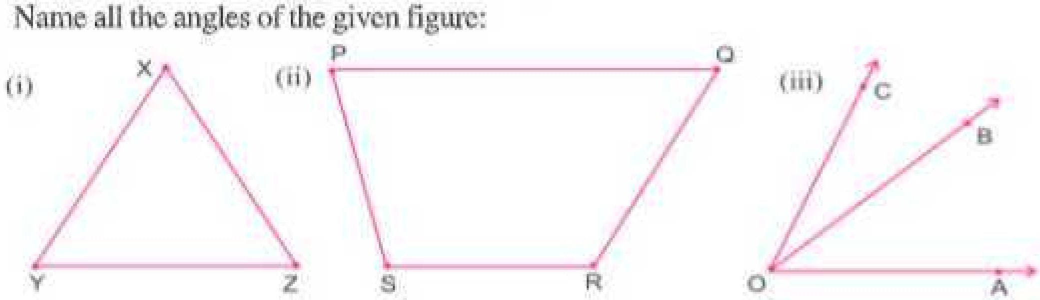
1. Name the given angles in all ways:



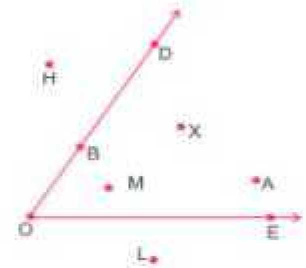
2. Name the vertex and the arms of given angles:



3. Name all the angles of the given figure:



4. In the given figure, name the points that lie

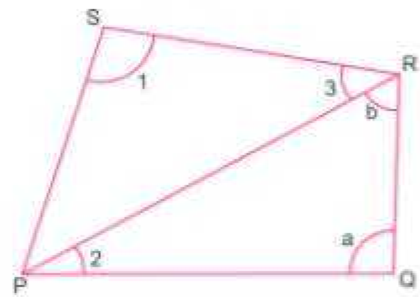


(i) In the interior of ∠DOE

(ii) In the exterior of ∠DOE

(iii) On the ∠DOE

5. In the given figure, write another name for the following angles:



(i) ∠1

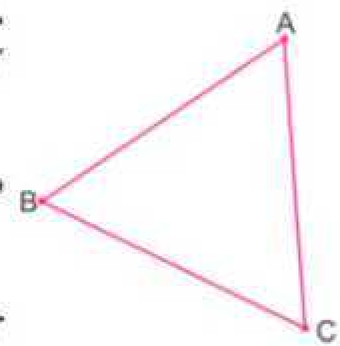
(ii) ∠2

(iii) ∠3

(iv) ∠a

(v) ∠b

8.12 Triangles



Take any three non-collinear points A, B and C. Join them as AB, BC and AC. The figure formed by joining three non-collinear points by line-segments is called a triangle.

A triangle is a closed figure made up of three line segments.

The symbol 'Δ' is used to denote a triangle. This triangle can also be written as ΔABC, ΔACB, ΔBCA, ΔBAC, ΔCAB.

(i) Three points A, B and C are called its **vertices.**

(ii) The line segments AB, BC are AC are called its three **sides.**

(iii) Three angles ∠ABC or ∠B, ∠BCA or ∠C, ∠BAC or ∠A are called its interior angles or simply **angles.**

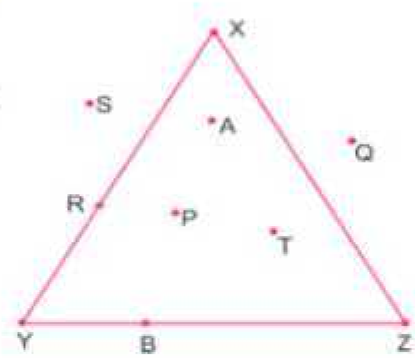
The three sides and the three angles of a triangle taken together are called the **six parts or elements** of a triangle.

Note that three vertices are not part of the triangle as these can not be measured.

In ΔABC, we observe that the sides AB and AC meet at vertex A and BC is the remaining side. So we can say that 'BC is the side opposite to vertex A and A is the vertex opposite BC."

A triangle is a polygon with least number of sides.

8.12.1 Interior and Exterior of a Triangle



A triangle divides all the points in a plane into three parts.

(i) The part of the plane made by all those points that are enclosed by the triangle is called the **interior** of the triangle.

In the figure, points A, P and T are in the interior of ΔXYZ.

(ii) The part of the plane made by the points that are on the triangle is called the **boundary** of the triangle.

In the figure, X, R, Y, B and Z are on the boundary of ΔXYZ.

The interior of ΔXYZ together with its boundary is called the **triangular region.**

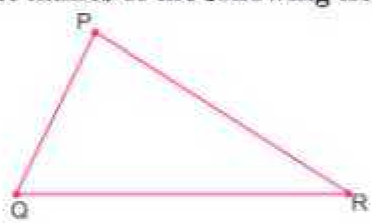
(iii) The part of the plane made by the points that are not enclosed by the triangle is called the exterior of the triangle.

In the figure Q and S are in the exterior of ΔXYZ.

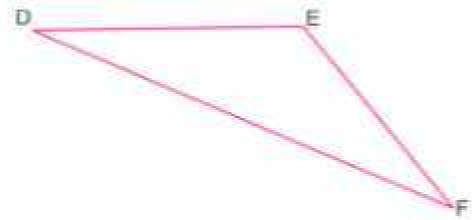
Let's consider some examples.

Example 9 : Write names of the following triangles in all orders:

(i)



(ii)



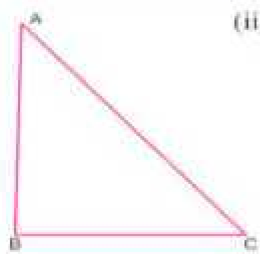
Solution : (i) Names of the given triangle are

ΔPQR, ΔPRQ, ΔQPR, ΔQRP, ΔRPQ or ΔRQP

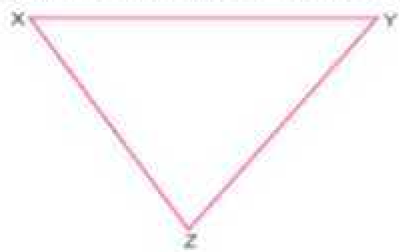
(ii) Names of the given triangle are Δ DEF , ΔDFE, ΔEDF, ΔEFD, ΔFED or ΔFDE.

Example 10 : Write the name of vertices, sides and angles of the following triangles:

(i)



(ii)



Solution : (i) (a) Vertices = A, B and C

(b) Sides = AB or BA, BC or CB, AC or CA

(c) Angles = ∠BAC or ∠A, ∠ACB or ∠C, ∠ABC or ∠B.

(ii) (a) Vertices = X, Y and Z

(b) Sides = XY or YX, YZ or ZY, XZ or ZX.

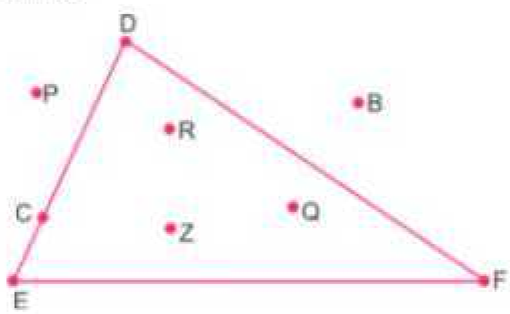
(c) Angles = ∠XYZ or ∠Y, ∠YZX or ∠Z, ∠ZXY or ∠X.

Example 11 : In the given figure, name the points that lie

(i) On the boundary of ΔDEF

(ii) In the interior of ΔDEF

(iii) In the exterior of ΔDEF



Solution : (i) Points on the boundary of ΔDEF = D, C, E, F

(ii) In the interior of ΔDEF = R, Z, Q

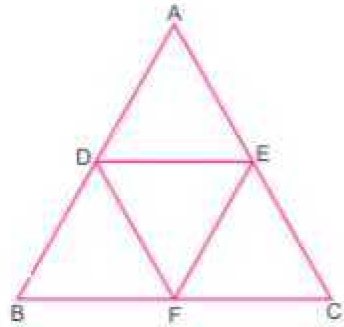
(iii) In the exterior of ΔDEF = P, B

Example 12 : In the given figure, write the name of

(i) All triangles

(ii) Triangles have A as the vertex.

(iii) Triangles having E as the vertex.



Solution : (i) There are five triangles, i.e.

ΔADE, ΔDEF, ΔDBF, ΔEFC and ΔABC

(ii) There are two triangles having A as the vertex.

ΔADE, ΔABC

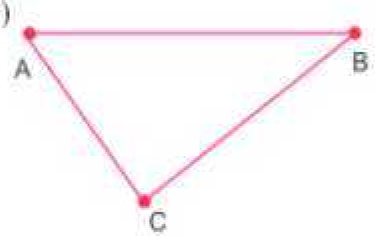
(iii) There are three triangles having E as the vertex.

ΔEDA, ΔEDF, ΔEFC

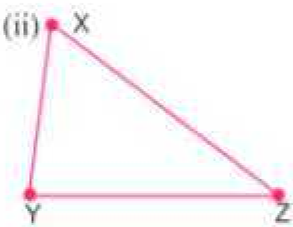
Exercise 8.4

1. Write all the names of the following triangles in all orders:

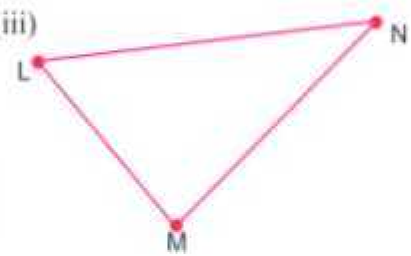
(i)



(ii)

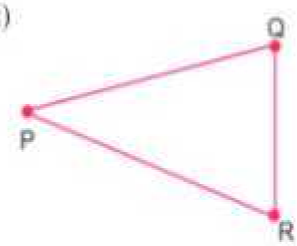


(iii)

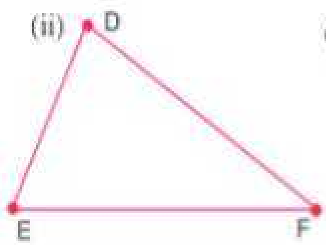


2.. Write the name of vertices. sides and angles of the following triangles:

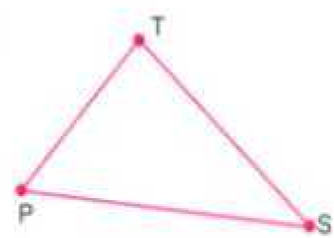
(i)



(ii)



(iii)

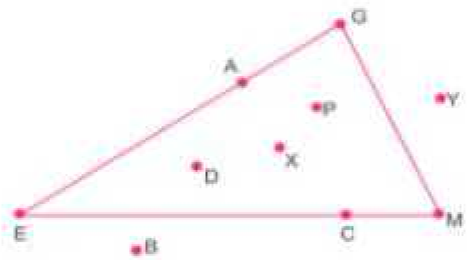


3. In the given figure, name the points that lie

(i) On the boundary of ΔGEM

(ii) In the interior of ΔGEM

(iii) In the exterior of ΔGEM

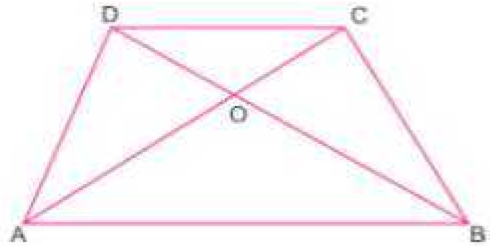


4. In the given figure, write the name of

   (i) All different triangles.

   (ii) Triangles having O as the vertex.

   (iii) Triangles having A as the vertex.



5. Fill in the blanks of the following:

   (i) A triangle has .... vertices.

   (ii) A triangle has .... angles.

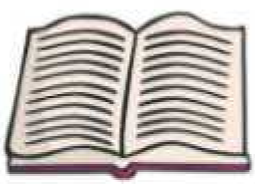
   (iii) A triangle has .... sides.

   (iv) A triangle divides the plane into .... parts.

   (v) A triangle has .... parts.

8.13. Quadrilateral

If you look around, you will find many objects which are geometrically four-sided closed figure. For example top of a book, front side of a door, surface of a blackboard etc. All these objects resembles a four-sided closed figure. This is called Quadrilateral.

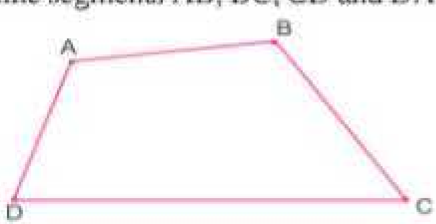


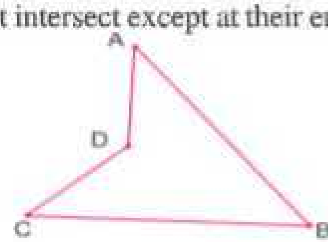
A quadrilateral is a four sided closed figure bounded by four line segments. It is a four-sided polygon. The word quadrilateral is derived from two words: **'Quadri' means 'four' and 'lateral' means 'side'.**

Let, A, B,C, D be four points in a plane such that

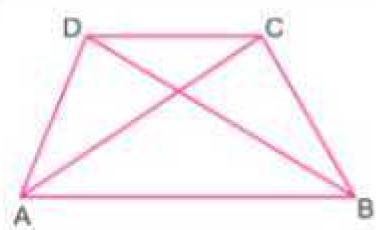
(i) At least three of them are non collinear.

(ii) The line segments AB, BC, CD and DA do not intersect except at their end points.





A quadrilateral is named by writing its vertices in a cyclic manner i..e ABCD or BCDA (not ABDC) Now a quadrilateral ABCD has



(i) Four vertices, namely A, B, C and D.

(ii) Four sides, namely AB, BC, CD and DA

(iii) Four angles $\angle A$, $\angle B$, $\angle C$ and $\angle D$.

(iv) Two diagonals AC and BD.

The line segments joining the opposite vertices of a quadrilateral are called its diagonals. Two diagonals divide the quadrilateral into four triangles.

Adjacent sides : Two sides of a quadrilateral are called its adjacent sides, if they have a common end point (vertex).

In the figure AB, BC; BC, CD ; CD, DA; DA, AB are four pairs of adjacent sides of quadrilateral ABCD.

Opposite sides : Two sides of a quadrilateral are called its opposite sides, If they do not have a common end-point.

In the figure, AB, CD and AD, BC are two pairs of opposite sides of the quadrilateral ABCD.

Adjacent Angles : Two angles of a quadrilateral are called adjacent angles if they have a common side (arm).

In the figure $\angle A$ and $\angle B$, $\angle B$ and $\angle C$, $\angle C$ and $\angle D$, $\angle D$ and $\angle A$ are pairs of adjacent angles.

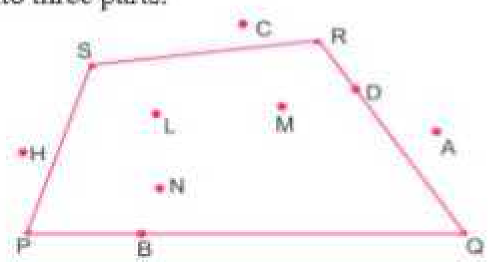
Opposite Angles: Two angles of a quadrilateral are called opposite angles if they are not adjacent angles.

In the figure, $\angle A$ and $\angle C$, $\angle B$ and $\angle D$ are pairs of opposite angles.

8.13.1 Interior and Exterior of A Quadrilateral

A quadrilateral divides all the points in a plane into three parts.

(i) The region inside the quadrilateral is called the **interior** of the quadrilateral. In the figure, L, M, N are the interior points of the quadrilateral PQRS.



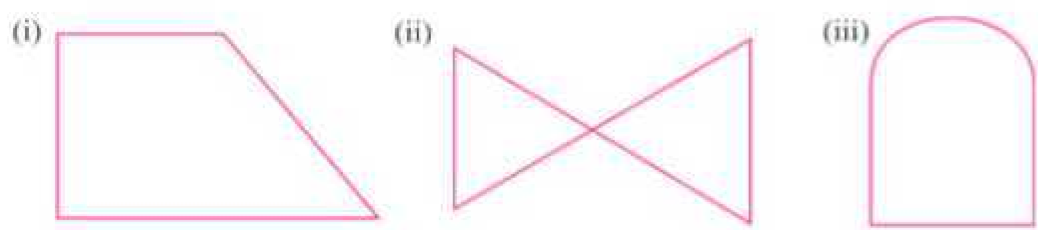
(ii) The points which lie on the boundary of the quadrilateral are said to be on the quadrilateral. In the figure points P, B, Q, D, R and S are on the quadrilateral.

The interior of the quadrilateral together with its boundary is called the **Quadrilateral region.**

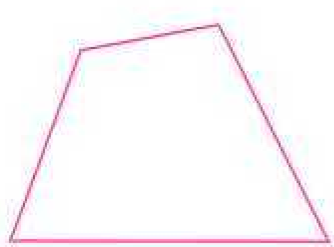
(iii) The region lying outside the boundary of the quadrilateral is called the **exterior** of the quadrilateral.

In the figure, points A, C and H are in the exterior of the quadrilateral PQRS.

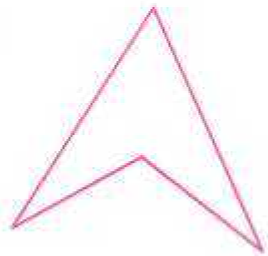
Example 13 : Out of the following, identify the quadrilateral.



(iv)



(v)



Solution : (i) This is a quadrilateral as all sides meet at end points.

(ii) Although it has four sides, but it is not a quadrilateral as some of its sides intersect each other at more than two points

(iii) This is not a quadrilateral as it has three sides and a curve.

(iv) This is a quadrilateral having four sides meet at end points.

(v) This is a quadrilateral having four sides meet at end points.

Example 14 : For the given quadrilateral EFGH, name

(i) All the vertices **(ii) All sides**

(iii) All angles **(iv) Side opposite to HE**

(v) Angles adjacent to G

Solution : (i) Vertices = E, F, G, H

(ii) Sides = EF, FG, GH, HE

(iii) Angles = $\angle E$, $\angle F$, $\angle G$, $\angle H$

(iv) Side opposite to HE is GF.

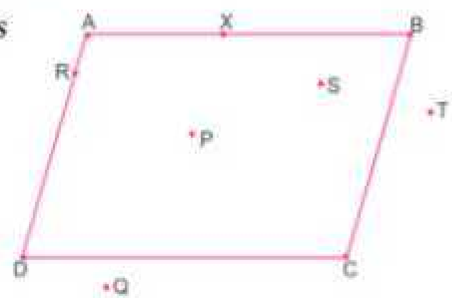
(v) Angles adjacent to G are $\angle H$ and $\angle F$.

Example 15 : In the given figure ABCD, name the points

(i) In its interior

(ii) In its exterior

(iii) On its boundary



Solution : (i) Points in interior of ABCD are P, S

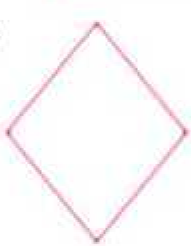
(ii) Points in exterior of ABCD are Q, T

(iii) Points on its boundary are A, X, B, C, D, R

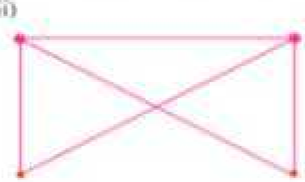
Exercise 8.5

1. Out of the following, Identify the quadrilateral :

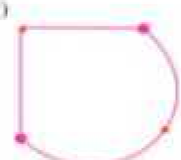
(i)



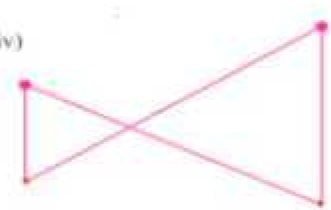
(ii)



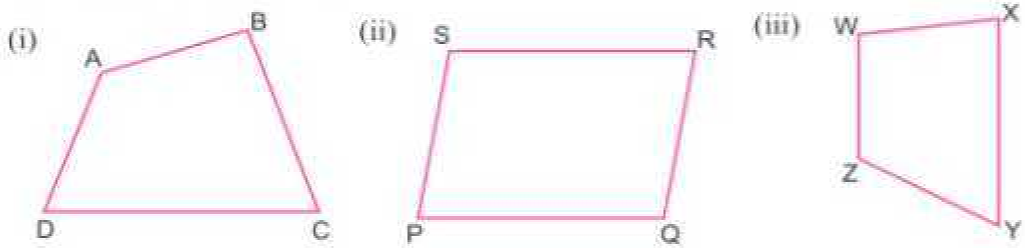
(iii)



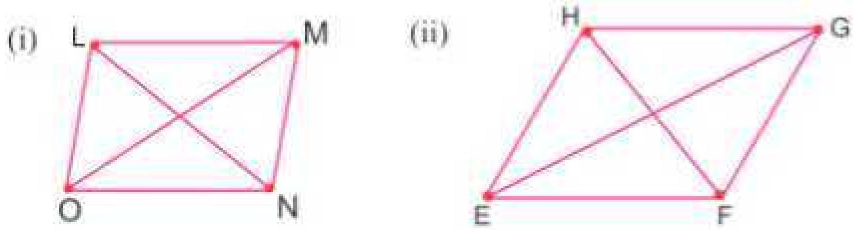
(iv)



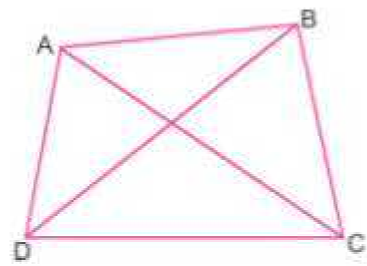
2. Name the given quadrilaterals:



3. Write the name of all vertices, angles, sides, diagonals of the following quadrilaterals:



4. For the given quadrilateral ABCD, name:



(i) Side opposite to AB

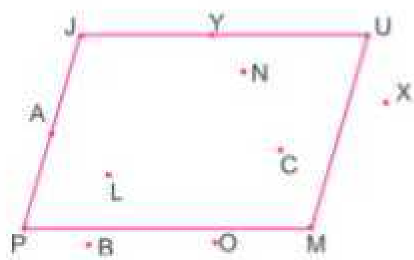
(ii) Angles adjacent to $\angle B$

(iii) Diagonal joining B and D

(iv) Angle opposite to $\angle A$

(v) Sides adjacent to CD

5. In the given quadrilateral JUMP, name the points.



(i) In its interior

(ii) In its exterior

(iii) On its boundary

6. Fill in the blanks:

(i) A quadrilateral has .... vertices.

(ii) A quadrilateral has .... sides.

(iii) A quadrilateral has .... angles.

(iv) A quadrilateral has .... diagonals.

(v) A diagonal divides the quadrilateral into .... triangles.

(vi) A line segment joining the opposite vertices of a quadrilateral is called its ....

(vii) The interior and the boundary of a quadrilateral together consitute the .... region.

7. State True or False:

(i) A diagonal divides quadrilateral into four triangles.

(ii) The angle that have a common vertex are called adjacent angles.

(iii) The sides that have a common vertex are called adjacent sides.

(iv) A quadrilateral has four diagonals.

(v) The quadrilateral region consists of the exterior and the boundary of the quadrilateral.

8.14. Circle

Think of circular shapes and you will find many shapes around you.

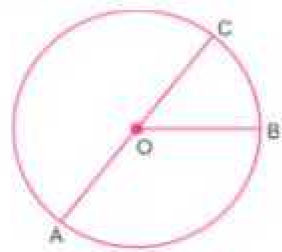
For Example : Bangle, Coin, Chapati etc.

A circle is one of the most common geometrical shape that always reminds you of a round shape. A circle is a simple closed curve not a polygon.

A circle is the set of all those points in a plane whose distance from a fixed point remains constant.

There is only one circle passing through three non-collinear points.

8.14.1 Parts of a circle

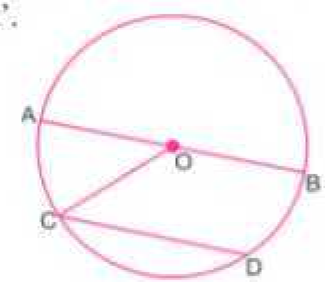


Centre : The fixed point inside a circle from which all the points on the circle are at equal distance is called the **centre** of the circle.

In the figure, O is the centre of the circle.

Radius : The distance from any point on the circle to its centre is called the radius of that circle. It is denoted by 'r'. In the figure OA, OB and OC are radius of the circle. and also OA=OB=OC.

Diameter : A line segment passing through the centre of the circle, connecting any two points on the circle is called the diameter of that circle. It is denoted by 'd'.



In the figure , AB is the diameter of the circle.

- Note that centre (O) is the mid point of the diameter.
- Diameter is twice of the radius of the circle.

i.e. Diameter = 2 × radius

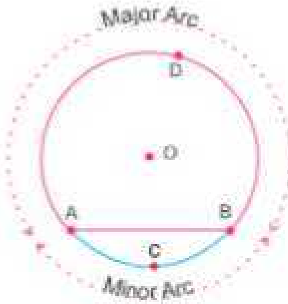
or d = 2r

Chord : A line segment joining any two points on a circle is called the chord of that circle.

In the figure CD and AB are chords.

- **The diameter of a circle is the longest chord of a circle.**

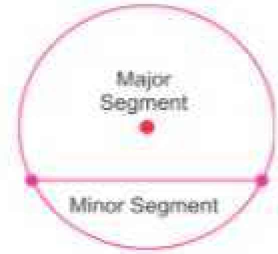
Arc : A part of circle is called the arc of a circle.



In the figure the portion ACB of the circle is an arc and written as $\overset{\frown}{ACB}$ or $\overset{\frown}{AB}$.

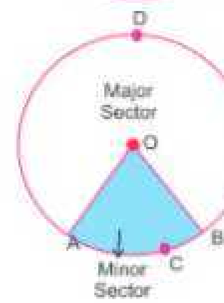
- A chord of a circle divides it into two parts. The smaller part is called the minor arc and the larger part is called the major arc. Here ACB is a minor arc of a circle and ADB is a major arc of a Circle.

Segment : The region enclosed by an arc and its corresponding chord is called a segment.



The segment which is formed by a minor arc of a circle and its corresponding chord is called a minor segment. The segment which is formed by a major arc of a circle and its corresponding chord is called a major segment.

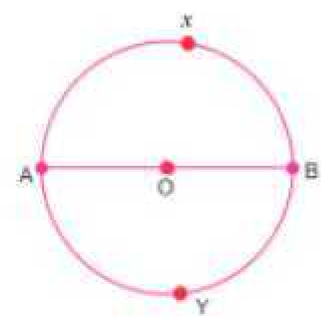
Sector : The region enclosed by an arc and two radii of a circle (which join the end points of the arc) is called a sector.



The sector formed by minor arc is called minor sector. The sector formed by the major arc is called the major sector.

In the given figure, $\overset{\frown}{AB}$ is an arc and OA and OB are two radii which divide the circular region into two parts. OACB is minor sector and OADB is major sector.

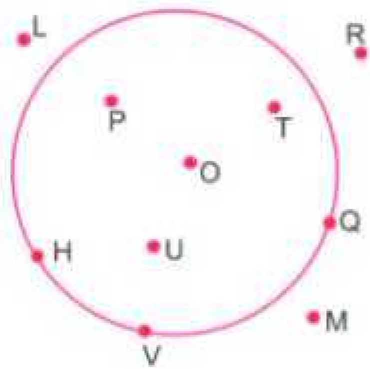
Semi-circle: The diameter of the circle divides the circle into two equal parts. Each part is called Semi-circle.



In the figure, AB is a diameter of the circle with centre O. Each part AXB and AYB into which the circle is divided, is a semi-circle.

8.14. Interior and Exterior of a circle

A circle divides all the points in the plane into three parts.



(i) The part of the plane made by all those points that are enclosed by the circle is called the interior of the circle.

In the figure, points O, T, P, U are in the interior of the circle.

(ii) The part of the plane made by all those points that are on the circle is called the boundary of the circle.

In the figure, points V, Q, H are on the boundary of the circle.

- The length of the boundary of a circle is also known as the circumference of the circle.
- The interior and the boundary of the circle constitute the circular region.

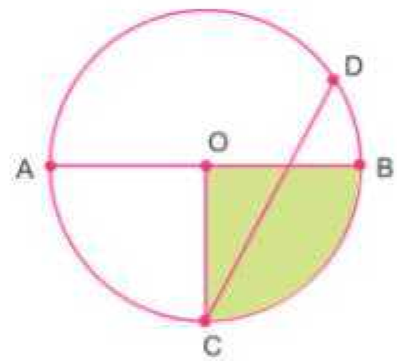
(iii) The part of the plane made by all those points that are not enclosed by the circle is called the exterior of the circle.

In the figure, points L, R and M are on the exterior of the circle,

Let us consider some examples.

Example 16 : See the adjoining figure and write the name of :

(i) Centre (ii) Radius (iii) Diameter (iv) Chord
(v) Arc of shaded portion (vi) Shaded Sector



Solution : (i) Centre : O

(ii) Radii OA, OB, OC

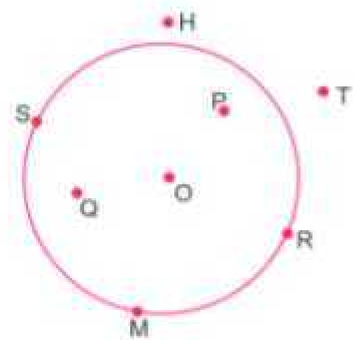
(iii) Diameter : AB

(iv) Chord : AB, CD

(v) Arc : $\overset{\frown}{CB}$ or $\overset{\frown}{BC}$

(vi) Sector : OBC

Example 17 : In the given figure name the points:



(i) In its interior

(ii) In its exterior

(iii) On its boundary

Solution : (i) Points in the interior of circle = O, P, Q

(ii) Points in the exterior of circle = T, H

(iii) Points on the boundary of circle = S, M, R

Example 18 : If the radius of the circle is 3cm, then find the diameter of the circle.

Solution : Given radius of the circle = 3cm

We know diameter = 2× radius

$$= 2 \times 3 = 6 \text{ cm}$$

Example 19 : If the diameter of the circle is 20cm, then find the radius of the circle.

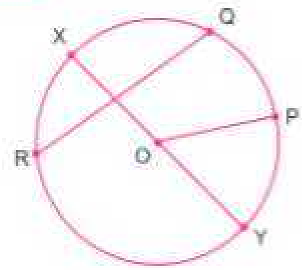
Solution : Given diameter of circle = 20cm

∴ Radius of circle = diameter ÷ 2

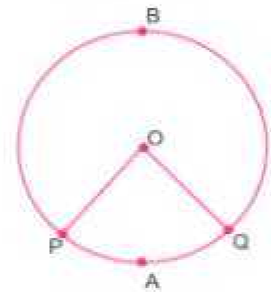
$$= 20 \div 2 = 10\text{cm}$$

Exercise 8.6

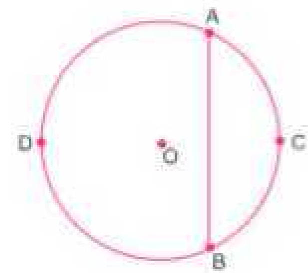
1. In the given figure, write the name of
   - (i) Centre (ii) Radii
   - (iii) Diameter (iv) Chord



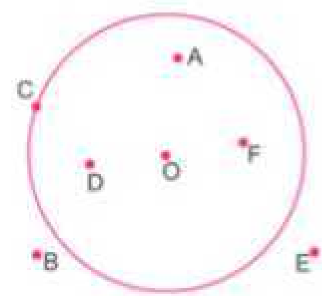
2. In the given figure, write the name of
   - (i) minor arc (ii) major arc
   - (iii) minor sector (iii) major sector



3. In the given figure, write the name of
   - (i) Minor segment
   - (ii) Major segment



4. In the given figure, name the points
   - (i) In its interior
   - (ii) On its boundary (circumference)
   - (iii) In its exterior



5. Find the diameter of the circle whose radius is
   - (i) 5cm (ii) 4m (iii) 10cm
6. If the diameter of a circle is 12cm. Find the radius.
7. Fill in the blanks:
   - (i) The distance around a circle is called ....
   - (ii) The diameter of a circle is .... times its radius.
   - (iii) The longest chord of circle is ....
   - (iv) All the radii of a circle are of .... length.
   - (v) The diameter of a circle passes through ....
   - (vi) A circle divides all the points in a plane into .... parts.
8. State true or false:
   - (i) The diameter of a circle is equal to its radius.
   - (ii) The diameter is a chord of circle.

(iii) A radius is a chord of the circle.

(iv) Every circle has a centre.

(v) The region enclosed by a chord and arc is called a segment.

Multiple Choice Questions

1. How many lines can pass through a point?

(a) 1 (b) 2 (c) 4 (d) Infinite

2. The number of points lie on a line are ....

(a) 2 (b) 4 (c) 1 (d) Infinite

3. The number of lines passes through two points are ....

(a) 1 (b) 2 (c) 3 (d) Infinite

4. In how many parts, a closed curve divides the plane?

(a) 1 (b) 2 (c) 3 (d) 4

5. A quadrilateral has .... diagonals.

(a) 1 (b) 2 (c) 3 (d) 4

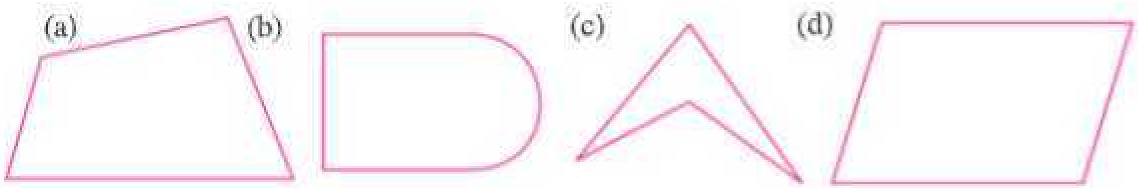
6. Which of the following is not a polygon?

(a) Triangle (b) Pentagon (c) Circle (d) Quadrilateral

7. A triangle has .... parts.

(a) 3 (b) 6 (c) 9 (d) 2

8. Which of the following is not a quadrilateral?



9. A line segment joining the opposite vertices of a quadrilateral is called its ....

(a) Diagonal (b) Side (c) Angle (d) Region

10. The radius of a circle is 4cm then the diameter is ....

(a) 8cm (b) 2cm (c) 6cm (d) 12cm

11. The diameter of a circle is 12cm then the radius is ....

(a) 24cm (b) 6cm (c) 18cm (d) 4cm

12. The longest chord of a circle is ....

(a) Arc (b) Perimeter (c) Diameter (d) Radius

Learning Outcomes

After completion of this chapter the students are now able to

(i) Know about point, line,line segment and ray.
(ii) Know about curve and its types.
(iii) Know about different types of polygons i.e triangle and quadrilateral etc and their parts.
(iv) Know about circle and its parts.
(v) Describe the geometrical concepts with surroundings.

ANSWER KEY

Exercise 8.1

2. AB, AC, AD, BC, CD, BD 3. Infinite 4. Infinite 5. One

6. (i) O, A, B, C, D, or E (ii) BE (iii) $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$, $\overrightarrow{OD}$ or $\overrightarrow{OE}$
   (iv) OA. OB, OC, OD, OE , DE

7. $\overrightarrow{PQ}$, $\overrightarrow{PR}$, $\overrightarrow{QR}$

8. (i) ℓ and m (ii) p and n, n and ℓ, n and m, p and ℓ, p and m.
   (iii) m and n (iv) P, Q, S and P, R, T

9. (i) n and p, q and p, n and q
   (ii) m and ℓ, m and n, m and p, m and q, ℓ and n, ℓ and p, ℓ and q
   (iii) p and ℓ. (iv) E (v) G, E, C, A and F, D, C, B

10. (i) ℓ, n (ii) ℓ and m (iii) B (iv) m and ℓ, n and ℓ.

11. (i) F (ii) T (iii) F (iv) F (v) T

Exercise 8.2

1. (a) Simple curves : (i), (iii), (iv), (vi), (vii), (viii)
   (b) Open curves : (iii), (vi), (viii)
   Closed curves : (i), (ii), (iv), (v), (vii)

2. (ii), (iii), (v)

4. (i) A, B, Q (ii)R, N (iii) P, M

5. (i) D, E, A, B, C (ii) AB, BC, CD, DE, EA
   (iii) AC, AD, BE, BD, CE (iv) AE and BC (v) A and D

Exercise 8.3

1. (i) ∠DEF, ∠FED, ∠E, ∠a (ii) ∠XOY, ∠YOX, ∠O, ∠1
   (iii) ∠NOM, ∠MON, ∠O, ∠x

2.

	(i)	(ii)	(iii)
Vertex	B	Q	O
Arm	$\overrightarrow{BC}, \overrightarrow{BA}$	$\overrightarrow{QP}, \overrightarrow{QR}$	$\overrightarrow{OS}, \overrightarrow{OP}$

3. (i) ∠X, ∠Y, ∠Z (ii) ∠P, ∠Q, ∠R, ∠S (iii) ∠AOB, ∠BOC, ∠AOC
4. (i) A, X, M (ii) H, L (iii) D, B, O, E
5. (i) ∠S or ∠PSR or ∠RSP (ii) ∠RPQ or ∠QPR
   (iii) ∠SRP or ∠PRS (iv) ∠Q or ∠RQP or ∠PQR (v) ∠PRQ or ∠QRP

Exercise 8.4

1. (i) ΔABC, ΔACB, ΔBAC, ΔBCA, ΔCAB, ΔCBA.
   (ii) ΔXYZ, ΔXZY, ΔYZX, ΔYXZ, ΔZXY, ΔZYX.
   (iii) ΔLMN, ΔLNM, ΔMNL, ΔMLN, ΔNML, ΔNLM.

2.

	(i)	(ii)	(iii)
Vertices	P, R, Q	D, E, F	T, P, S
Sides	PR, QR, PQ	DE, EF, DF	TP, PS, TS
Angles	∠P, ∠R, ∠Q	∠D, ∠E, ∠F	∠T, ∠P, ∠S

3. (i) G, A, E, C, M (ii) P, X, D (iii) Y, B
4. (i) ΔAOD, ΔDOC, ΔBOC, ΔAOB, ΔABD, ΔBCD, ΔACD, ΔABC
   (ii) ΔAOB, ΔBOC, ΔCOD, ΔAOD
   (iii) ΔAOB, ΔAOD, ΔABD, ΔABC, ΔACD
5. (i) 3 (ii) 3 (iii) 3 (iv) 3 (v) 6

Exercise 8.5

1. (i) 2. (i) ABCD (ii) PQRS (iii) XYZW
3. (i) Vertices = O, N, M, L ; Angles = ∠O, ∠N, ∠M, ∠L
   Sides = ON, NM, ML, LO ; Diagonals = OM, NL
   (ii) Vertices = H, G, F, E ; Angles = ∠H, ∠G, ∠F, ∠E
   Sides = HG, GF, FE, EH ; Diagonals = EG, FH

4. (i) CD (ii) ∠A and ∠C (iii) BD (iv) ∠C (v) AD and BC

5. (i) L, N, C (ii) B, O, X (iii) P, M, U, Y, J, A

6. (i) 4 (ii) 4 (iii) 4 (iv) 2 (v) 2 (vi) Diagonal
(viii) Quadrilateral

7. (i) F (ii) F (iii) T (iv) F (v) F

Exercise 8.6

1. (i) O (ii) OP, OX, OY (ii) XY (iv) XY and QR

2. (i) PAQ (ii) PBQ (iii) OPAQ (iv) OPBQ

3. (i) ACBA (ii) ADBA

4. (i) A, O, F, D (ii) C (iii) B, E

5. (i) 10cm (ii) 8m (iii) 20cm

6. 6cm

7. (i) Circumference (ii) Two (iii) Diameter (iv) Equal
(v) Centre (vi) 3

8. (i) F (ii) T (iii) F (iv) T (v) T

Multiple Choice Questions

(1) d (2) d (3) a (4) c (5) b (6) c

(7) b (8) b (9) a (10) a (11) b (12) c

W1I2R9

9

UNDERSTANDING ELEMENTARY SHAPES

Objectives

In this chapter you will learn

- To compare line segments in different ways.
- To measure line segments, angles etc.
- To understand angles by examples in the surroundings.
- To understand about polygons.
- To understand about 3D shapes from the surroundings.

9.1 Introduction

In the previous chapter, we have studied some basic geometrical concepts such as point, line, ray, line segments, angle, triangle etc.

The basic shapes around us are either made up of straight lines or curved lines. They have corners, edges, planes, they may be open or closed curves. We can classify them into line segments, angles, polygons, circles etc. All these shapes have different sizes and measurements. Let us learn to measure and compare these shapes.

9.2 Measuring And Comparing line Segments

We know that a line segment is a part of line with two end points.

Thus, two points in a plane determine exactly one segment. The measure of line segment i.e. shortest distane between these two points is called its length. It is measured in metres, centimetres, millimeteres etc. A line segment has fixed length of a line but no breadth or thickness. The fixed length of a line segment makes its measurement and comparison possible.

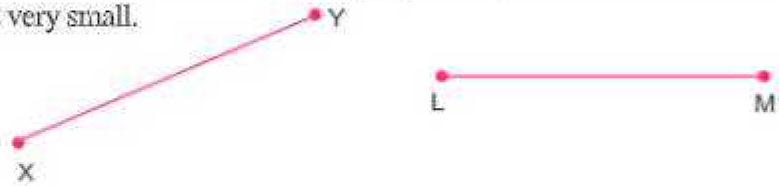
9.2.1 Comparing line segments

Comparing two line segments means finding the shorter or longer line segment among them. We can compare two line segments by different methods.

Method 1. **Comparison by observation :**

Look at the line segments AB and CD.

Just by observing them, we can easily find out that line segments AB is shorter than CD. i.e. AB<CD. But this is not always possible, if the difference between their lengths is very small.



Here both the line segments XY and LM appear to be of the same length and It is difficult to say which one is longer or shorter by just looking at them.

So we need some accurate methods for comparison.

Method 2. **Comparison by Tracing : Let us compare AB and CD by tracing method**

Trace AB on a tracing paper and place it on CD in such a way that the point A coincides with point C.

There can be three posibilities

(i) B is between C and D. We say that AB is shorter than CD i.e. AB<CD

(ii) B is exactly on D. We say that AB is equal to CD i.e. AB = CD.

(iii) B is beyond D. We say that AB is longer than CD i.e. AB>CD.

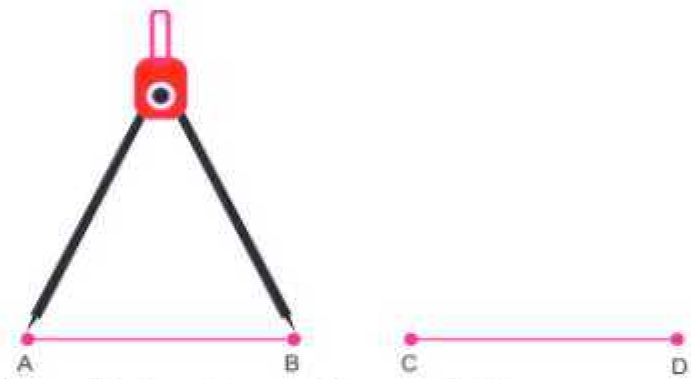
Method 3. **Comparison by Divider :**

Look in your gemoetry box you will notice an object with two pointed arms, hinged together with the help of a knob,this object is known as a divider.

Let us compare the two line segments AB and CD using a divider.

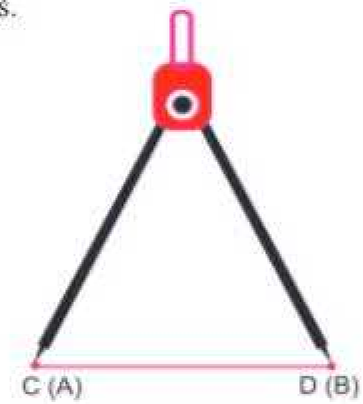
Place the needle of one hand of the divider at A and open other hand carefully so that it coincides with B.

Now, lift the divider carefully so that the opening of two arms remains unchanged. Place one of the needle at C of line segments CD and other arm is free to fall at any

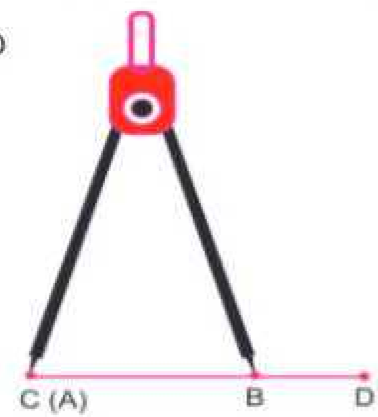


point on CD. Now, there are three possibilities.

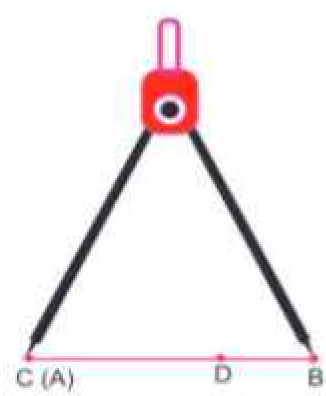
(i) The other arm falls exacity at D, then AB = CD.



(ii) The other arm falls between C and D such that AB< CD.



(iii) The other arm falls beyond D such that AB>CD.



These methods are not useful where we want to know by how much a line segment is longer or shorther than the other. Now let us learn to measure the lengths of the line segments.

9.2.2. Measurement of line segment

Method 1. **Measuring by ruler**

To measure line segments, we use a scale that has centimetre marks on one edge and inch marks on the other edge.

Observe that each centimeter (cm) is divided into ten equal parts and each part is called millimetre (mm).

To measure AB, keep the ruler in such a way that point A of the line segment coincides with the 'o' mark of the ruler. Then read the mark on the ruler against point B.

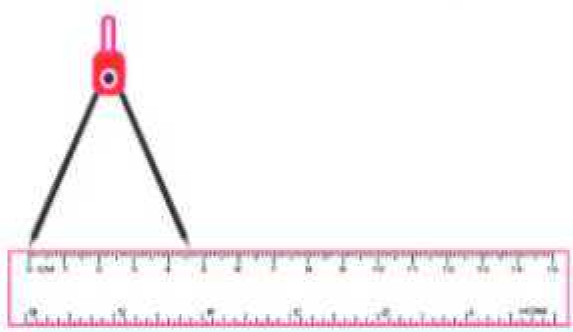
Hence, the length of line segment AB = 6cm

Method 2. **Measuring by both ruler and divider**

Let us use a ruler and a divider to measure the length of AB.

Open the arms of divider in such a way that one of its arms is at A and the other is at B.

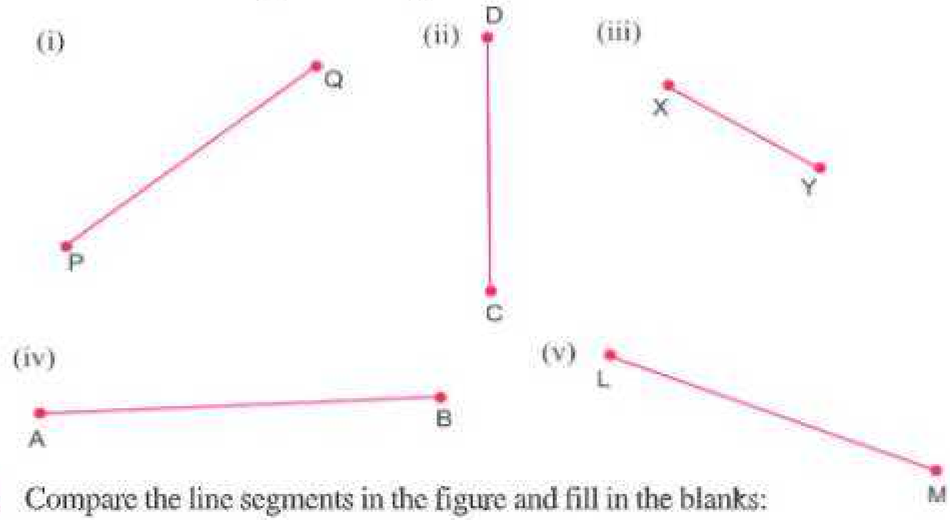
Now, lift the divider without disturbing its arms and place it on the ruler such that one of its arms is at mark '0'. Read the mark against the other arm of the divider.



The other arm of the divider is at 4.5 cm mark of the ruler. Thus lenght AB = 4.5cm.

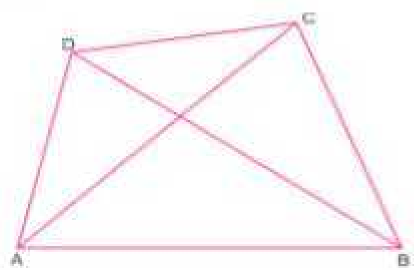
Exercise 9.1

1. Measure the line segments using a ruler and a divider:



2. Compare the line segments in the figure and fill in the blanks:

   (i) AB ____ AB
   (ii) CD ____ AC
   (iii) AC ____ AD
   (iv) BC ____ AC
   (v) BD ____ CD

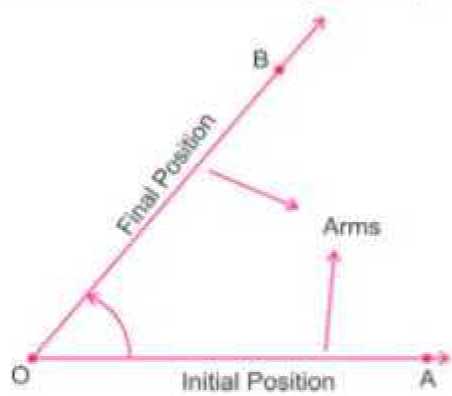


3. Draw any line segment AB. Take any point C between A and B. Measure the lengths of AB, BC and AC. Is AB = AC + CB ?

4. Draw a line segment AB = 5cm and AC = 9cm in such a way that points A, B, C are collinear. What is the length of BC?

9.3 Measuring Angles

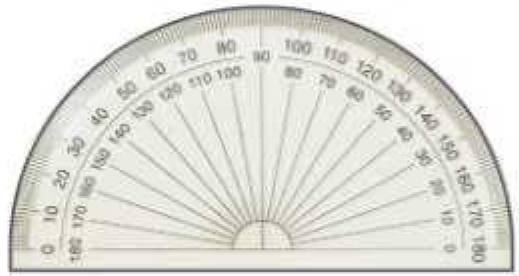
In the previous chapter, we have learnt that an angle is a figure formed by two rays with the same initial point. An angle can also be described by rotating a ray over another ray.



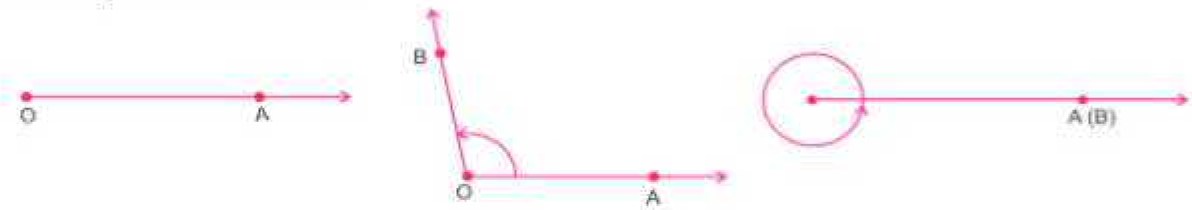
The magnitude or the size of an angle is the amount of rotation through which one of the arms must be rotated about the vertex to bring it to the position of the other arm.

The magnitude of an angle depends upon the opening or inclination between the two rays that form the angle. If two angles have different inclination, then we say that they have different magnitudes. The magnitudes of an angle can be measured with the help of Protactor in degrees.

Protactor : Look into your geometry box, there is a geometrical instrument that looks like the letter D. The angles are marked from 0° to 180° on the edge in clockwise direction as well as in anticlockwise direction.



Degree measure of angles : Consider a ray OA. Rotate this ray starting from its initial position, keeping the point O fixed. When the ray comes back to its initial position, we say that the ray has completed one revolution.



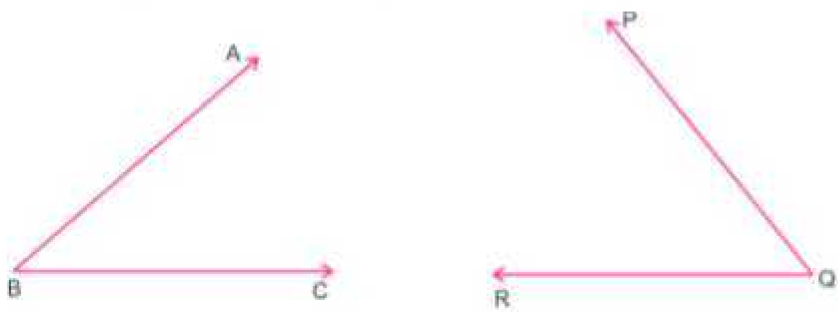
(i) (ii) (iii)

One revolution is divided into 360 equal parts and each point is called 'one degree.'

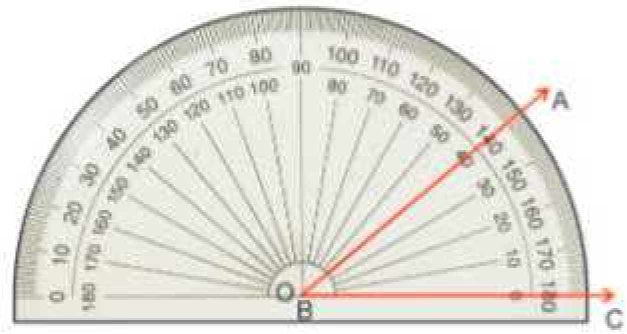
The standard unit for measuring an angle is 'degree'. It is denoted as : 'o'

Thus, we say that one complete revolution or complete angle is 360°.

Let us measure angle ∠ABC and ∠PQR.

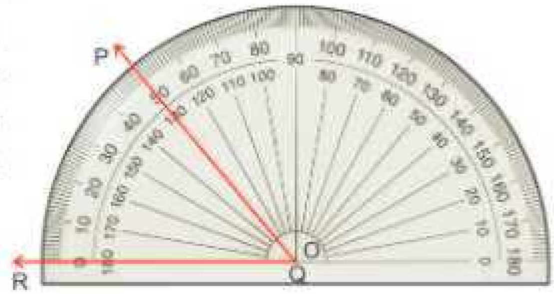


Place the protractor in such a way that the mid point O of the baseline coincides with B and the baseline exactly overlaps on ray $\overrightarrow{BC}$. Since $\overrightarrow{BC}$ is on the right of vertex (mid point of baseline) O. Start counting from 0° on the right side of B and read the mark with which arm AB coincides. It coincides with 40° mark. So ∠ABC = 40°.



Similarly to measure ∠PQR. Place the protractor in such a way that the mid point O at the baseline coincides with point Q and the baseline overlaps exactly on QR. Since QR is on the left of Vertex O, so start counting from left side of Q and read the mark with which arm PQ coincides. It coincides with 50° mark.

$\therefore \angle PQR = 50^\circ$

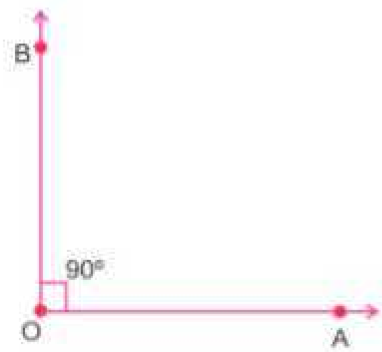


9.3.1. Types of Angles

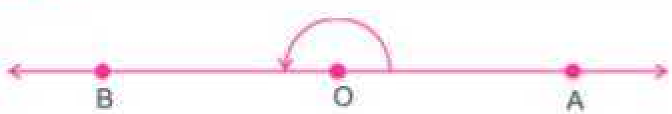
In geometry, angles can be classified according to their magnitude.

Zero Angle : An angle whose measure is 0° called a zero angle. When a ray does not move at all, we say, it has moved through an angle of 0°.

Right Angle : An angle whose measure is 90° called a right angle. Two lines that meet at a Right angle are said to be perpendicular. It is also written as OB⊥OA. '⊥' is the symbol of perpendicular.

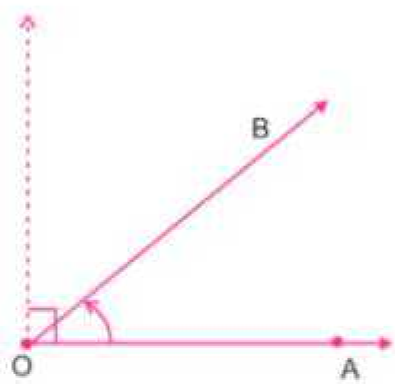


Straight Angle : An angle whose measure is 180° called a straight angle.

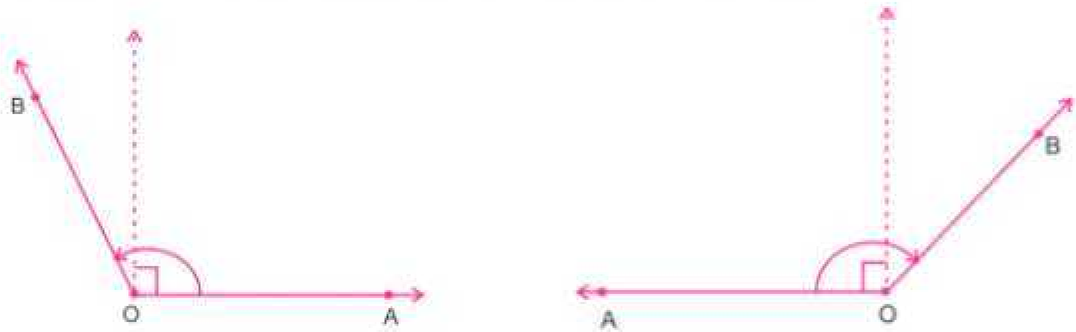


Two right angles make one straight line.

Acute Angle : An angle whose measure is between 0° and 90° is called an acute angle. Thus, an acute angle is more than a Zero angle but less than a right angle.

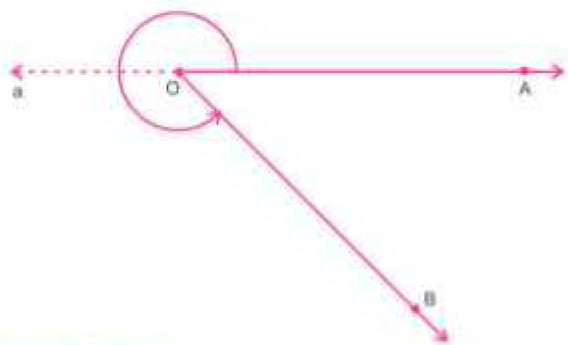


Obtuse Angle : An angle whose measure is between 90° and 180° is called an obtuse angle. Thus an obtuse angle is more than a right angle but less than a straight angle.



Complete Angle : An angle whose measure is 360° called a complete angle. When a ray completes one full revolution. It has moved through an angle of 360°.

Reflex Angle : An angle whose measure is between 180° and 360° is called a reflex angle. Thus, a reflex angle is more than a straight angle but less than a complete angle.



Let us consider some examples:

Example 1. **Classify the following angles as acute, right, obtuse, straight or reflex angle:**

(i) 89°	**(ii) 101°**	**(iii) 62°**	**(iv) 180°**
(v) 91°	**(vi) 215°**	**(vii) 90°**	**(viii) 181°**
(ix) 18°	**(x) 130°**		

Solution : (i) 89° is between 0° and 90°.

$\therefore$ It is an acute angle.

(ii) 101° is between 90° and 180°.

$\therefore$ It is an obtuse angle.

(iii) 62° is between 0° and 90°.

$\therefore$ It is an acute angle.

(iv) 180° is a straight angle.

(v) 91° is between 90° and 180°.

∴ It is an obtuse angle.

(vi) 215° is between 180° and 360°.

∴ It is a reflex angle.

(vii) 90° is a right angle.

(viii) 181° is between 180° and 360°.

∴ It is a reflex angle.

(ix) 18° is between 0° and 90°.

∴ It is an acute angle.

(x) 130° is between 90° and 180°.

∴ It is an obtuse angle.

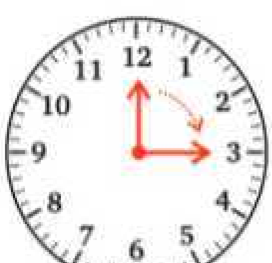
9.4. Angles in terms of revolution

Let us express the angle in terms of revolution on clock face by an activity.

Types of angles through wall clock.

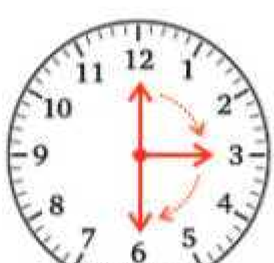
When the minute hand of a clock is at 12 and has not moved, we say that the minute hand has turned by zero angle. Thus, zero angle involves no revolution.

The movement of the minute hand from 12 to 12 is given below.



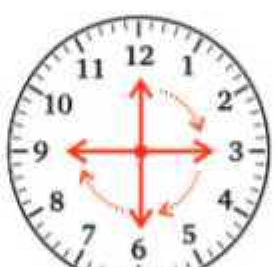
12 to 3

1 right angle = $\frac{1}{4}$ of a revolution.



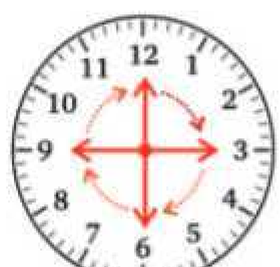
12 to 6

2 right angles = ⊥ straight angle = $\frac{1}{2}$ of a revolution.



12 to 9

3 right angles = $\frac{3}{4}$ of a revolution

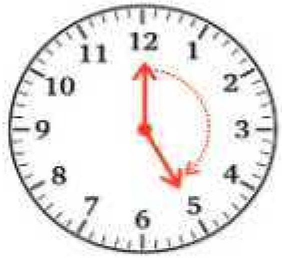


12 to 12

4 right angles = complete angle = $\frac{4}{4}$ or 1 complete revolution.

12 to 2

acute angle (less than $\frac{1}{4}$ of a revolution.)



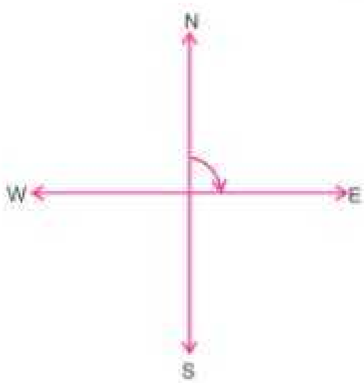
12 to 5

obtuse angle (more than $\frac{1}{4}$ but less than $\frac{1}{2}$ of a revolution).

12 to 8

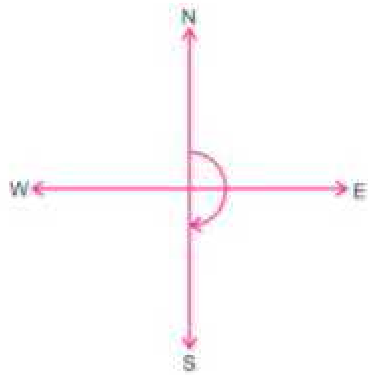
reflex angle (more than $\frac{1}{2}$ but less than 1 complete revolution).

Let us explain the kind of angles through direction. A person is facing north. The turns he takes to face the other direction are given below:



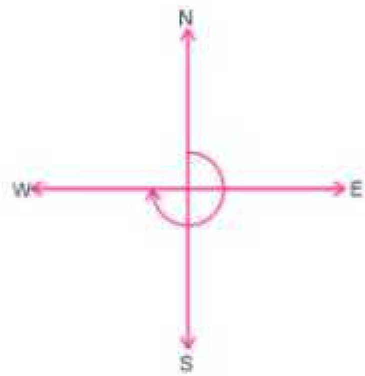
North to East

1 right angle = $\frac{1}{4}$ of a revolution



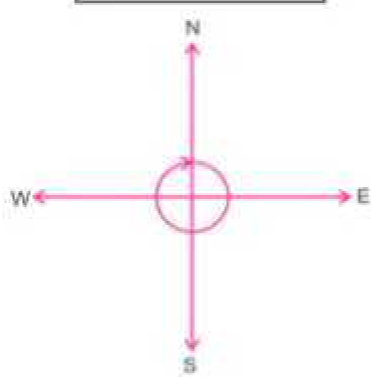
North to South

2 right angles = $\frac{1}{2}$ of a revolution



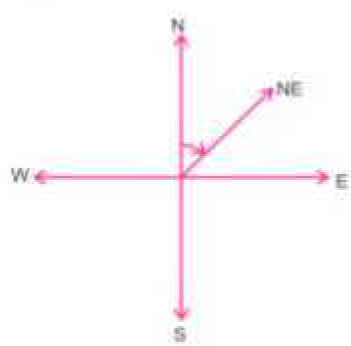
North to West

3 right angles = $\frac{3}{4}$ of a revolution



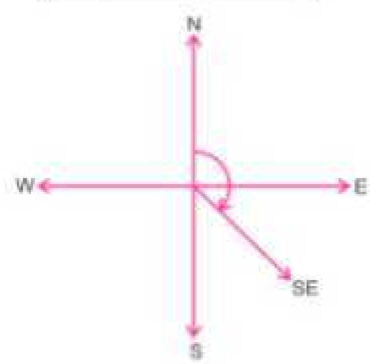
North to North

4 right angles = $\frac{4}{4}$ of a revolution = 1 Complete Revalution



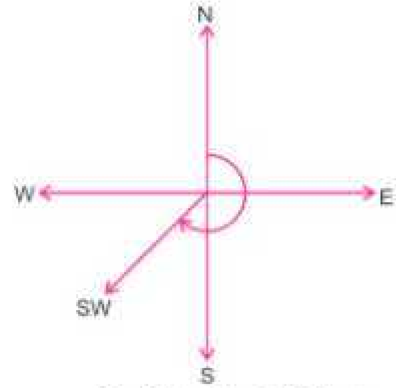
North to North-East

acute angle = less than $\frac{1}{4}$ of a revolution



North to South-East

obtuse angle = more than $\frac{1}{4}$ but less than $\frac{1}{2}$ of a revolution.



North to South-West

Reflex angle = more than $\frac{1}{2}$ but less than 1 complete revolution.

Example 2. **By what fraction of a revolution does the minute hand of a clock move, when it goes from (i) 12 to 3 (ii) 2 to 8 (iii) 3 to 12**

Solution : (i) 12 to 3 : Quarter or $\frac{1}{4}$ (ii) 2 to 8 : Half or $\frac{1}{2}$

(iii) 3 to 12 : 3 Quarters or $\frac{3}{4}$.

Example 3. **At which point does the hour hand of a clock stop if it starts at:**

(i) 12 and make $\frac{1}{2}$ revolution clockwise.

(ii) 4 and make $\frac{1}{4}$ revolution clockwise.

(iii) 7 and make $\frac{3}{4}$ revolution clockwise.

Solution : (i) For 1 revolution. the hour hand takes 12 hours.

For $\frac{1}{2}$ of a revolution, the hour hand takes

$\frac{1}{2} \times 12 = 6$ hours

$\therefore$ If hour hand starts at 12 and make $\frac{1}{2}$ revolution clockwise it will stop at 6.

(ii) For 1 revolution, the hour hand takes 12 hours. For $\frac{1}{4}$ of a revolution, the hour hand takes $\frac{1}{4} \times 12 = 3$ hours

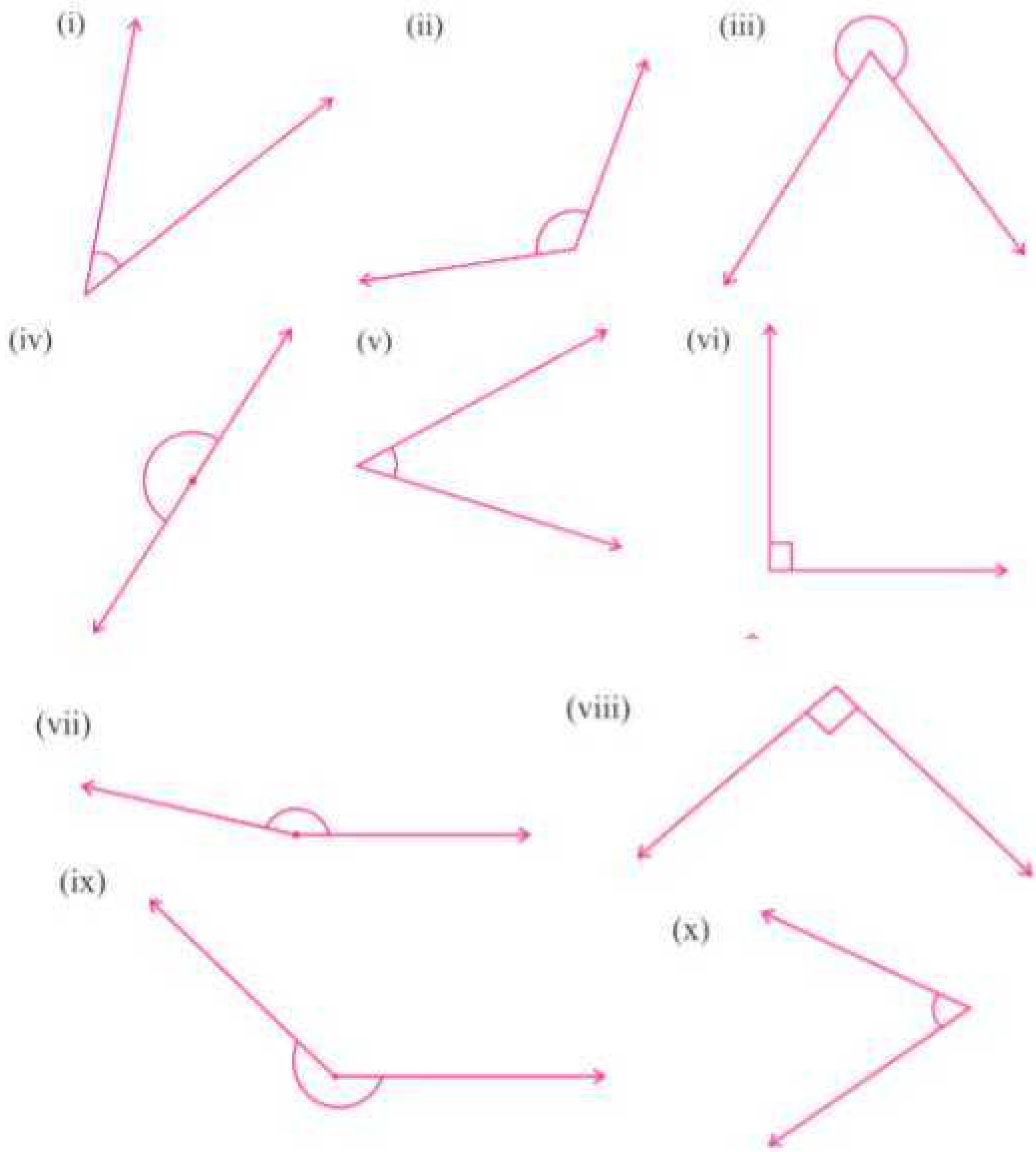
So, if hour hand starts at 4 and makes $\frac{1}{4}$ revolution clockwise, it will stop at 7.

(iii) For 1 revolution, the hour hand takes 12 hours. For $\frac{3}{4}$ of a revolution, the hour hand takes $\frac{3}{4} \times 12 = 9$ hours.

$\therefore$ If hour hand starts at 7 and make $\frac{3}{4}$ of a revolution clockwise it will stop at 4.

Exercise 9.2

1. Classify the angles as acute, obtuse, right, straight or reflex angles.



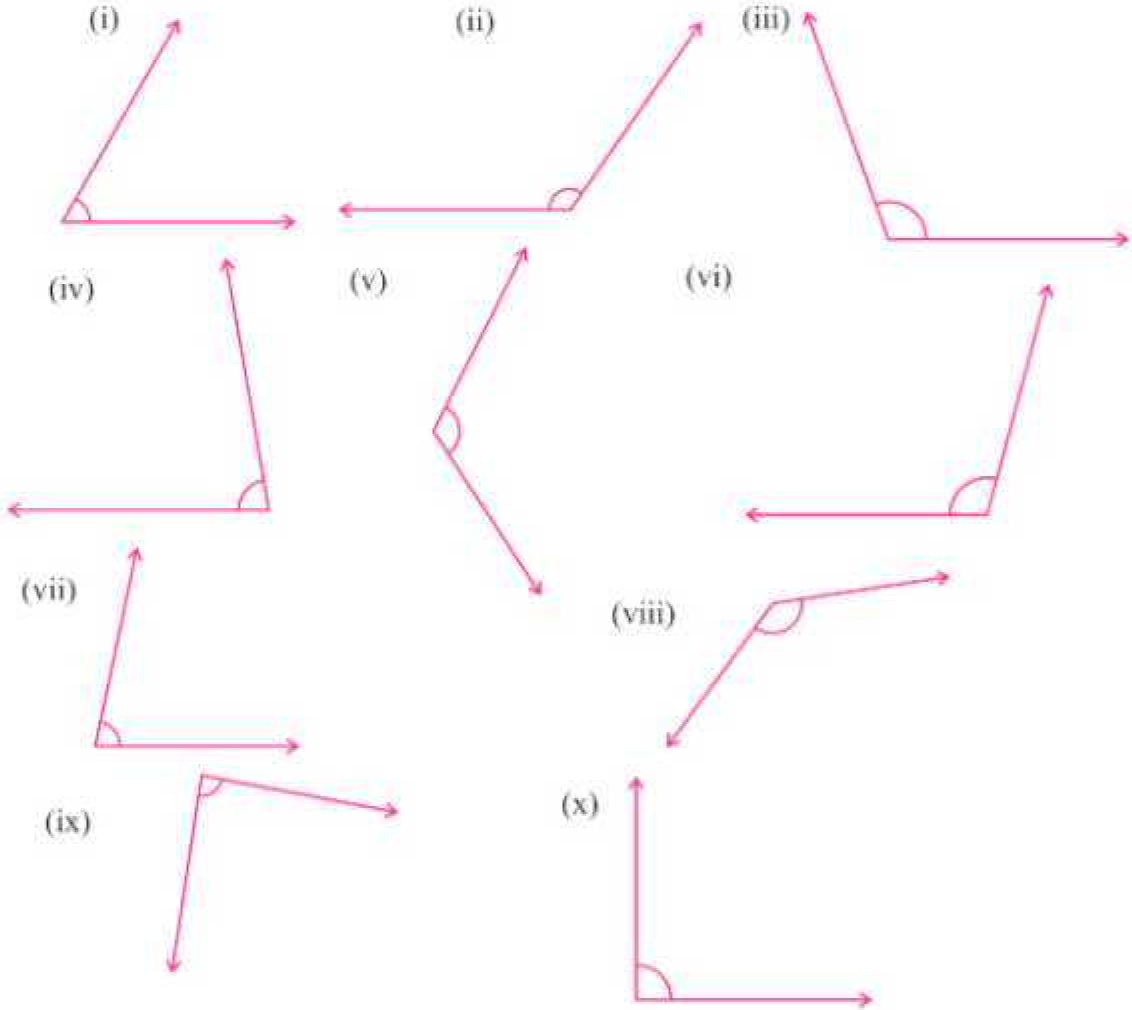
2. Classify the angles:

(i) 80° (ii) 172° (iii) 90° (iv) 0°

(v) 179° (vi) 215° (vii) 360° (viii) 350°

(ix) 15° (x) 180°

3. Measure the following angles with protractor and write their measurement:



4. How many degrees are there in

(i) Two right angles (ii) $\frac{2}{3}$ right angles

(iii) four right angles

5. What fraction of a clockwise revolution does the hour hand of a clock turn through when it goes from:

(i) 3 to 9 (ii) 5 to 8 (iii) 10 to 4

(iv) 2 to 11 (v) 6 to 3 (vi) 2 to 7

6. Find the number of right angles turned through by the hour hand of a clock when it goes from:

(i) 5 to 8	(ii) 1 to 7	(iii) 4 to 10
(iv) 9 to 12	(v) 11 to 2	(vi) 9 to 6
(vii) 2 to 11	(viii) 10 to 1	(ix) 12 to 6
(x) 5 to 2		

7. Where will be the hand of a clock stop if it starts at:

(i) 12 and make $\frac{1}{4}$ revolution clockwise.

(ii) 2 and make $\frac{1}{2}$ revolution clockwise.

(iii) 5 and make $\frac{1}{4}$ revolution clockwise.

(iv) 5 and make $\frac{3}{4}$ revolution clockwise.

8. What part of revolution have you turned through if you stand facing.

(i) East and turn clockwise to north.

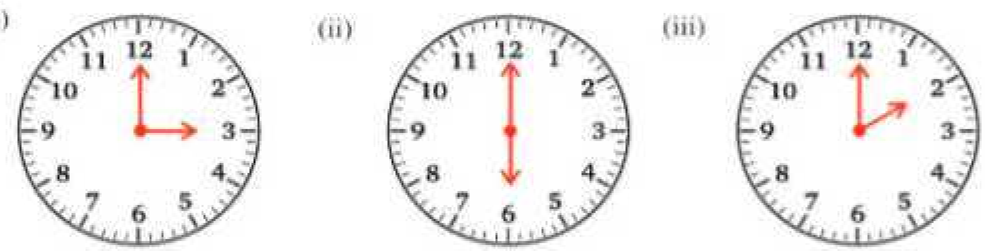
(ii) South and turn clockwise to north.

(iii) South and turn clockwise to east.

(iv) West and turn clockwise to east.

9. Find the angle measure between the hands of the clock in each figure:

(i) (ii) (iii)



3.00 am 6.00 am 2.00 am

10. Draw the following angles by protractor:

(i) 40° (ii) 75° (iii) 105° (iv) 90° (v) 130°

11. State true or false:

(i) The sum of two right angles is always a straight angle.

(ii) The sum of two acute angles is always a reflex angle.

(iii) The obtuse angle has measurement between 90° to 180°.

(iv) A complete revolution has four right angles.

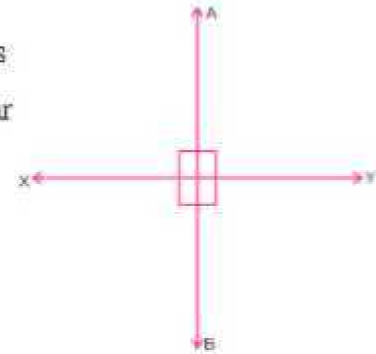
12. Fill in the blanks:

(i) The angle which is greater than 0° and less than 90° is called ....

(ii) The angle whose measurement equal to two right angle is ....

(iii) The angle between 90° and 180° is ....

9.5 Perpendicular Lines



If two lines intersect each other at a right angle, then the lines are called perpendicular lines. The symbol for a perpendicular line is ⊥ .

In the given figure, line AB is perpendicular to XY.

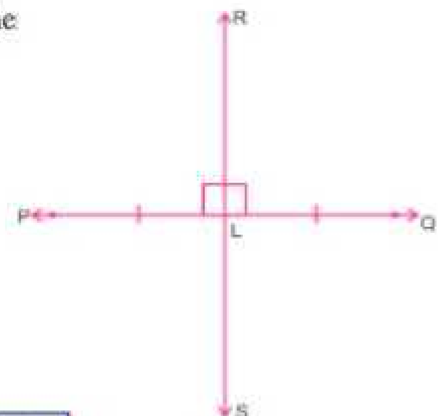
i.e. $AB \perp XY$

We can also say that line XY is perpendicular to AB.

i.e. $XY \perp AB$

Two adjacent edges of a book are pendicular to each othe

9.6 Perpendicular Bisector



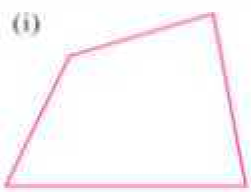
A line which is perpendicular to a line segment at its mid point, is called perpendicular bisector of that line segment.

As in figure $RS \perp PQ$ and RS bisects PQ.

Exercise 9.3

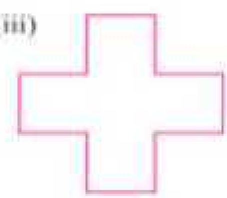
1. Identify the shape having perpendicular lines:

(i)



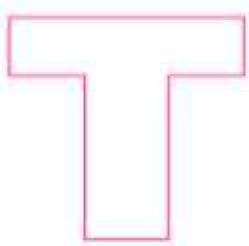
(ii)

(iii)



(iv)

(v)



2. Identify the examples having perpendicular lines:

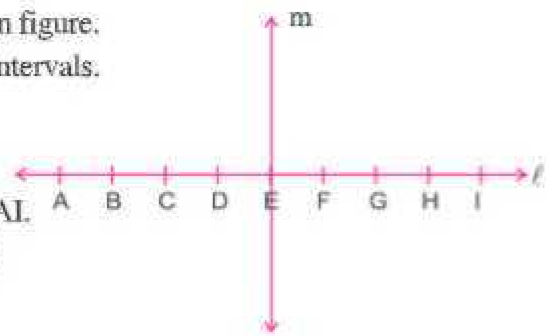
   (i) Lines of railway track.

   (ii) Adjacent edges of a table.

   (iii) Line segment forming letter 'L'.

3. Let $\overleftrightarrow{AB}$ be perpendicular to $\overleftrightarrow{PQ}$ and they intersect at O. What is the measure of $\angle AOP$.

4. Line m is perpendicular to line ℓ in the given figure. Each point on the line ℓ is marked at equal intervals. Study the diagram and state true or false.



   (i) Line m is $\perp$ bisector of line segment AI.

   (ii) CE = EG (iii) DF = 2DE

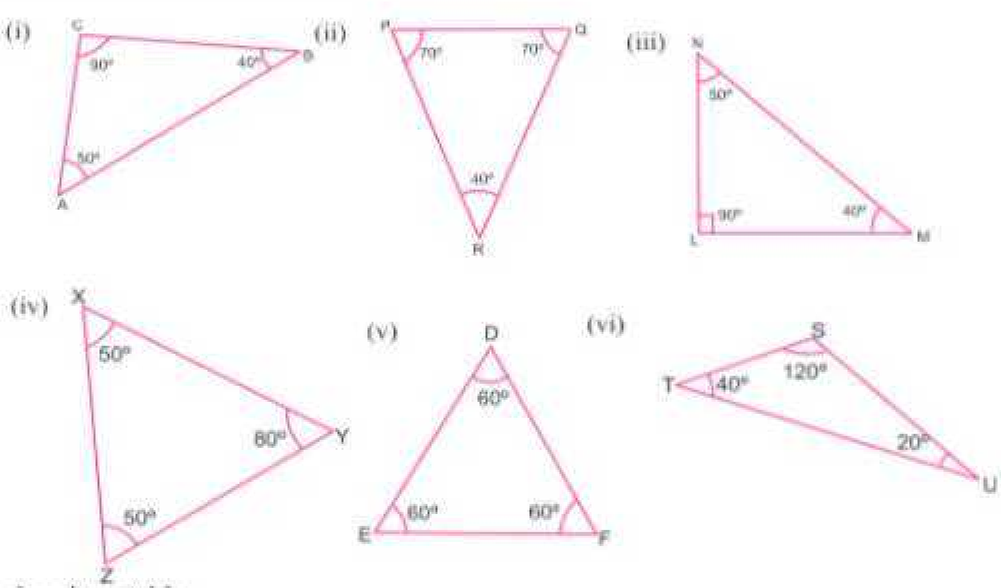
9.7 Classification of Triangles

In last chapter, we have learnt about triangle. 'Triangle is a plane closed figure formed by three non-prallel line segments.

It has six parts or elements which contains three sides and three angles. So we, can classify the triangles on the basis of the measure of their sides and angles.

9.7.1 On the basis of sides

Measure the sides of the following triangles:



Complete the given table:

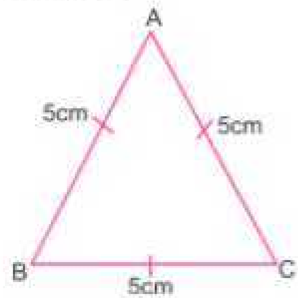
Property of triangle	Name of the triangle
(a) Triangle with all sides equal in length	ΔDEF
(b) Triangle with two sides equal in length.	....
(c) Triangle with all sides different in length	....

The Triangles can be classified based on the measurement of their sides.

Equilateral triangle : A triangle whose all sides are equal in length, is called an equilateral triangle.

In the given ΔABC,

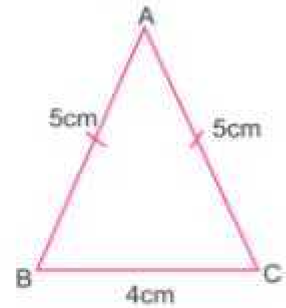
$AB = BC = AC = 5cm$, So it is an equilateral triangle.



Isosceles triangle : A triangle whose two sides are equal in length, is called an isosceles triangle.

In the given ΔABC,

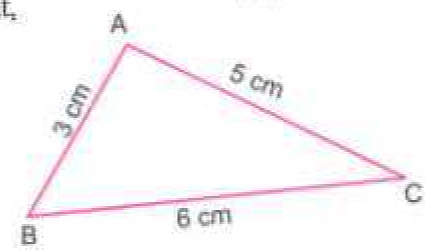
$AB = AC = 5cm$, So ΔABC is an isosceles triangle.



Scalene triangle : A triangle whose all sides are different, is called a scalene triangle.

In the given ΔABC,

$AB \neq BC \neq AC$, So ΔABC is a scalene triangle.



9.7.2 On the Basis of Angles

After Observing the angles of the triangle whose sides you have measured earlier in 9.7.1 and complete the following table:

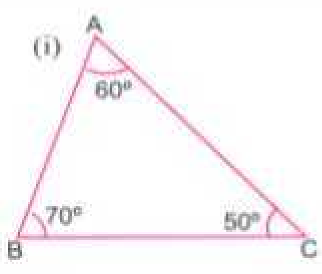
Property of triangle	Name of the triangle
Triangle with all acute angles	ΔXYZ, ΔDEF, ΔPQR
Triangle with one right angle	....
Triangle with one obtuse angle	....

The triangle can be classified on the measurement of their angles.

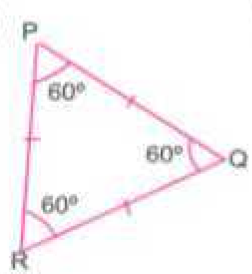
Acute triangle or acute angled triangle : A triangle whose all angles are acute is called an acute triangle or acute angled triangle.

In the following figures all triangles are acute triangles.

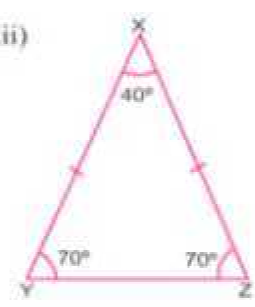
(i)



(ii)



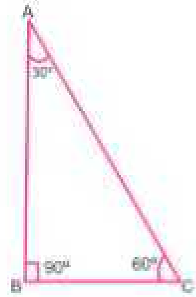
(iii)

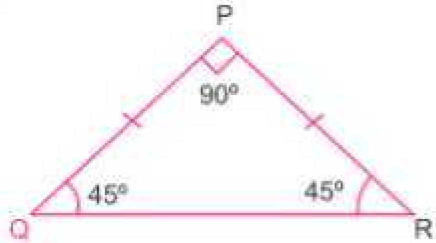


Acute triangle can be a scalene, equilateral or isosceles triangle.

Right triangle or Right Angled triangle : A triangle whose one angle is a right (or 90^0)angle is called a right triangle or right angled triangle.

In the following figure, all triangles are right triangle.

(i) 

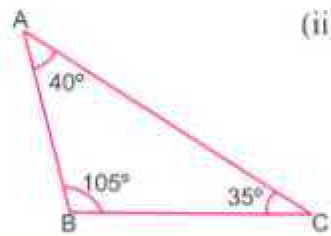
(ii) 

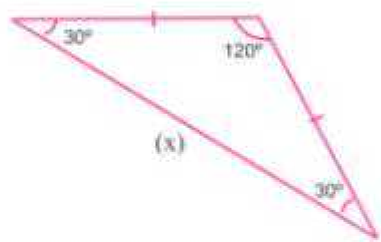
The side opposite to right angle is called 'Hypotenuse' and other two sides are called its 'legs'

In the figure, AC and QR are respective Hypotenuse.

* A right triangle can scalene or isosceles triangle:
* In right triangle one angle is right angle and the other two are acute angles.

Obtuse Triangle or Obtuse Angled Triangle : A triangle whose one angle is obtuse, is called an obtuse triangle or obtuse angled triangle. In the following figure, both triangles are obtuse triangles.

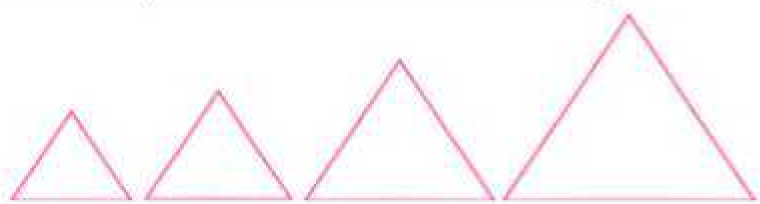
(i) 

(ii) 

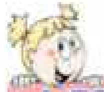
* **An obtuse triangle can only be scalene or isosceles triangle.**
* **In obtuse triangle, one angle is obtuse and the other two angles are acute angles.**

From the above discussion of triangles we can observe that

* The size of a triangle does not effect the measure of angles.



* The side opposite to the greater angle is greater than the side opposite to the smaller angle.
* If angles of a triangle are equal then sides opposite to equal angles are also equal in length.
* Sum of all angles of a triangle is 180^o.
* In a triangle sum of any two sides is always greater than its third side.

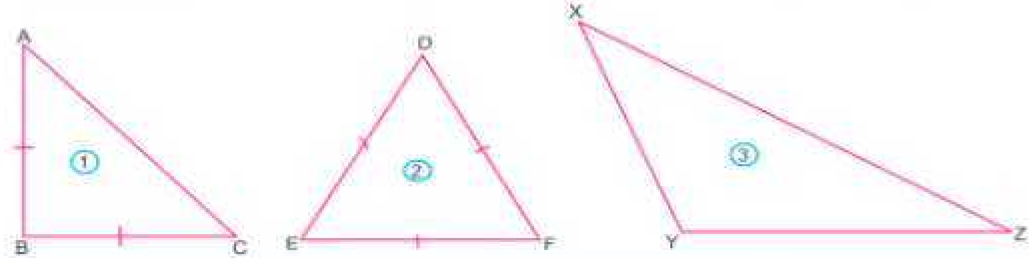


ACTIVITY

To Classify the triangles on the basis of sides and angles.

Material Required: Scale, Protractor and Pen etc.

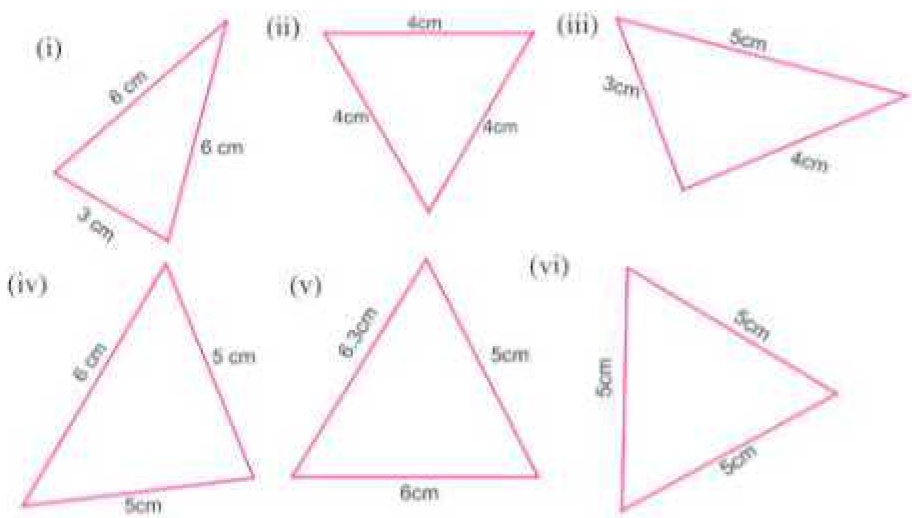
Procedure:- Measure each angle and length of each side of given triangle.



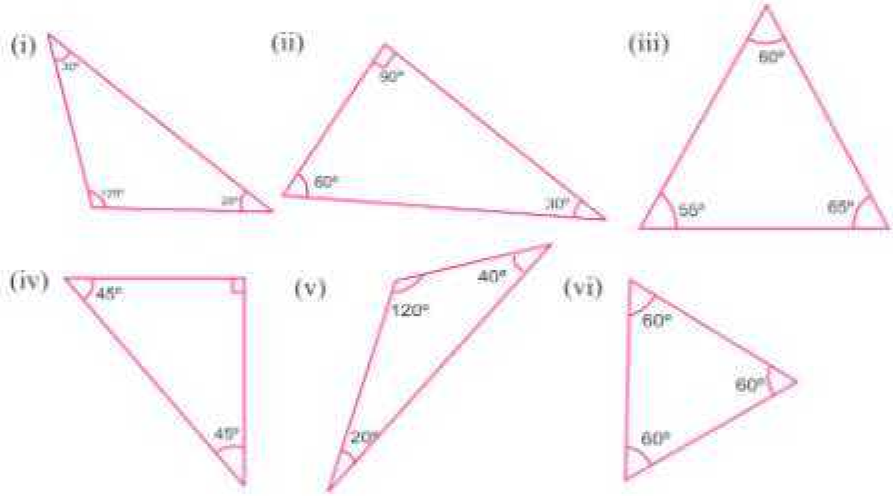
Sr.No.	Triangle Name	Classification on the basis of sides	Classification on the basis of angles
1	ΔABC	$AB = BC \neq AC$ Isosceles Triangle	$\angle B = 90°$ Right angled triangle
2	ΔDEF	$DE = EF = DF$ Equilateral Triangle	$\angle E = \angle F = \angle D = 60°$ Actue angled triangle
3	ΔXYZ	$XY \neq YZ \neq ZX$ Scalene Triangle	$\angle Y = 120°$ Obtuse angled triangle

Exercise 9.4

1. Classify each of the following triangles as scalene, isosceles or equilateral:



2. Classify each of the following triangles as acute, obtuse or right triangle:



3. Which of the following triangles are possible with the given angles ?

(i) 60°, 60°, 60° (ii) 110°, 50°, 30° (iii) 65°, 55°, 60°

(iv) 90°, 40°, 50° (v) 48°, 62°, 50° (vi) 90°, 95°, 30°

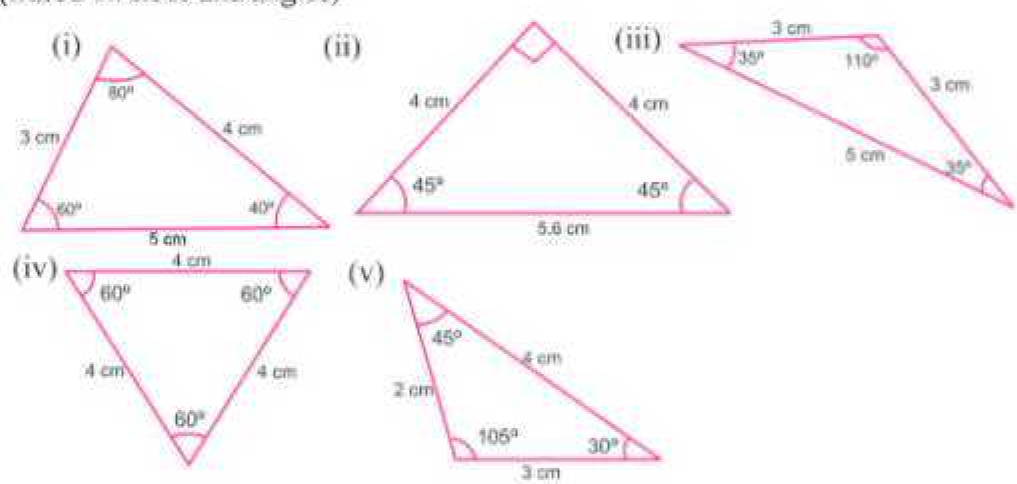
4. Classify each of the following triangles as scalene, isosceles or equilateral triangle:

(i) 4cm, 5cm, 6cm (ii) 5cm. 7cm, 5cm (iii) 4.2m, 5.3m, 6.1m

(iv) 3.5cm, 3.5cm, 3.5cm (iv) 8cm, 4.2cm, 4.2cm (v) 2cm, 3cm, 4cm

5. Name the following triangles in both ways:

(Based on sides and angles)



6. Fill in the blanks:

(i) A triangle has .... sides.

(ii) A triangle has .... vertices.

(iii) A triangle has .... angles.

(iv) A triangle has .... parts.

(v) A triangle whose all sides are different is known as ....

(vi) A triangle whose all angles are acute is known as ....

(vii) A triangle whose two sides are equal is known as ....

(viii) A triangle whose one angle is obtuse is known as ....

(ix) A triangle whose all sides are equal is known as ....

(x) A triangle whose one angle is right angle is known as ....

7. State True or False:-

(i) Each equilateral triangle is an isosceles triangle.

(ii) Each acute angled triangle is a scalene triangle.

(iii) Each isosceles triangle is an equilateral triangle.

(iv) There are two obtuse angles in an obtuse triangle.

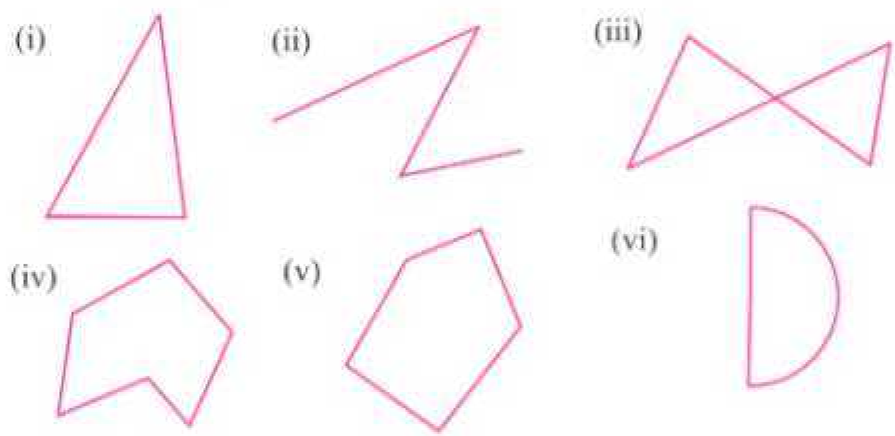
(v) In right triangle, there is only one right angle.

(vi) Right triangle can never be isosceles.

9.8 Polygons

We have already learnt in last chapter that a polygon is a plane figure that is made by joining the line segments, where each line segment meets exactly two other line segments. The inter section point of two line segments is called the vertex of the polygon.

Observe the following figures:



Here (i), (iv), (v) are polygons but (ii), (iii) & (vi) are not polygons. As (ii) is not closed, in (iii) sides of a figure intersect each other at three places and (vi) is not made of line segments.

Classification of Polygons

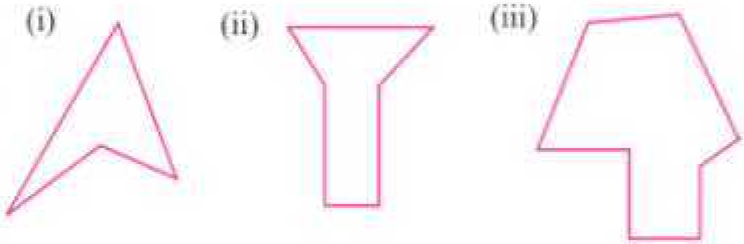
Regular and Irregular Polygons : If all the sides and angles of a polygon are equal then the polygon is called a regular polygon. Otherwise it is called an irregular polygon. Equilateral triangle and square are examples of regular polygon and scalene triangle, rectangle etc. are called irregular polygons.

Convex and Concave Polygons : A polygon is called a convex polygon if the measure of its each angle is less than 180^o.

In a convex polygon, the line containing any side of the polygon has the remaining vertices on the same side of it. Square, rectangle, parallelogram, all triangles etc. are examples of it.

A polygon is called a concave polygon if the measure of atleast one of the angles is a reflex (more than 180°) angle.

If a line segment joining any two points in the interior of a polygon does not lie within it, then it is a concave polygon.



These are figures of concave polygons.

Play way Activity on Concave and Convex Polygons :

⇒ Take one thread and hold it in the form of a circle or make a circle by tying its both ends.

⇒ Call one student and say him/her to insert fingers inside the circle and stretch the thread in outward direction.

You will get a convex polygon.

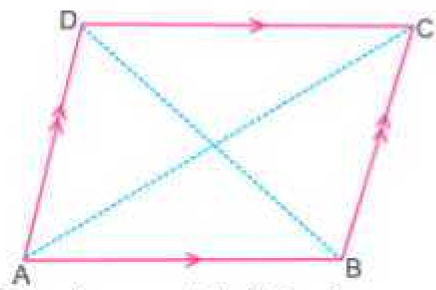
Call one more student and say him/her to place one or more fingers in the outer of thread and push it inside.

You will get a concave polygon.

9.9 Types of Quadrilateral

We have already studied in the previous chapter about the basic elements of a quadrilateral, such as sides, angles, diagonals, vertices, interior and exterior part etc. Here we will study the types of quadrilateral.

* **Parallelogram:-** A quadrilateral in which both pairs of opposite sides are equal or parallel is called parallelogram.
  In the given figure. ABCD is a parallelogram in which.
  AB = DC, AD = BC
  or AB ll DC, AD ll BC



Observe it carefully and measure its sides angles and diagonals, you get the following properties of parallelogram

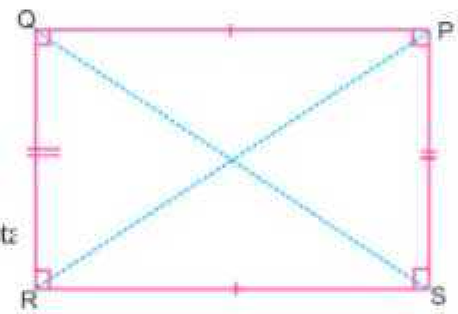
* Opposite sides are equal.
* Opposite sides are parallel.
* Opposite angles are equal.
* Diagonals bisect each other.

* **Rectangle:-** A parallelogram in which each angle is right angle is called rectangle.

In the given figure, PQRS is a rectange in which

PQ = RS, PS = RQ and $\angle P = \angle Q = \angle R = \angle S = 90^o$

Observe it carefully and measure its sides, angles and diagonals, you get the following properties of recta

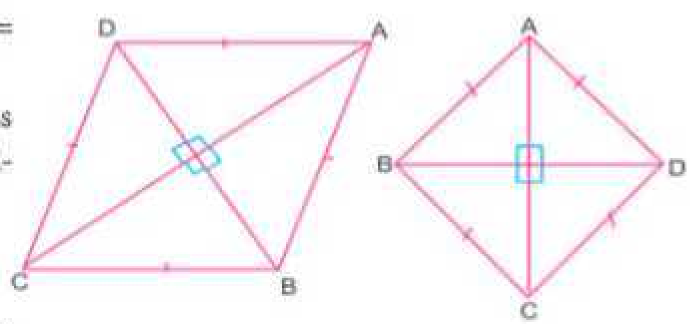


* Opposite sides are equal.
* Each angle is 90^o.
* Diagonals bisect each other.
* Diagonals are equal in length.

* **Rhombus:-** A quadrilateral having all sides are equal is called a rhombus. **Or**
  A parallelogram having adjacent sides equal is called rhombus.

In the given figures, ABCD is a rhombus in which AD ll BC, AB ll CD and AB = BC = CD = DA

Observe it carefully and measure its sides angles and diagonals, you get the following properties of rhombus.

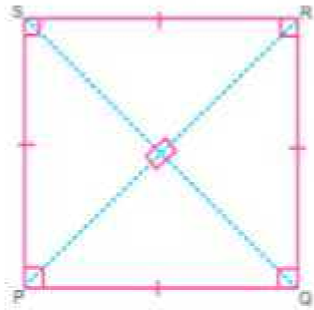


* All sides are equal.
* Opposite angles are equal.
* Diagonals bisect each other at 90^o.

* **Square:-** A rhombus with each angle is 90° is called a square

In the given figure, PQRS is a square in which PQ = QR = RS = SP and

$\angle P = \angle Q = \angle R = \angle S = 90°$

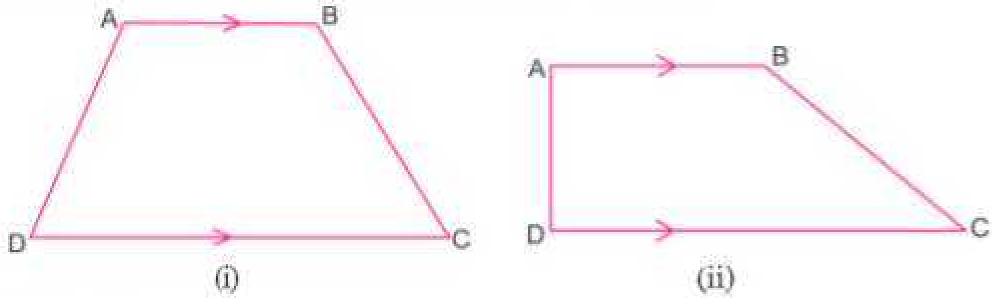


Observe it carefully and measure its sides angles and diagonals, you get the following properties of square.

* All sides are equal.
* Each angle is right angle 90°.
* Diagonals are equal in length..
* Diagonals bisect such other at right angle.

* **Trapezium:-** A quadrilateral in which one pair of oppsite sides is parallel, is called a trapezium.

In the given figure, ABCD is a trapezium in which AB ∥ DC.

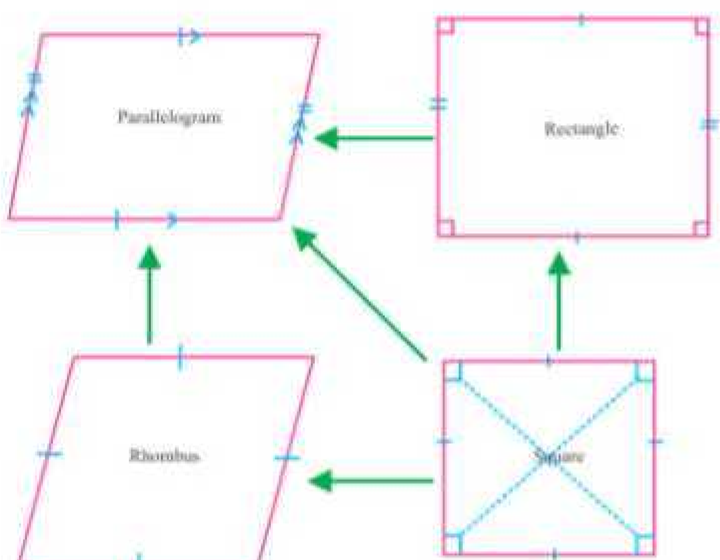


Isosceles Trapezium:- A quadrilateral in which a pair of opposite sides is parallel and the other two sides are equal is called an isosceles trapezium In above figures (ii), if AB ∥ DC and AD = BC then ABCD is an isosceles trapezium

Observe it carefully and measure its sides angles and diagonals

* A pair of opposite sides is parallel
* Diagonals do not bisect

Table of Quadrilaterals



ACTIVITY

To make closed geometrical shapes like triangle, quadrilateral pentagon and Hexagon, using match sticks.

Pre-requisite: Knowledge of geometerical shapes.

Material Required :- Match sticks, glue, paper.

Procedure : Paste match sticks as shown in following Figures.

Observation:

1. Three closed match sticks makes Triangle.
2. Four closed match sticks makes Quadrilatral.
3. Five closed match sticks makes Pentagon.
4. Six closed match sticks makes Hexagon.

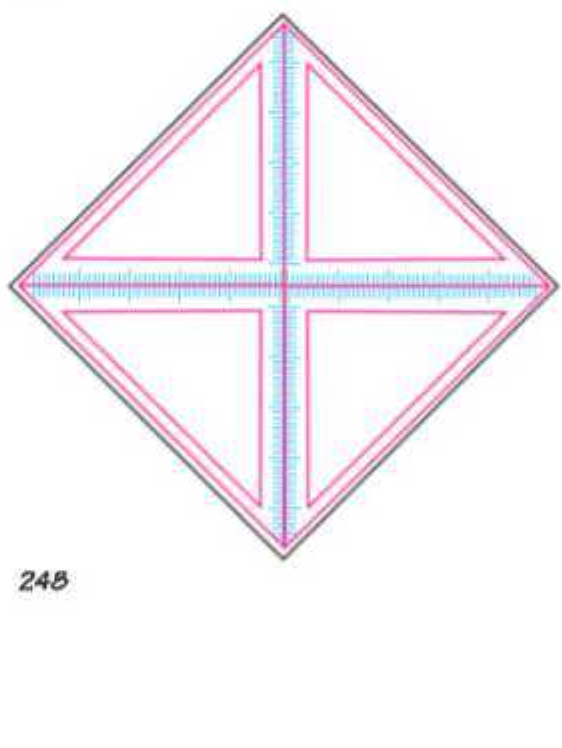
ACTIVITY

Make the following shapes using a pair of set squares (i) square (ii) rectangle (iii) parallelogram (iv) rhombus (v) trapezium

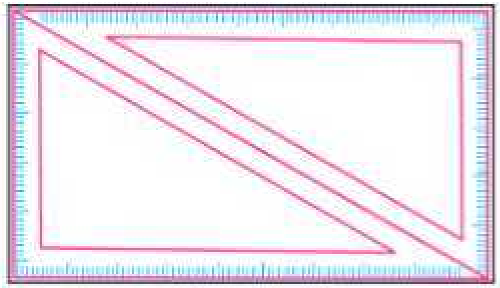
Material Required:- Set squares.

Procedure:-

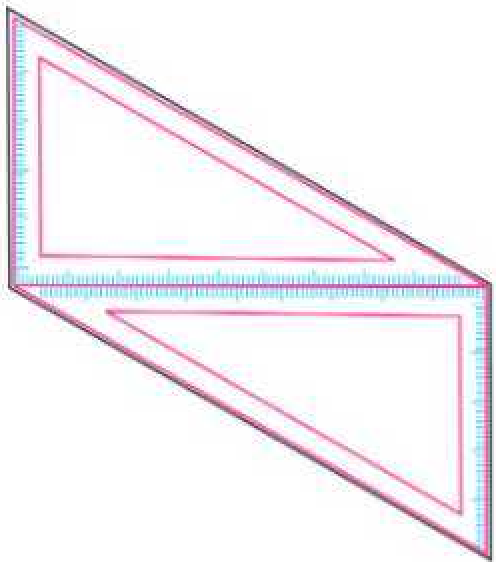
(i) **Square:-** Take four set squares of 45° – 45° – 90° and place them as shown.



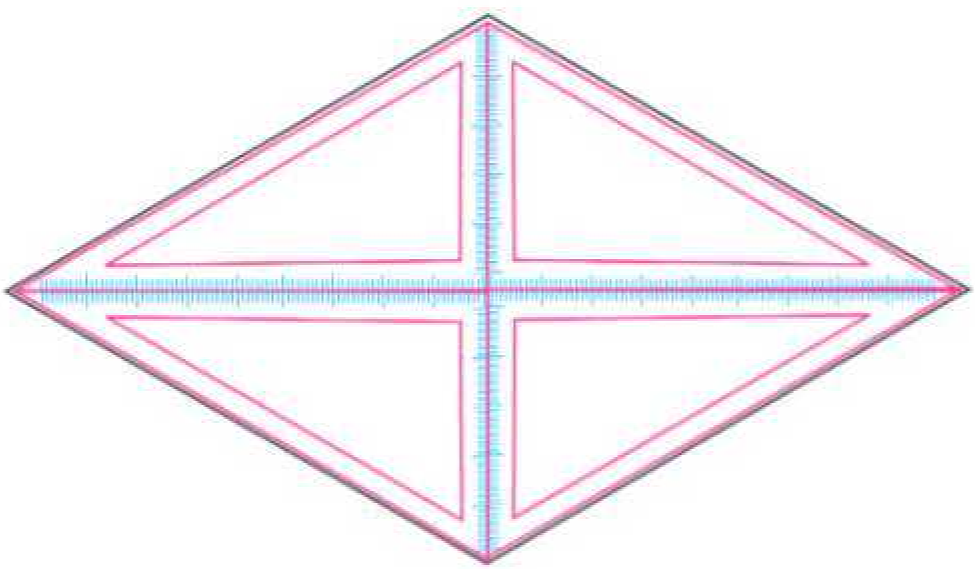
(ii) **Rectangle:-** Take two set suqares of $30^{\circ} - 60^{\circ} - 90^{\circ}$ and place them as shown.



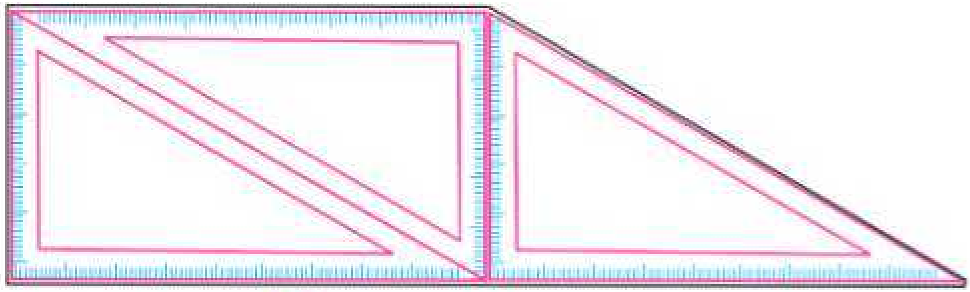
(iii) **Parallelogram:-** Take two set squares of $30^{\circ} - 60^{\circ} - 90^{\circ}$ in a different position and place them as shown.



(iv) **Rhombus:-** Take four set squares of $30^{\circ} - 60^{\circ} - 90^{\circ}$ and place them as shown.

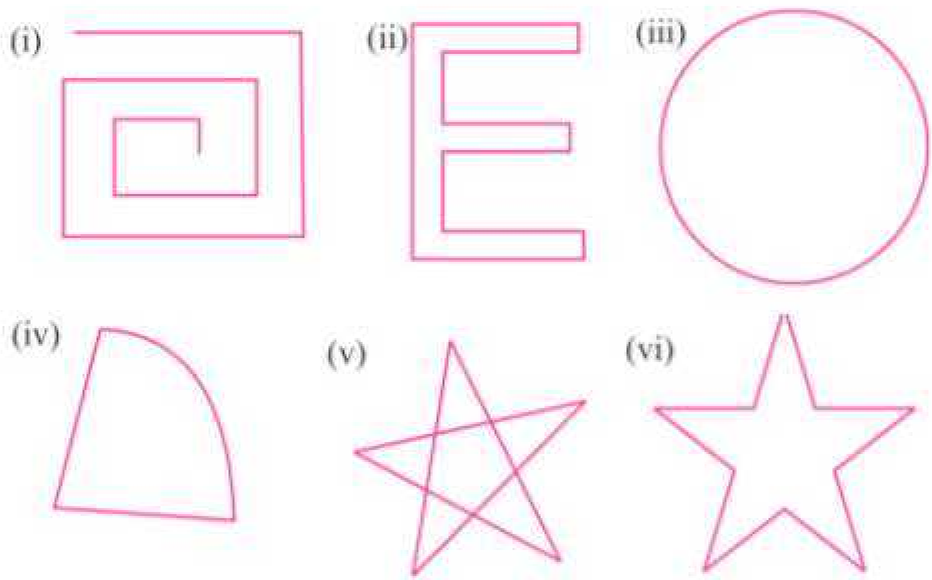


(v) **Trapezium :-** Take three set squares of 30º-60º-90º or 45° – 45°-90º and place them as shown.

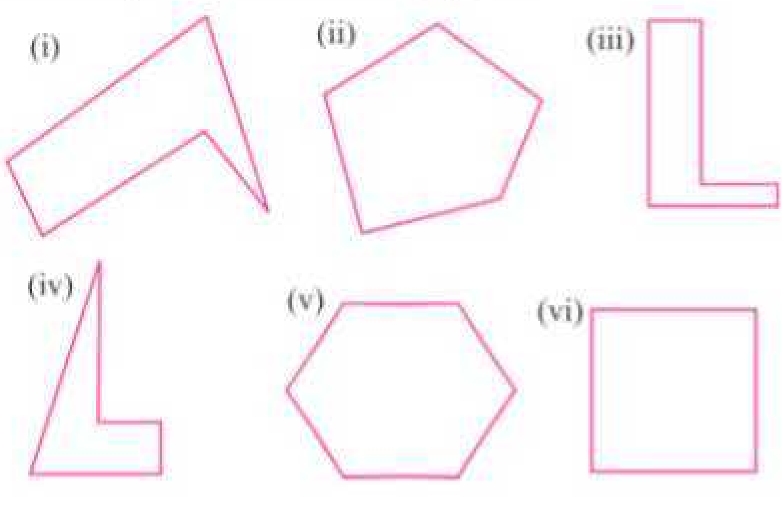


Exercise 9.5

1. Which of the followings are polygons or not. Give the reason :



2. Classify the following as concave or convex polygons :



3. Tick in the boxes, if the property holds true for a particular quadrilateral otherwise croes out 'x'

Quadrilateral Properties	Rectangle	Paralelogram	Rhombus	Trapezium	Square
All sides are equal					
Only opposite are sides equal					
Diagonals are equal					
Diagonals bisect each other					
Diagonals are perpendicular to each other					
Each angle is 90°					

4. Fill in the blanks:-
   (i) .... is a quadrilateral with only one pair of opposite sides parallel.
   (ii) .... is a quadrilateral with all sides equal and diagonals of equal length.
   (iii) A polygon with atleast one angle is reflex is called ....
   (iv) .... is a regular quadrilateral.
   (v) .... is a quadrilateral with opposide sides equal and diagonals of unequal length.

5. State True or False:-
   (i) A rectangle is always a rhombus.
   (ii) The diagonals of a rectangle are perpendicular to each other.
   (iii) A square is a parallelogram.
   (iv) A trapezium is a parallelogram.
   (v) Opposite sides of a parallelogram are parallel.

9.10 Three Dimensional Shapes

You have learnt about flat shapes or two dimensional shapes or plane figure such as triangle, quadrilateral circle etc. These shapes lie in one plane. Now you will learn about solid shapes or three dimensional shapes. These shapes lie in more than one plane.

In our daily life, we see many solid shapes, some of them are.



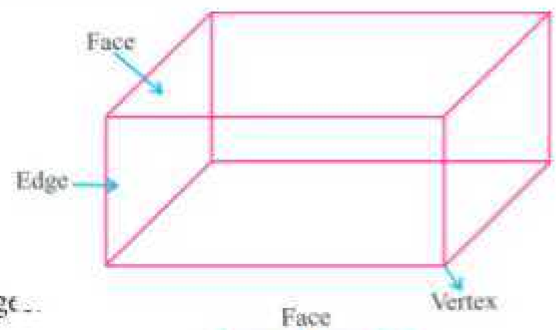

Three dimensional figures have 3 dimensions i.e length, breadth and height. A three dimensional figure or shape can be better described, if its faces (flat surfaces) , vertices and edges are known.

- The surface of a solid shape is called a face.
- The line where two faces meet is called an edge.
- The point where three edges meet is called a corner or vertex.

The various types of three dimensional figures are as follows:-

* **Cuboid:-** Objects such as a match box, brick, geometry box etc. look like a cuboid. These are made of six rectangular plane regions.

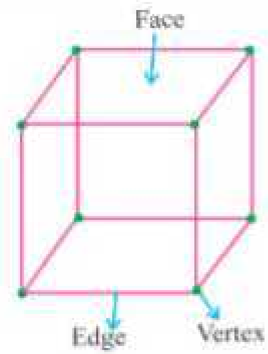
"A cuboid is a solid bounded by six rectangular plane regions."



A cuboid has 6 faces, 8 vertices and 12 edge..

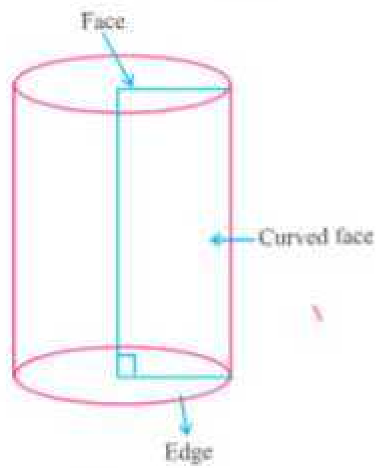
* **Cube:-** A cuboid whose length, breadth and height are and equal is called a cube. Objects such, as dice, sugar cubes etc. look like a cube.

A cube has 6 faces, 8 vertices and 12 edges. All the faces of cube are equal.



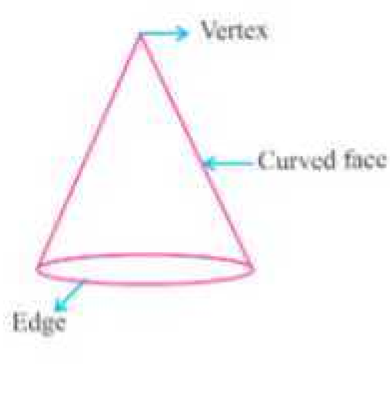
* **Cylinder:-** Objects such as drum, glass, circular pipes, look like a cylinder. These solids have a curved (lateral) surface with identical circular ends. Such solids are right circular cylinders.

A cylinder has 2 plane faces (top and bottom) and 1 curved face. It has 2 circular edges. It has no vertex.



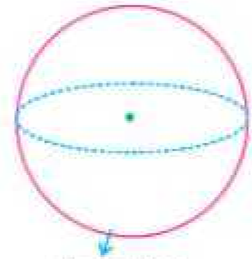
* **Cone:-** Objects such as ice-cream cone, joker cap, birthday cap look like a cone.

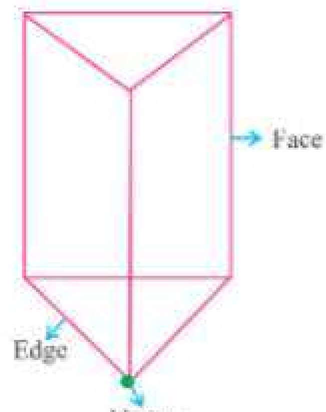
A cone has 1 plane face which is its base and 1 curved faces It has 1 edge and 1 vertex.



* **Sphere:-** Objects such as globe, ball, sun etc. look like a sphere. A sphere is the set of all points in a 3-dimensional space which are at equal distance from a fixed point.

  It has one curved surface, no vertex and no edge.





* **Prism:-** A solid object with ends that are parallel and of the same size and shape, and with faces whose opposite edges are equal and parallel

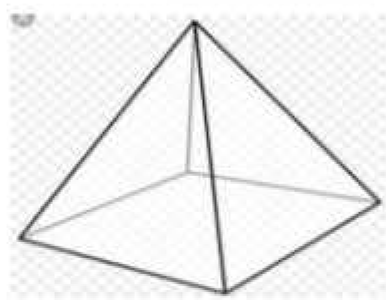
  A triangular prism is made up of two congurent triangles at each end and three congurent rectangles

  It has 5 faces, 9 edges and 6 vertices.

  Cubes and Cuboids are also prisms. They are called square prism and rectangular prism respectively.

* **Pyramid:-** A pyramid is a solid figure having a rectilinear base and the side faces as triangles.

  Side faces have a common point which is called the vertex of the pyramid.



- A pyramid having a triangular base is called a triangular pyramid. It has 4 faces, 4 verties and 6 edges.
- A pyramid having a square base is called a square pyramid. It has 5 faces, 8 edges, 5 vertices.

  A triangular Pyramid which has a base and all three lateral surfaces are equilateral triangles is called a tetrahedron.

Exercise 9.6

1. Give two examples of each of the following shapes from your surroundings:-

   (i) Cube (ii) Cuboid (iii) Cone (iv) Cylinder (v) Sphere

2. Classify the following as plane figures and solid figures:

   (i) Rectangle (ii) Sphere (iii) Cylinder (iv) Circle (v) Cube
   (vi) Cuboid (vii) Triangle (viii) Cone (ix) Square (x) Prism

3. Write the name of shapes in the base of the following solids:

   (i) Cube (ii) Cylinder (iii) Tetrahedron (iv) Cuboid (v) Square Pyramid

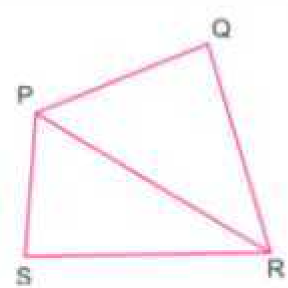
4. Fill in the table:-

	Shape	Number of flat faces	Number of Curved faces	Number of Vertices	Number of Edges
(i)	Cuboid				
(ii)	Cube				
(iii)	Cylinder				
(iv)	Cone				
(v)	Sphere				
(vi)	Triangular Prism				
(vii)	Square Pyramid				
(viii)	Tetrahedron				

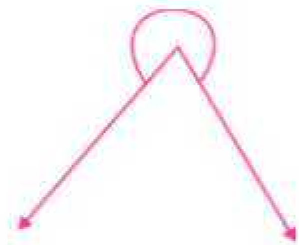
Multiple Choice Questions

1. In the given figure, which of the following is true?

   (a) PR = PQ (b) PR > QR
   (c) PS > PR (d) PR < PQ



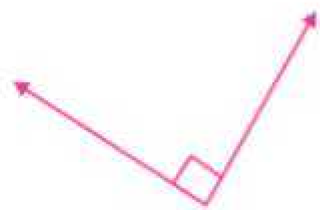
2. Which angle is represented in the given figure?



(a) Reflex (b) Acute

(c) Obtuse (d) Right angle

3. Which angle is represented in the given figure?



(a) Acute (b) Right angle

(c) Obtuse (d) Reflex

4. Which of the following is the example of perpendicular lines?

(a) Railway lines (b) Line Segment forming letter 'x'

(c) Adjacent edges of a table (d) Line segment forming line 'M'

5. Which of the following forms triangles?

(a) 60°, 72°, 48° (b) 73°, 54°, 59°

(c) 60°, 51°, 70° (d) 100°, 42°, 39°

6. Which of the following are sides of a triangle?

(a) 1, 2, 3 (b) 2, 2, 7 (c) 3, 4, 2 (d) 5, 6, 12

7. A parallelogram having adjacent sides equal is called a ....

(a) Trapezium (b) Rhombus (c) Rectangle (d) Square

8. Which of the following is not true for rectangle?

(a) Diagonals are equal (b) Diagonals bisect each other

(c) Each angle is 90° (d) All sides are equal

9. Which of the following is not true?

(a) Every rhombus is a parallelogram

(b) Each square is a rhombus.

(c) Each rectangle is a square.

(d) Each square is parallelogram

10. A cuboid has .... edges.

(a) 10 (b) 6 (c) 12 (d) 8

Learning Outcomes

After completion of this chapter the students are now able to

- Compare line segments in different ways
- Measure line segments, angles etc.
- Understand about angles with examples from the surroundings
- Understand about polygons
- Understand about 3D shaps from surroundings

ANSWER KEY

Exercise 9.1

1. (i) 4.4 cm (ii) 3.6 cm (iii) 2.5 cm (iv) 5.8 cm (v) 5cm, 14cm
2. (i) AB = AB (ii) CD < AC (iii) AC > AD (iv) BC < AC (v) BD > CD

Exercise 9.2

1. (i) Acute angle (ii) Obtuse angle (iii) Reflex angle (iv) Straight angle (v) Acute angle (vi) Right angle (vii) Obtuse angle (viii) Right angle (ix) Reflex angle (x) Acute angle
2. (i) Acute angle (ii) Obtuse angle (iii) Right angle (iv) Zero angle (v) Obtuse angle (vi) Reflex angle (vii) Complete angle (viii) Reflex angle (ix) Acute angle (x) Straight angle
3. (i) 60° (ii) 125° (iii) 110° (iv) 80° (v) 120° (vi) 105° (vii) 80° (viii) 135° (ix) 88° (x) 90°
4. (i) 180° (ii) 60° (iii) 360°
5. (i) $\frac{1}{2}$ (ii) $\frac{1}{4}$ (iii) $\frac{1}{2}$ (iv) $\frac{3}{4}$ (v) $\frac{3}{4}$ (iv) $\frac{5}{12}$
6. (i) 1 (ii) 2 (iii) 2 (iv) 1 (v) 1 (vi) 3 (vii) 3 (viii) 1 (ix) 2 (x) 3
7. (i) 3 (ii) 8 (iii) 8 (iv) 2
8. (i) $\frac{3}{4}$ (ii) $\frac{1}{2}$ (iii) $\frac{3}{4}$ (iv) $\frac{1}{2}$
9. (i) 90° (ii) 180° (iii) 60°

11. (i) T (ii) F (iii) T (iv) T

12. (i) acute angle (ii) straight angle or 180° (iii) obtuse angle

Exercise 9.3

1. (ii), (iii), (v) 2. (ii), (iii) 3. 90° 4. (i) T (ii) T (iii) T

Exercise 9.4

1. (i) Isosceles (ii) Equilateral (iii) Scalene (iv) Isosceles
(v) Scalene (vi) Equilateral

2. (i) Obtuse angled (ii) Right angled (iii) Acute anlged (iv) Right angled
(v) Obtuse angled (vi) Acute angled

3. (i), (iii), (iv)

4. (i) Scalene (ii) Isosceles (iii) Scalene (iv) Equilateral
(vi) Isosceles (vi) Scalene

5. (i) Scalene, acute angled (ii) Isosceles, right angled
(iii) Isosceles, obtuse angled (iv) Equilateral, acute angled
(v) Scalene, obtuse angled.

6. (i) 3 (ii) 3 (iii) 3 (iv) 6
(v) Scalene triangle (vi) Acute angled (vii) Isosceles triangle (viii) obtuse angled triangles
(ix) Equilateral triangle (x) Right angled triangle

7. (i) T (ii) F (iii) F (iv) F (v) T (vi) F

Exercise 9.5

1. (ii), (vi) 2. Concave (i), (iii), (iv) Convex (ii), (v), (vi)

4. (i) Trapezium (ii) Square (iii) Concave polygon (iv) Square (v) Parallelogram

5. (i) F (ii) F (iii) T (iv) F (v) T

Exercise 9.6

2. Plane figures (i), (iv), (vii), (ix)
Solid figures (ii), (iii), (v), (vi), (viii), (x)

3. (i) Square (ii) Circle (iii) Equilaterial triangle
(iv) Rectangle (v) Square

Multiple Choice Questions

(1) b (2) a (3) b (4) c (5) a (6) c (7) b (8) d (9) c (10) c

P9Z4N7

10

PRACTICAL GEOMETRY

Objectives

In this chapter you will learn

- About geometrical tools.
- To describe the understanding of angles and their constructions.
- To classify the angles according to their measures.
- About Construction of angle bisectors, perpendicular bisectors etc.

10.1 Introduction

Look at the shape and design of your rooms, bathrooms, floor, garden, verandahs etc. Architects, masons need to draw these shapes with accurate measurements, as the construction of entire work is based upon it. Such constructions are known as **geometrical constructions**.

The skill of geometrical constructions of various shapes is not only useful for a mason or architect,it is also useful in many other occupations like tailoring, fashion designing, engineering etc.

In this chapter, you will learn about the construction of geometrical shapes. Let us first learn more about the **tools** in your **geometrical box** which will be used in constructing these shapes.

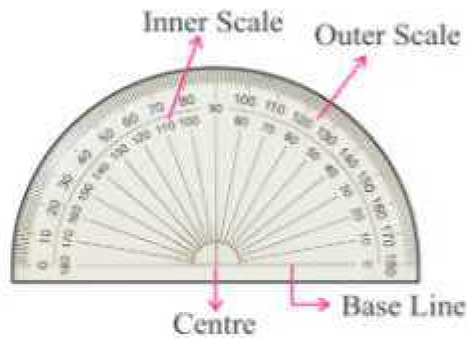
10.2 Basic Geometrical Tools

*** The Ruler:-** A ruler is smaller than a metre scale. It has straight edges and is usually of length **30 cm (a feet app.)** or **15cm (6 inches app)**. Each centimetre is further divided into 10 small equal small divisions called **millimetres**.

It has centimetre and millimetre marks on one edge and has inch marks on other edge. The marks on the ruler are called **graduations** and the ruler is called a **graduated ruler**.

We use the ruler **to draw and measure the line segments**.

*** The Protractor:-** A protractor is a **semicircular** device, with a straight edge and a curved edge which is marked with 180 equal small divisions, each denoting 1°. It has two scales, outer scale and inner scale. The **inner scale** is marked from 0° to 180° from **right to left**. The **outer scale** is marked from 0° to 180° from **left to right**. It has a 0-180 line parallel to the straight edge, is called a **base line**. The mid point of base line is called the **centre** of the protractor.It looks like a english alphabet D. So usually it is called 'D' also.



We use a protractor to **draw and measure the angles**.

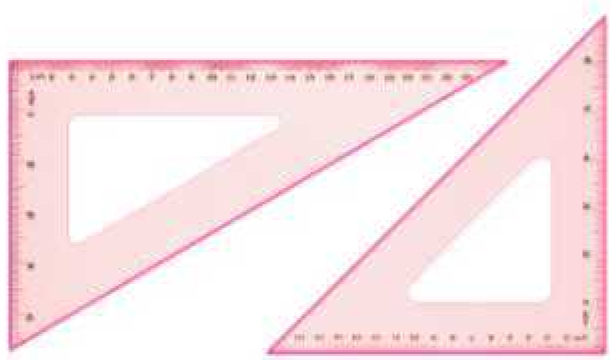
*** The Compasses:-** The instrument compasses has two metal arms, which are hinged together. One of the arms has a metal end point (pointer) and the other arm has screw arrangement which can hold a pencil tightly. The end point of the pencil can be adjusted at any distance from the metallic end point.

A compasses is used to **mark off equal lengths, draw arcs and circles**.

*** The Divider:-** The instrument divider has two pointed metal arms hinged at one end with the helps of a knob. The distance between the arms can be adjusted by opening and closing the arms.

A divider is used to **compare line segments of different lengths**.

*** Set-Squares:-** In the geometrical box, there are two triangular shaped insturments called set squares. In one, the angles are of 30°–60°–90° called **30° set square** and in other, the angles are of 45° – 45° – 90° called as **45° set square**. Two perpendicular edges in these set-squares are usually graduated, one in centimetre and other in inches.



Set-squares are used to draw **perpendicular, parallel lines and many angles** such as 30°, 45°, 60°, 75° etc.

While constructing geometrical shapes, the following points should be kept in mind.

- Use rulers with fine edges and pencils with fine tip.
- Always draw thin lines and mark the points lightly.
- Preferably, Keep two pencils in your tool box one small for the compasses and other for drawing lines and marking points.
- Make sure that the pencil tip and the metal tip of the compasses are at the same level, while using the compasses.

Exercise 10.1

1. What is the use of insturment ruler?
2. What is the use of protractor?
3. What is the use of Compasses?
4. Construct the following angles using set-squares:

   (i) 30° (ii) 45° (iii) 60° (iv) 75° (v) 90°

10.3. Construction of a line segment

In the previous chapter, we have already learnt the technique of measuring line segments and comparing line segments by observation and by using a divider. Here, we shall construct a line segment in two ways.

(i) By using a ruler (ii) By using compasses.

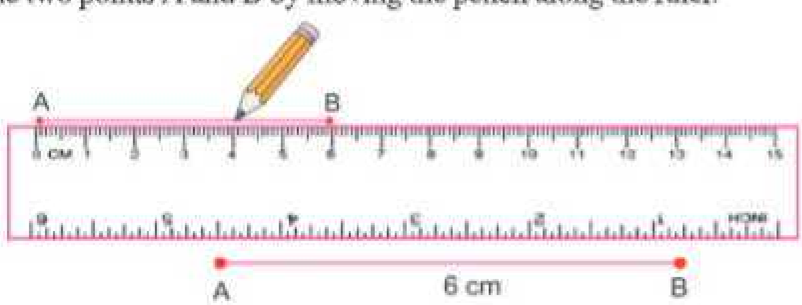
(i) Construction of a line segment using a ruler:- A simple method to construct a line segment by using a ruler and a pencil given below :

Let us draw a line segment of length 6cm.

Step of Construction:-

1. Place the ruler on a paper and hold it firmly.
2. Mark a point A with the pencil against 0 of the ruler and another point B against 6cm mark of the ruler.

3. Join the two points A and B by moving the pencil along the ruler.



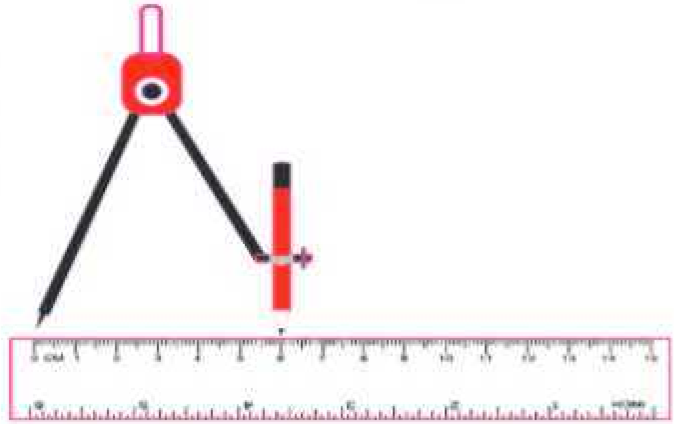
Thus AB = 6cm is the required line segment.

The another method would be to use compasses for the construction of a line segment.

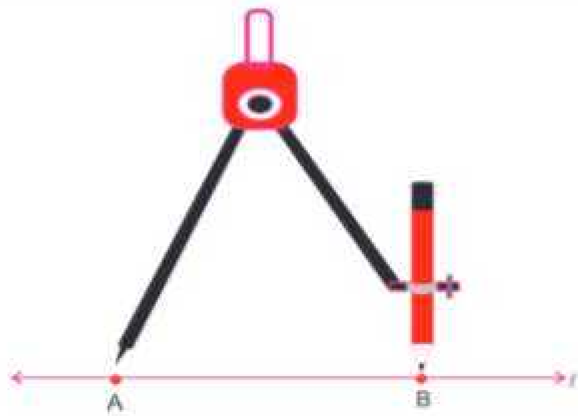
(ii) Construction of a line segment using a ruler and Compasses:- To construct a line segment of given length using compasses, we follow the following steps:-

1. Draw a line ℓ and mark a point A on it.

2. Place the metal point of the compasses at zero mark on the ruler and open it out that the pencil point is on the mark 6.



3. Without disturbing the opening of the compasses, place its needle at point A and draw an arc to cut the line ℓ at point B.



4. AB is the required line segment of length 6cm.

10.3.1 Construction of a copy of a given line segment

Suppose a line segment XY is given

We want to draw a line segment AB whose length is equal to line segment XY.

Normally we measure the length of line segment XY with the ruler and then draw another line segment AB of the same measurement.

(But this method do not always give a line segment of accurate length.) We use ruler and compass to draw an accurate copy of a line segment.

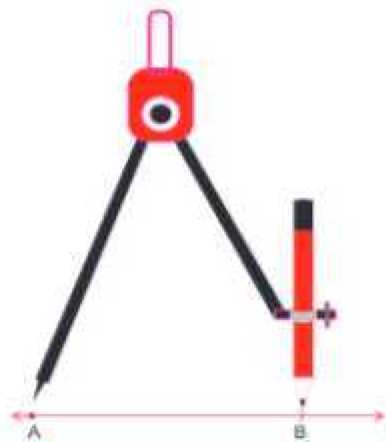
• **Construction using a ruler and compasses**

Given a line segment XY.

Step 1:- Draw a line ℓ and mark a point A on it.

Step 2:- Take the compasses and measure XY.

Step 3:- Without disturbing the compasses, place the needle of the compasses at point A on line ℓ and draw an arc, which intersects the line at point B.



Step 4:- AB is the required line segment which is equal to the length of XY.

Thus AB = XY

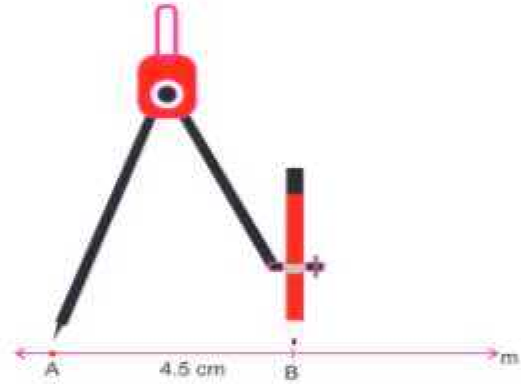
10.3.2. Construction of a line segment whose length is the sum of the length of the two given segments

Let AB and BC be two line segments of length 4.5cm and 3cm respectively. We can construct a line segment AC of length AB + BC i.e. 4.5 + 3 = 7.5cm using a ruler and compasses.

Steps of Construction:-

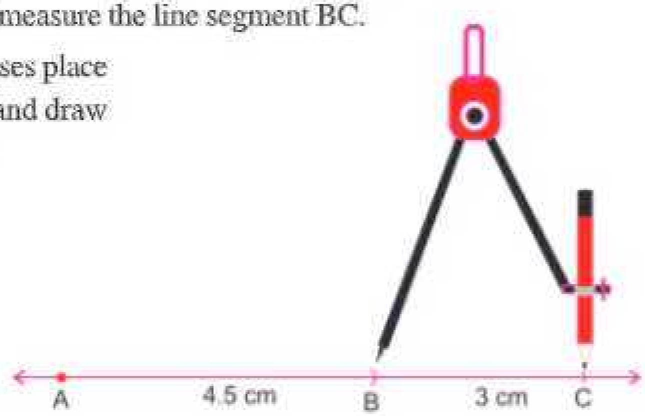
Step 1:- Draw a line m longer than combined length of AB and BC. Mark a point A on it.

Step 2:- Take the compasses and measure AB. Without disturbing the compasses place its needle at A and draw an arc intersecting line m at B.



Step 3:- Again adjust the compass and measure the line segment BC.

Step 4:- Without disturbing the compasses place the pointer at B on the line m and draw an arc cutting the line m at C.

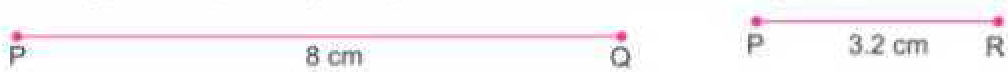


Step 5:- Thus AC is the required line segment whose length is equal to the sum of lengths of line segments AB and BC.

i.e. AC = AB + BC = 4.5 + 3 = 7.5cm.

10.3.3 Construction of a line segment equal to the difference of the lengths of two given line segments

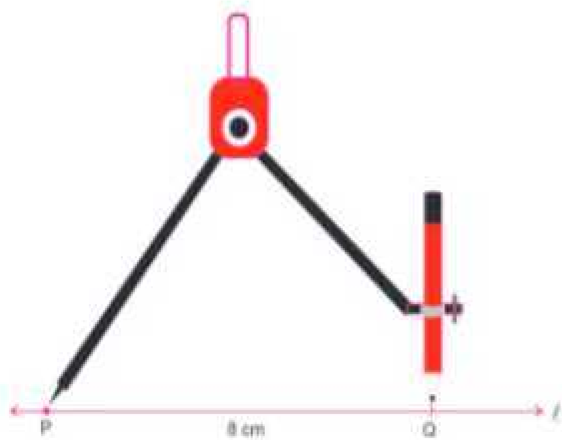
Let PQ and PR be two line segments of length 8cm and 3.2cm respectively. We can construct a line segment RQ of length PQ – PR i.e. 8 – 3.2 = 4.8cm using a ruler and compasses.



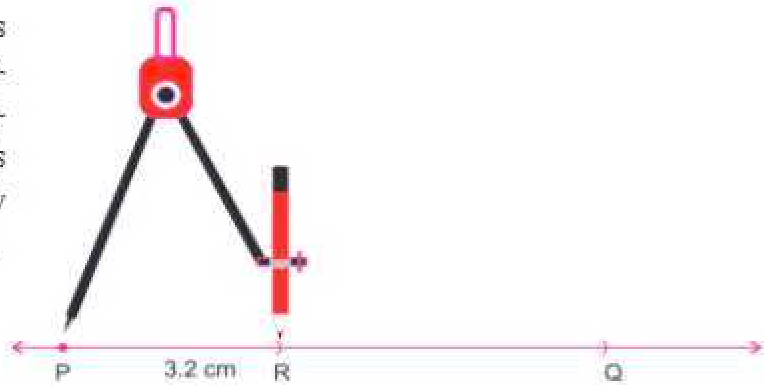
Steps of Construction

Step 1:- Draw a line ℓ and mark point P on it.

Step 2:- Take the compasses and measures PQ. Without disturbing the compasses place its needle at P and draw an arc intersecting ℓ at Q.



Step 3:- Again adjust the compasses and measure the line segment PR. Without disturbing the compasses, place its needle (pointer) at P draw an arc intersecting ℓ at R.



Step 4:- RQ is the required line segment whose length is equal to the difference of lengths of line segments PQ and PR

ie. RQ = PQ – PR = 8 – 3.2 = 4.8cm

Exercise 10.2

1. With the help of a ruler, construct line segments of given lengths:

   (i) 5cm (ii) 6.5cm (iii) 5.2cm (iv) 6.8cm (v) 9.7cm (vi) 8.4cm

2. Draw line segments given in Q.1 by using a ruler and compasses.
3. Construct AB of length 8.4cm. From it cut off AC of length 5.3cm. Measure BC.
4. Draw two line segments AB and CD of lengths 8.4cm and 4.5cm respectively. Construct the line segments of the following lengths:-

   (i) AB + CD (ii) AB – CD (iii) 2CD

5. Draw two line segments PQ and RS of lengths 6.4cm and 3.6cm respectively. Construct the line segment of the following lengths:

   (i) PQ + RS (ii) PQ – RS (iii) 2PQ (iv) 2RS (v) 3RS

6. Draw a line segment PQ of any length Now without measuring it, draw a copy of PQ.

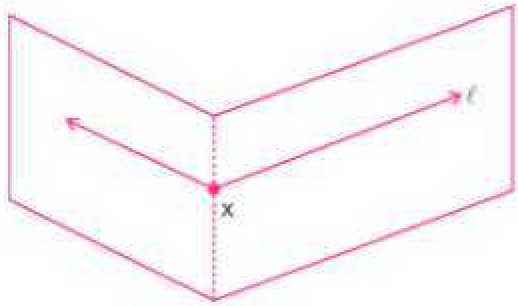
10.4. Construction of a Perpendicular at a point on the line

Given a line ℓ with a point X on it. Let us draw a perpendicular passing through this point X on line ℓ.

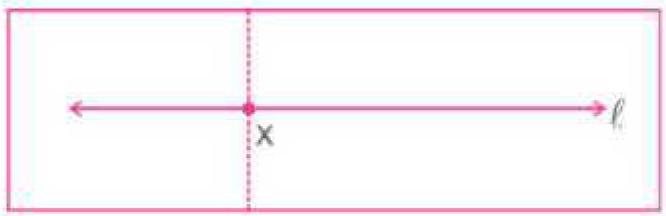
There are many methods to draw a perpendicular on a line which are as follows:

(a) By Paper Folding

1. Draw this line on a trace paper.
2. Now, fold the tracing paper in such a way that the lines across the folding exactly overlap each other.

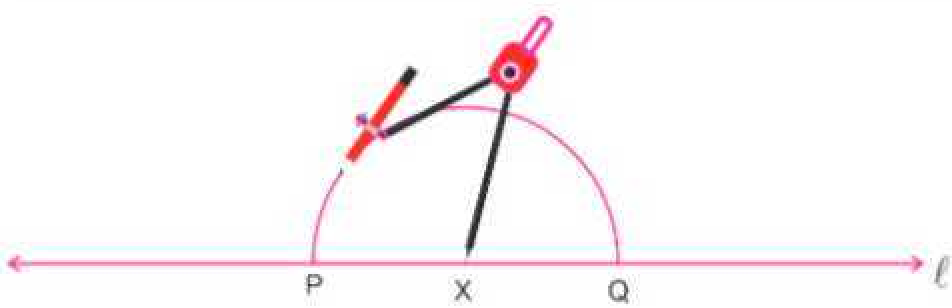


3. Adjust the fold such that the crease passes through the point X.
4. On opening the paper, the crease you get is the required perpendicular to the line ℓ passing throughst X.

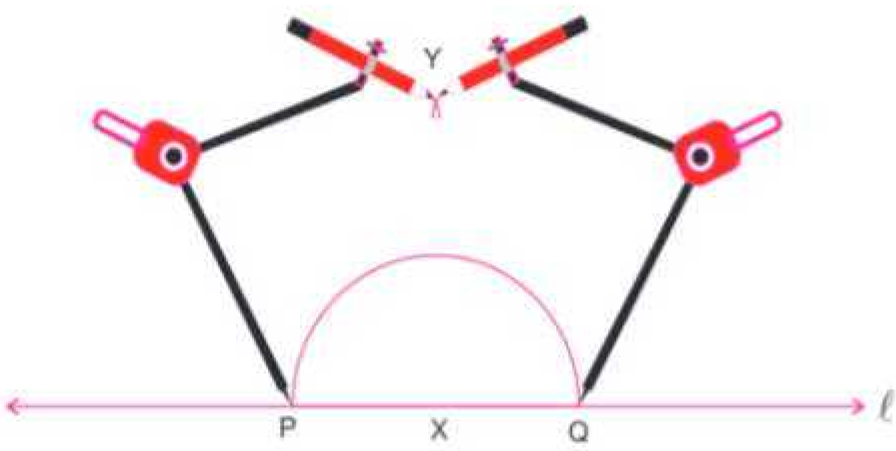


(b) By ruler and compasses :

1. Draw a line ℓ and mark a point X on it.
2. Draw an arc from X to the line ℓ of any suitable radius which intersects line ℓ at Pand Q.

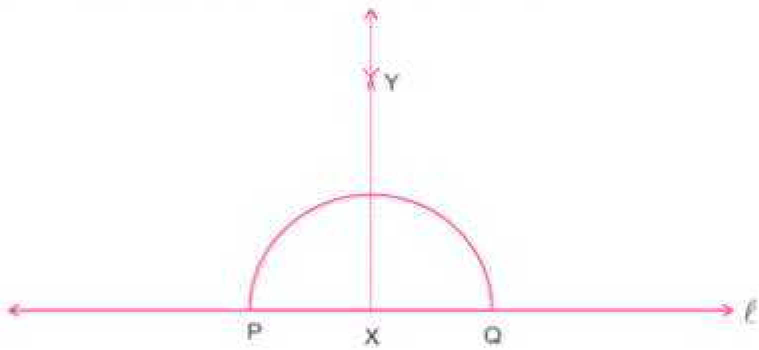


3. Draw arcs of any radius which is more than half of arc made in step (2) from P and Q which intersect at Y.



4. Join XY.

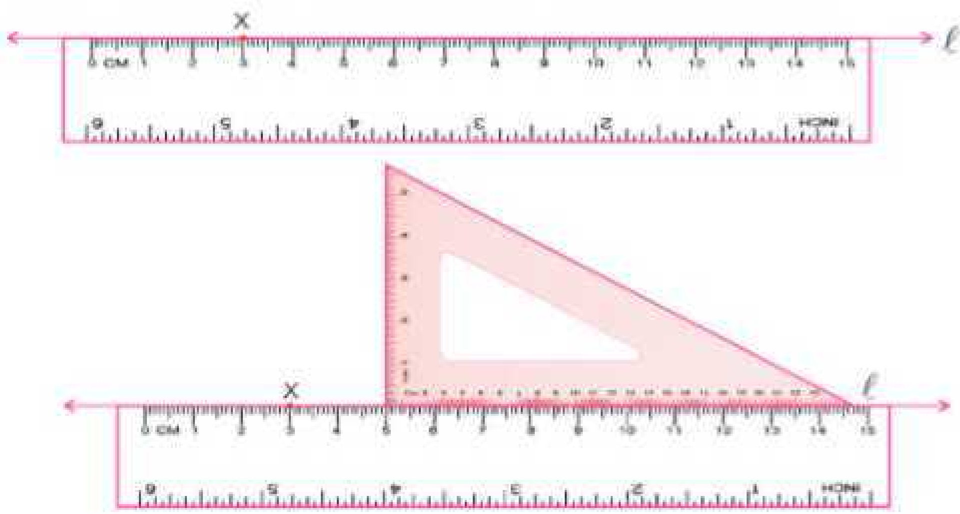
Thus XY is perpendicular to PQ or line ℓ or $XY \perp PQ$



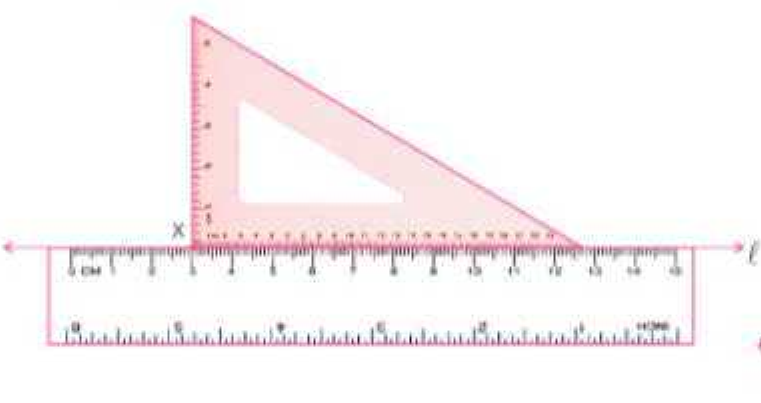
Here X is called **foot of perpendicular**.

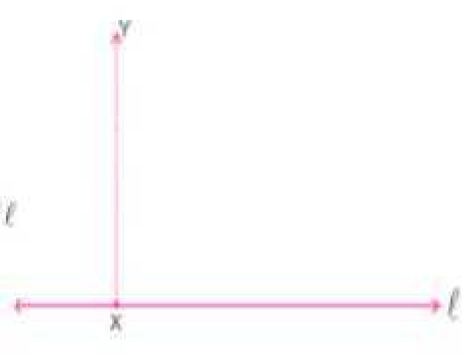
(c) Construction using a set square and a ruler:

1. Draw a line ℓ and mark a point X on it.
2. Place one of the edges of a ruler along the line ℓ and hold if firmly.



3. Place the set square in such a way that one of its edges contaning the right angle coincides with the ruler.
4. Holding the ruler, slide the set square along the line ℓ till the vertical side reaches the point X.





5. Firmly hold the set square in this position. Draw XY along its vertical edge.

   Now XY is the required perpendicular to ℓ. ie. $XY \perp \ell$.

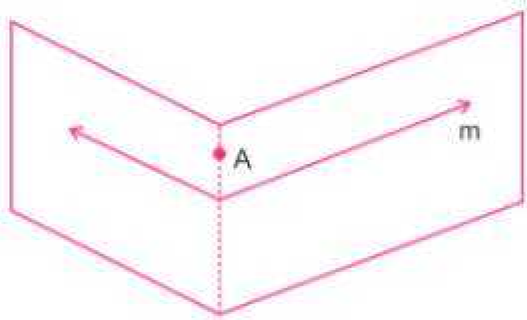
10.4.1. Construction of an Altitude to a line from a given point which is not on the line

Given a line m with a point A which is not lying on it. Let us draw an altitude passing through this point A on line m.

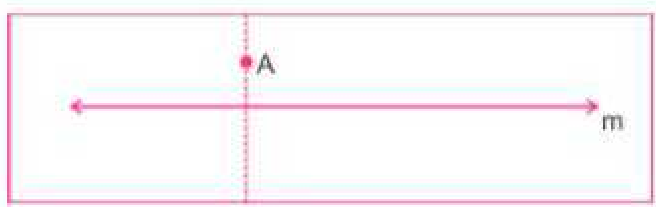
There are many methods to draw altitude which are as follows:

(a) By Paper Folding

1. Draw this line m on a trace paper.
2. Now, fold the trace paper in such a way that the lines across the folding exactly overlap each other

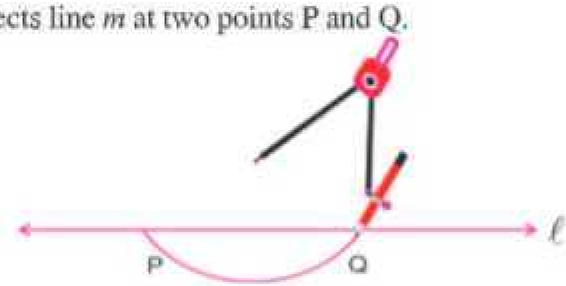


3. Adjust the fold such that the crease passes through the point A.
4. On opening the paper, you get a crease which is the required altitude to the line m passing through A.

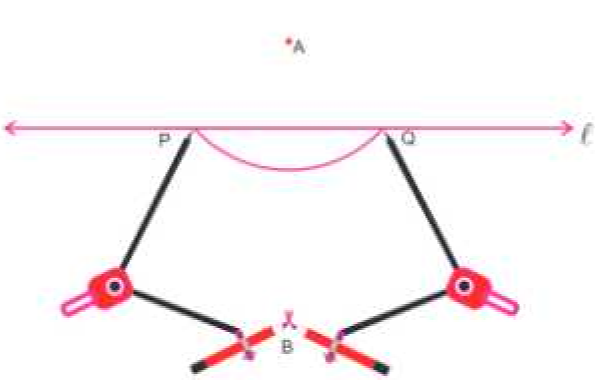


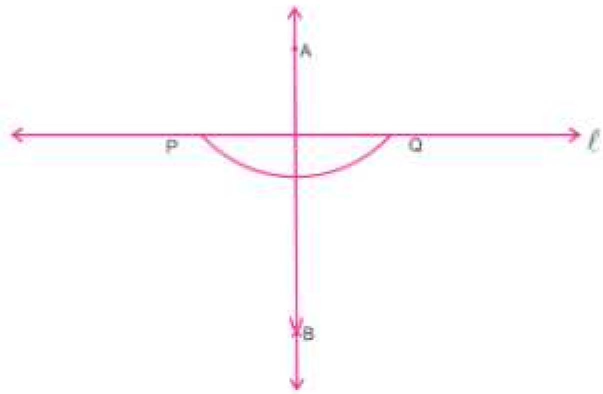
(b) By Ruler and Compasses

1. Draw a line m and mark a point A not lying on it.
2. From point A draw an arc which intersects line m at two points P and Q.



3. Using any radius and taking P and Q as centre, draw two arcs that intersect at point say B. on the other side (as shown in figure).
4. Join AB to obtain altitude to the line m.



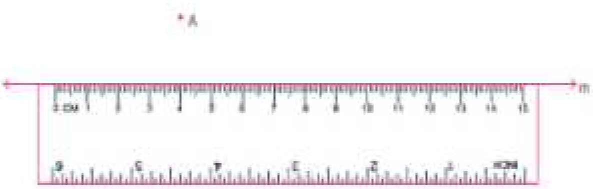


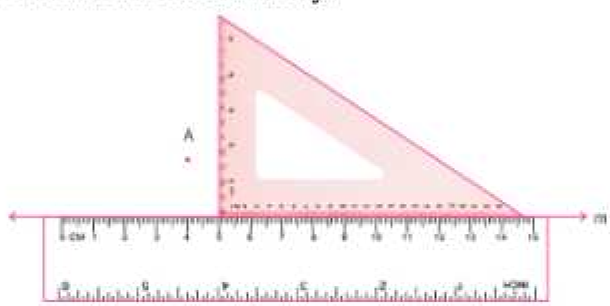
Thus AB is altitude to line m.

i.e $AB \perp m$

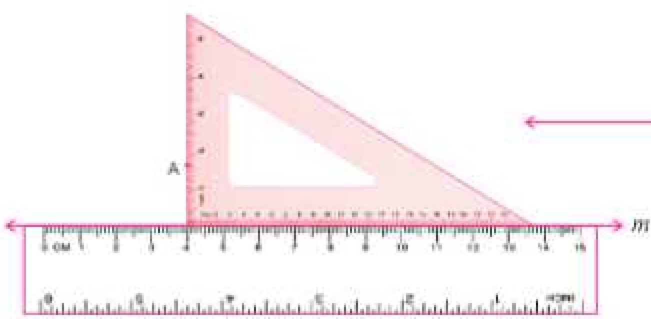
(c) Construction using a set square and a ruler

1. Draw a line m and mark a point A which is not lying on it.
2. Place one of the edge of a ruler along the line m and hold it firmly.





3. Place the set square in such a way that one of its edges containing the right angle coincides with the ruler.
4. Holding the ruler firmly, slide the set square along the line m till its vertical side reaches the point A.

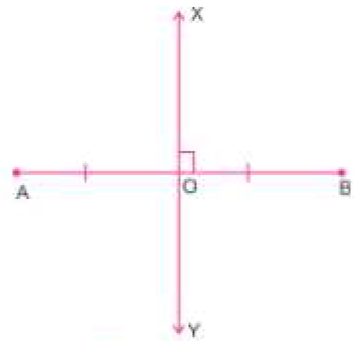


5. Firmly hold the set square in this position, Draw AB along its vertical edge

Now AB is the required altitude to m i.e. $AB \perp m$.

10.5 Perpendicular Bisector

A line which **is perpendicular to a line segment at its mid point is called perpendicular bisector** of that line segment. It is also called the **line of symmetry**.



In the given figure, line XY is perpendicular bisector of line segment AB i.e. AO = OB and $\angle XOB = \angle XOA = 90^o$

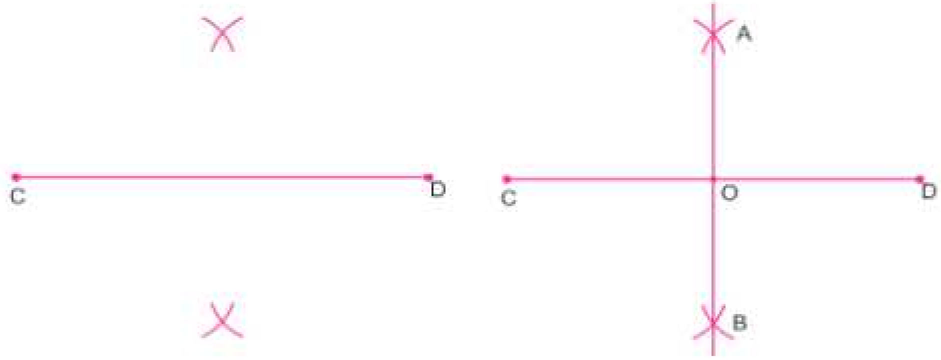
10.5.1. Construction of the perpendicular bisector of a line segment

Given a line segment CD of 6.4cm. Let us draw perpendicular bisector of this line segment.

Step of Construction:

1. Draw a line segment CD = 6.4cm

2. Taking C as centre, draw arcs on both sides of CD, by taking radius more than half of CD. (as shown in figure)
3. Now take D as a centre and draw arcs of same radius as before, intersecting the previous drawn arcs at A and B respectively.
4. Join AB intersecting CD at O. Then O bisects the line segment as shown.



Thus AB is the required perpendicular bisector of CD.

Exercise 10.3

1. Draw a line *r* and mark a point P on it. Construct a line perpendicular to *r* at point P.
   (i) Using a ruler and compasses
   (ii) Using a ruler and a set square.
2. Draw a line *p* and mark a point *z* above it. Construct a line perpendicular to p, from the point *z*.

(i) Using a ruler and composses.

(ii) Using a ruler and a set square.

3. Draw a line AB and mark two points P and Q on either side of line AB , Construct two lines perpendicular to AB, from P and Q using a ruler and compasses.
4. Draw a line segment of 7cm and draw perpendicular bisector of this line segment.
5. Draw a line segment PQ = 6.8cm and draw its perpendicular bisector XY which bisect PQ at M. Find the length of PM and QM. Is PM = QM?
6. Draw perpendicular bisector of line segment AB =5.4cm. Mark point X anywhere on perpendicular bisector Join X with A and B. Is AX = BX?
7. Draw perpendicular bisectors of line segment of the following lengths :

(i) 8.2cm (ii) 7.8cm (iii) 6.5cm

8. Draw a line segment of length 8cm and divide it into four equal parts using compasses. Measure each part.

10.6 Circle

We have already studied about circle and its parts radius, chord, centre, diameter etc.

A circle is a closed plane curve in which every point on the circle is at a constant distance from a fixed point inside the circle. The fixed point is known as the **centre** and the constant distance from centre to the circle is called **radius**.

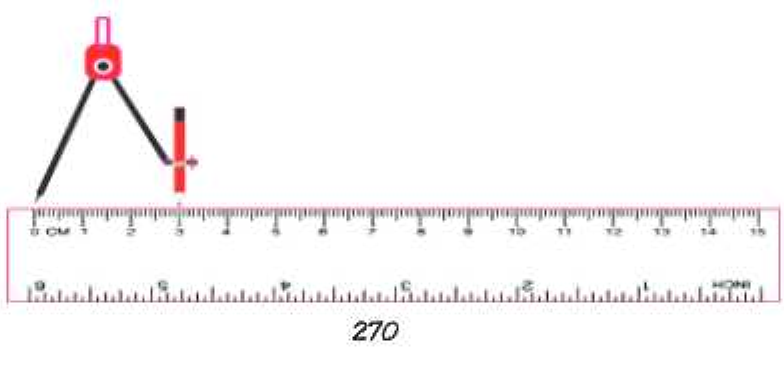
Bangle, coin, wheel, chapatti etc are all circular in shape.

Here, we will learn the construction of a circle with given centre.

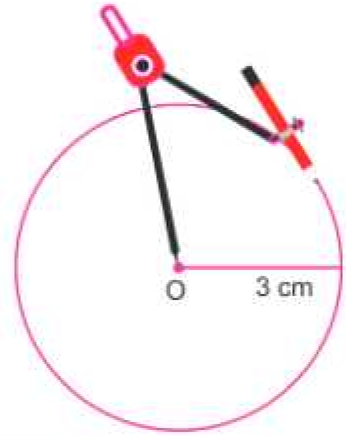
10.6.1. Construction of a Circle of a given radius

Let us construct a circle of radius 3cm.

1. Mark a point O on a paper or notebook, where a circle is to be drawn.
2. Take Compasses fixed with sharp pencil and measure 3cm using a scale as shown in figure.



3. Without disturbing the opening of the compasses, keep the needle at point O and draw complete arc by holding the compasses from its knob. After completing one round, we get the desired circle.



Exercise 10.4

1. Draw a circle of the following radius :

   (i) 3.5cm (ii) 4cm (iii) 2.8cm (iv) 4.7cm (v) 5.2cm

2. Draw a circle of the diameter 6cm.
3. With the same centre O, draw two concentric circles of radii 3.2cm and 4.5cm.
4. Draw a circle of radius 4.2cm with centre at O. Mark three points A, B and C such that point A is on the circle, B is in the interior and C is in the exterior of the circle.
5. Draw a circle of radius 3cm and draw any chord. Draw the perpendicular bisector of the chord. Does the perpendicualr bisector passes through the centre?

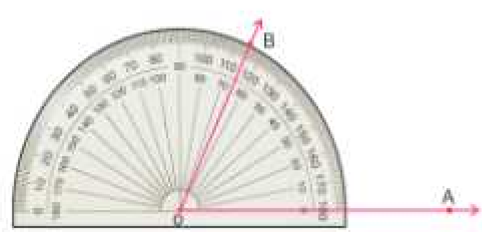
10.7. Angles

You have already learnt about the various types of angles. In this section, you will learn about the construction of angles of given measure by using a protractor and the construction of some specific angles with the help of compasses.

Let us construct an angle of 65°.

Step of Construction

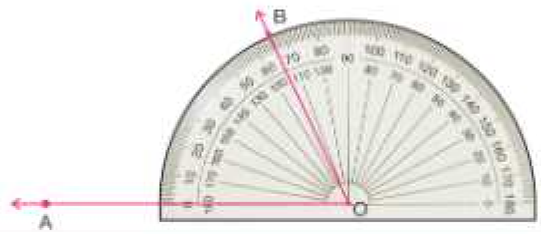
1. Draw a ray OA.
2. Place the protractor on OA such that its centre lies on the initial point O and 0-180 base line along OA.



3. Mark a point B on the paper against the mark of 65° (inner scale) on the protractor.
4. Remove the protractor and Join OB.

Thus required angle $\angle AOB = 65^\circ$

If ray $\overrightarrow{OA}$ lies to the left of the centre (mid point) of the baseline, start reading the angle on the outer scale from 0° and mark B at 65°. Join OB, then $\angle AOB = 65^\circ$



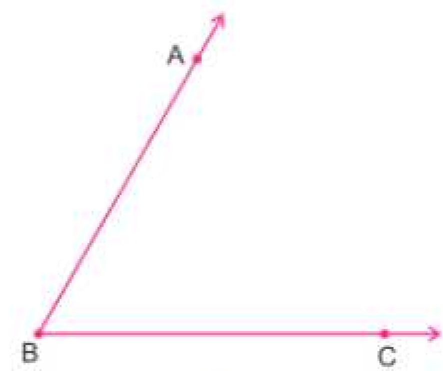
Remember word LORI - Left Outer Right Inner

When the direction of ray is towards left, read the outer scale and when the direction of ray is towards right, read the inner scale.

10.7.2. Construction of an angle equal to a given angle

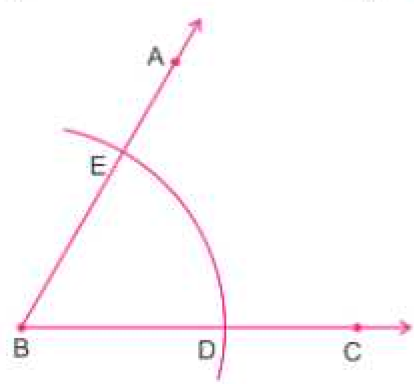
Suppose you are asked to copy an angle, whose measure is not known. A better method is to use compasses and a ruler.

Let us construct $\angle PQR$ equal to given $\angle ABC$.

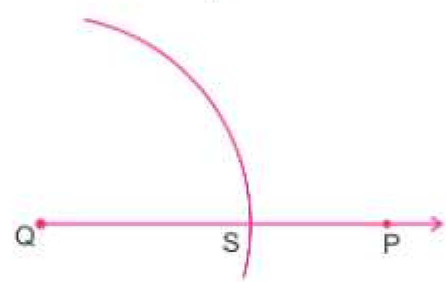


Step of Construction

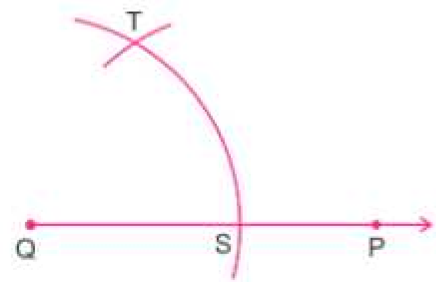
1. Taking B as centre and draw an arc of any radius intersecting the arms AB and BC at points E and D respectively.



2. Draw a ray QP, with Q as centre and the same radius as above, draw an arc intersecting ray QP at S.

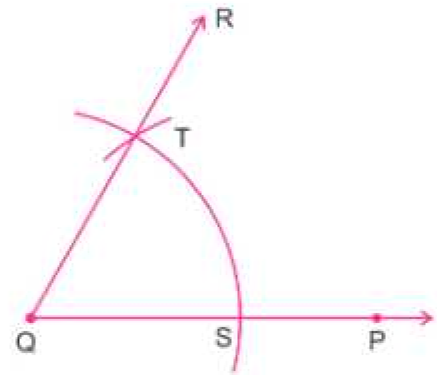


3. Now measure the distance between D and E with compasses, taking S as centre and take radius equal to DE, draw an arc to intersect the arc drawn above at a point T.



4. Join QT and produce it to form a ray QR.

5. $\angle PQR$ is the required angle equal to $\angle ABC$.



6. Verify it by measuring the angles with the help of protractor.

10.8. Angle Bisector

Any ray that divides an angle into two equal parts is known as the angle bisector of the given angle. In this section you will learn its construction.

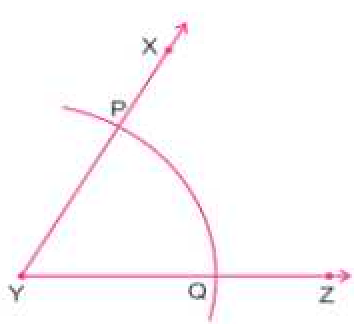
10.8.1. Construction of an Angle Bisector

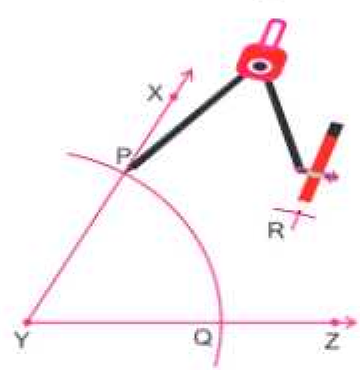
Bisecting an angle means drawing a ray in the interior of the angle, with its initial point at the vertex of the angle such that it divides the angle into two equal parts.

Now construct the angle bisector of $\angle XYZ$.

Steps of construction

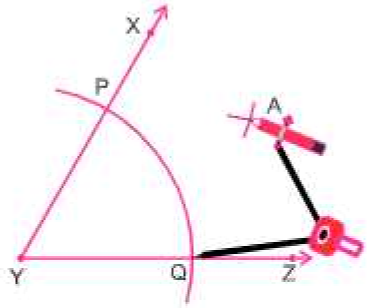
1. Draw $\angle XYZ$ of any measure.
2. Take Y as centre and any convenient radius, draw an arc intersecting XY and YZ at P and Q respectively.

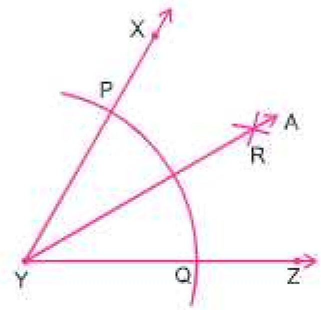




3. Draw an arc from P with radius more than half of PQ.

4. Draw an arc from Q with same radius as in step 3, which intersects the arc of step 3 at R.





5. Join YR and produce to any point A. Thus ray YA is angle bisector of $\angle XYZ$
Measure $\angle XYA$ and $\angle AYZ$, you will find that $\angle XYA = \angle AYZ$

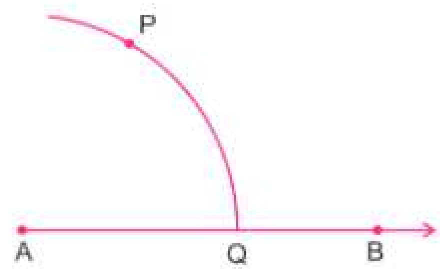
10.9. Construction of angles of special measures by compasses

In this section, we will learn the construction of some angles of special measures like 30°, 45°, 60°, 90°, 120° etc with the help of ruler and compasses.

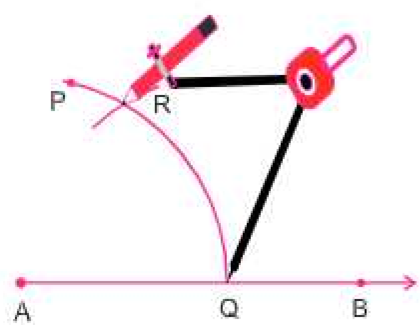
* Construction of 60° angles

In order to construct angle of 60° with the help of ruler and compasses only, we follow the following steps :

1. Draw any ray AB.
2. Draw an arc PQ of any radius from point A, which intersect AB at Q.

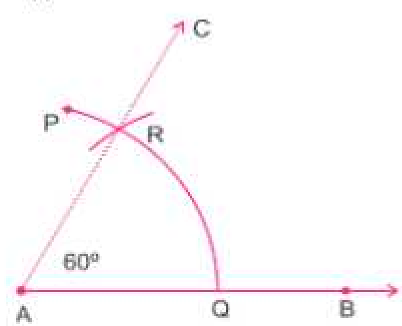


3. By taking same radius, draw an arc from point Q which intersect arc PQ at R.



4. Join AR and produce it to get AC.

5. Thus $\angle BAC = 60^\circ$

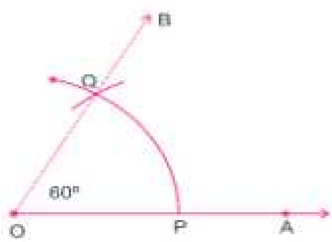


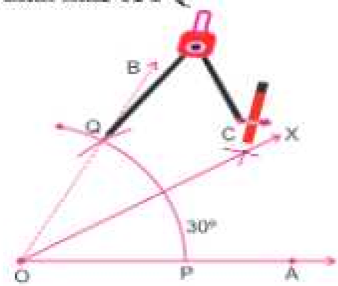
* Construction of 30° angles

Since 30° is half of 60° i.e. $30° = \frac{1}{2} \times 60°$ So to construct 30° angle, we bisect the angle 60°.

Step of Construction :

1. Draw an angle $\angle AOB = 60°$ as shown in figure with the help of compasses.
2. Draw an arc from P by taking radius greater than half of PQ





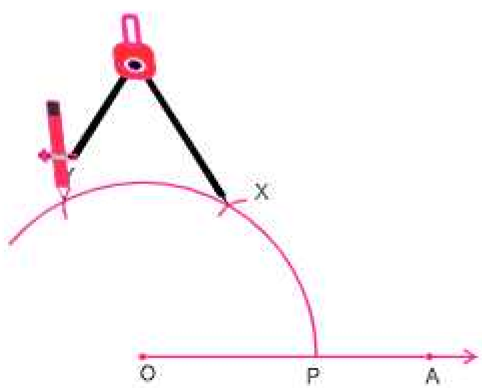
3. Similarly draw an arc from Q with same radius which intersect the previous arc at C.
4. Join OC and produce it to X.
5. Thus $\angle AOC = 30°$

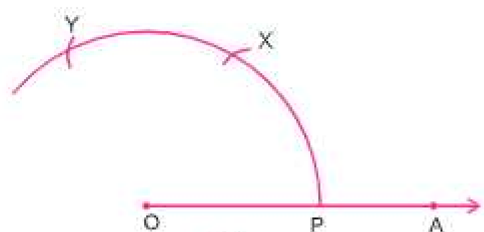
* Construction of 90° angle :

Since 90° is exactly in the mid of 60° and 120°. So for construction of 90°, we bisect the angle between 60° and 120°.

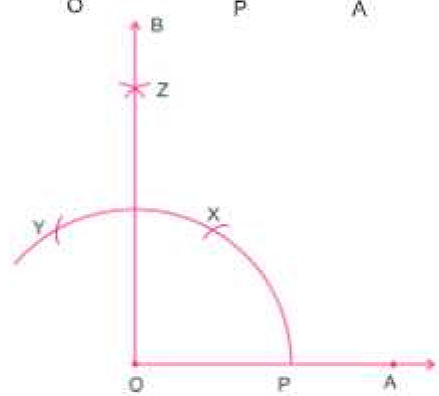
Step of Construction :

1. Draw arcs X and Y of angles 60° and 120° respectively as discussed above.





2. With X as centre and radius more than half of arc 'XY', draw another arc.
3. With Y as centre and same radius, draw an arc cutting the previous arc at Z.
4. Join OZ and produce it to B.



5. Thus $\angle AOB = 90^\circ$

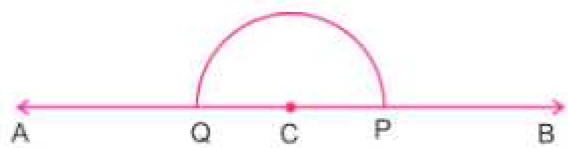
Aliter :-

There is one more method to construct 90° by a ruler and compasses.

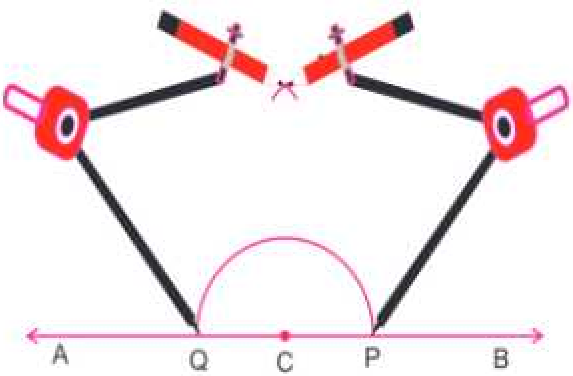
Since 90° is half of 180° i.e $90^\circ = \frac{1}{2} \times 180^\circ$ So we bisect angle between 0° and 180°.

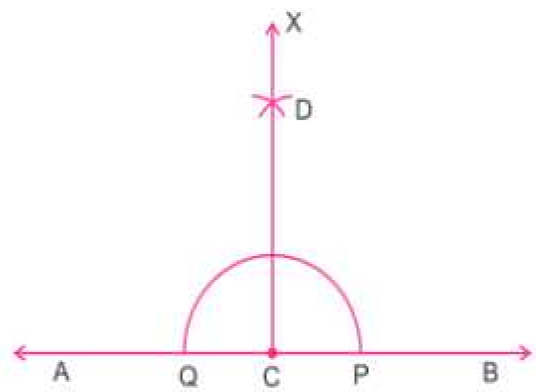
Step of Construction :

1. Draw a line AB and mark a point C on it.
2. Taking C as centre and with any suitable radius, draw an are PQ cutting AB at P and Q.



3. Here $\angle ACB = 180^\circ$ (It is a straight line)
4. Taking P and Q as the centres and with any convenient radius, draw arcs intersecting each other at D.





5. Join CD and produce it to X.
6. Thus $\angle ACX = \angle BCX = 90^\circ$

Exercise 10.5

1. Draw the following angles in both directions (Left and right) by protractor.

(i) 75° (ii) 110° (iii) 62° (iv) 165° (v) 170°

(vi) 32° (vii) 128° (viii) 25° (ix) 80° (x) 135°

2. Bisect the following angles by compasses :

(i) 48° (ii) 140° (iii) 75° (iv) 64° (v) 124°

3. Draw an angle of 80° and bisect it in to four equal parts by compasses.

4. Draw a right angle and bisect it.

5. Draw the following angles by ruler and compasses :

(i) 30° (ii) 45° (iii) 135° (iv) 180° (v) 120° (vi) 75°

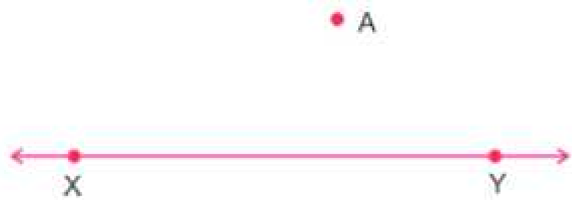
6. Draw an angle of 30° by protractor and bisect it by a ruler and compasses.

10.10. Construction of a Parallel Line through the point lying outside a line

We have studied about parallel lines in previous chapter that parallel lines are those lines which never intersect each other. Here we will construct a parallel line through a given point lying either side to a given line.

(a) By a Ruler and Compasses

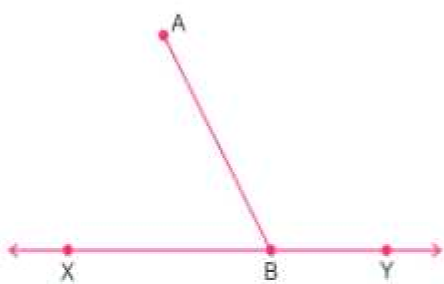
Let us consider a line XY and a point A not lying on it.

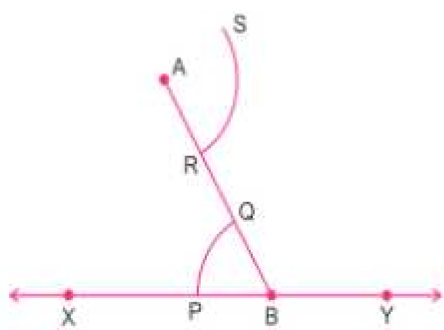


Steps of Construction

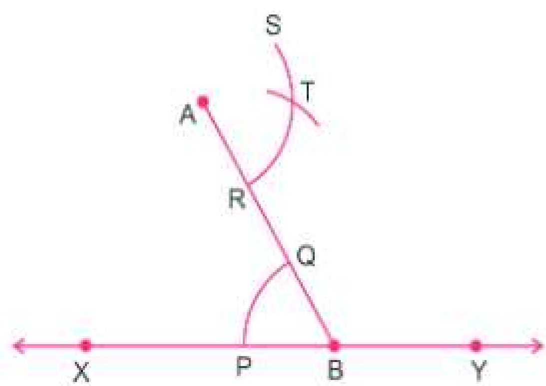
1. Take any point, say B, anywhere on line XY.

2. Join AB.





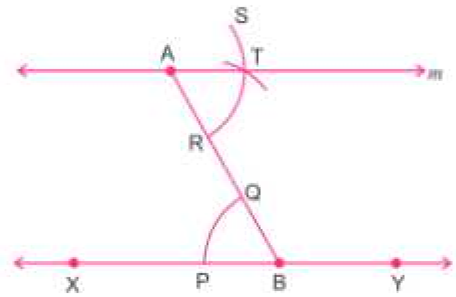
3. Now take B as centre, draw an arc PQ of any radius on XY. similarly draw an arc RS of same radius on line segment AB from point A.



4. Measure arc PQ with compasses.

5. Draw an arc equal to radius PQ from point R which intersect RS on T.

6. Join AT and produce it both sides.



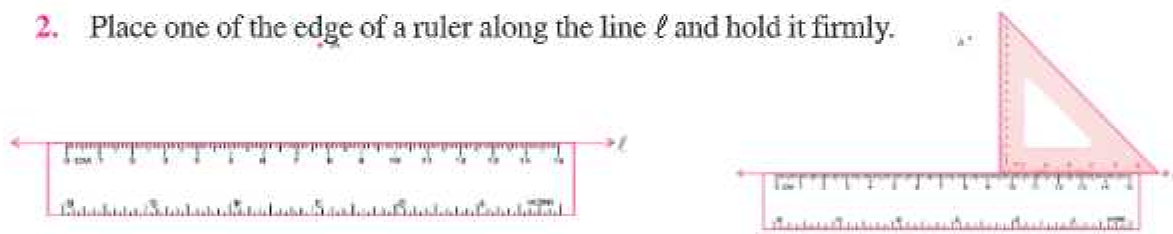
So the line m is the required line parallel to XY.

(b) By Set Squares

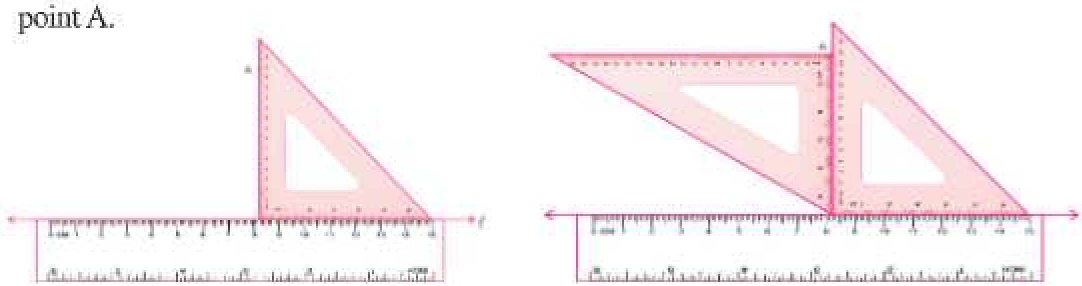
We can draw parallel line with the help of set squares also. This is the accurate method to construct a parallel line.

Step of Constructions :

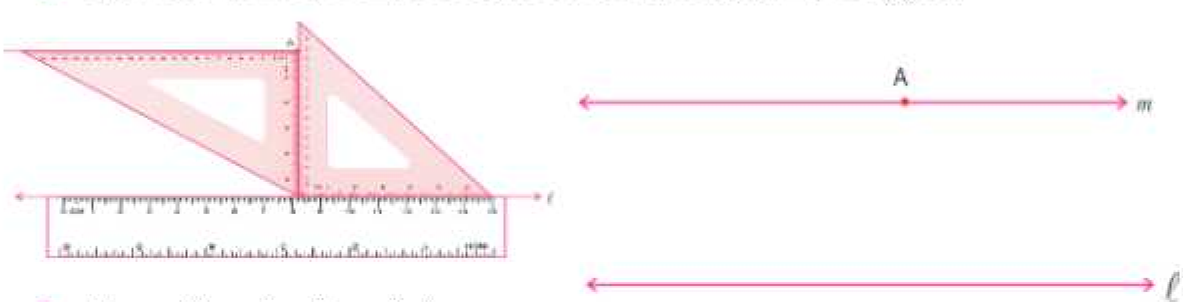
1. Given a line ℓ with point A not lying on it.
2. Place one of the edge of a ruler along the line ℓ and hold it firmly.



3. Place the set square in such a way that one of its edges containing the right angle coincides with the ruler.
4. Hold the ruler firmly, slide the set square along the line ℓ till its vertical side reaches the point A.



5. Firmly hold the set square in this position, take another set square and place it in such a way that one of its edges containing right angle concides with previous set square as shown.
6. Now draw a line m along side of second set square passing through A.



7. Thus $m \| \ell$ passing through A.

Exercise 10.6

1. Draw a line XY and point P not lying on XY. Draw a line parallel to XY passing through P with the help of ruler and compasses.
2. Draw a line *p* parallel to line *m* passing through a point A which is not lying on line *m* with the help of set squares.
3. Given a line AB and the point X is not lying on it Draw a line parallel to AB passing through X.
   (i) By a ruler and compasses (ii) By set squares

Learning Outcomes

After completion of this chapter the students are now able to

- Know about geometrical tools.
- Draw different types of angles.
- Classify the angles according to their measurment.
- Construct the bisector of an angle.
- Draw the perpendicular bisector of a line segment.

T3X5V6

11

RATIO AND PROPORTION

Objectives

In this chapter you will learn

- To compare quantities using ratios.
- To use ratio in different situations.
- To capable of using proportion in practical life.
- To use unitary method in daily life problems.

11.1 Introduction

$\frac{\text{Paras's marks}}{\text{Charan's marks}}$

In our daily life, we come across many situations where we have to compare two quantities of same type in terms of their magnitudes/measurements.

For Example, In an examination Paras got 84 marks and Charan got 70 marks. Generally their performance can be compared in two ways:

(i) Comparison by difference

Difference of marks = 84 – 70 = 14

i.e. Paras got 14 marks more than Charan.

Such a comparison is known as comparison by difference.

(ii) Comparison by Division

$$\frac{\text{Paras's marks}}{\text{Charan's marks}} = \frac{84}{70} = \frac{6}{5}$$

i.e. Paras's marks are $\frac{6}{5}$ times the marks of Charan.

Or $$\frac{\text{Charan's marks}}{\text{Paras's marks}} = \frac{70}{84} = \frac{5}{6}$$

i.e. Charan's marks are $\frac{5}{6}$ times the marks of Paras.

Such a comparison is known as comparison by division.

Thus, If we want to observe that how much more (or less) one quantity is than the other' then we compare by difference. On the other hand, if we want to observe how many times more (or less) one quantity is than the other, we compare by division.

Infact, when we compare two quantities of the same kind by division, we say that we form a ratio of two quantities.

i.e. The **comparison by division** is also known as **ratio**.

11.2 Ratio

Ratio is the relation used to describe the number of times a quantity is, of the other. Thus, the ratio of two numbers a and b ($b \neq 0$) is $\frac{a}{b}$ or $a \div b$ and is represented as a:b (read as "a ratio b" or 'a is to b"), where a and b are called the terms of the ratio.

- The first term 'a' is called **antecedent** and the second term 'b' is called **consequent**.

11.2.1 Properties of Ratio

- The two quantities to be compared should be of same kind or in the same units (of length, volume or quantity etc.)

  i.e. we can not compare18 boys and 12 horses but we can compare the number of boys and the number of horses,.
- The order of the terms in a ratio is important. i.e. the ratio 3 is to 4 or 3:4 is quite different from the ratio 4 is to 3 or 4:3.
- Ratio of two quantities is just a number and has no unit at all.
- **Equivalent Ratios :** The terms of ratio can be multiplied or divided by same non-zero number then there is no change in its value.

  i.e. 2 : 3

  $2 \times 2 : 3 \times 2 = 4:6$ (Multiply both terms by 2)

  or $2 \times 4 : 3 \times 4 = 8:12$ (Multiply both terms by 4)

  and 18:24

  or $18 \div 2 : 24 \div 2 = 9:12$ (Divide both terms by 2)
- **Ratio in the simplest form :** A ratio a:b is said to be in the simplest form if its antecedent a and consequent b have no common factor other than **1**.

  A ratio in the simplest form is also called the ratio in the lowest terms. For this, Divide both terms by their HCF.

  e.g. 32 : 24

  or HCF of 32 and 24 is 8.

  or $32 \div 8 : 24 \div 8$

  or 4 : 3

Note : Difference between a fraction and a ratio

- A fraction is a number that represents part of a whole or part of a group. The denominator represents the total number of equal parts the whole is divided into.
- A ratio is a comparison of two quantities

  for example In a group of 8 students, there are 5 boys and 3 girls, the fraction of the number of boys is $\frac{5}{8}$ and of girls is $\frac{3}{8}$. The denominator will always be 8, because the whole group consists of 8 students.

  While, the ratio of number of boys to number of girls is 5:3, The ratio of number of boys to total students is 5:8 and the ratio of number of girls to total students is 3:8.

  Ratio depends on the numbers that are being compared not the whole group.

Let's discuss some examples.

Example 1: **Express the following ratio in the simplest form:**

(i) 18:24 (ii) 15:45 (iii) 36:28

Solution : (i) 18:24

To express this ratio in the simplest form, we shall have to divide both terms by their HCF.

So HCF of 18 and 24 is 6.

$\therefore \quad 18 : 24$

$= \quad (18 \div 6) : (24 \div 6) = 3:4$

Hence, the simplest form of 18:24 is 3:4.

Aliter : (Prime Factorisation)

We have 18:24

Prime factors of both terms

$2\times3\times3 : 2\times2\times2\times3$

Divide both terms with common factors 2×3, We have

$3:2\times2$ or $3:4$

Aliter : 18 : 24

In this, we divide both terms one by one by common factors.

- Both terms are divisible by 2, so divide both terms by 2, We have

  9:12
- Now both terms are divisible by 3, so divide both terms by 3, we have

  3 : 4 → Which is simplest form

(ii) 15:45

To express this ratio in the simplest form, we shall divide both terms by their HCF.

So, HCF of 15 and 45 is 15.

$\therefore$ 15 : 45

$\therefore$ $\frac{15}{45} = \frac{15 \div 15}{45 \div 15} = \frac{1}{3}$ or $(15 \div 15) : (45 \div 15) = 1:3$

Hence, the simplest form of 15:45 is 1:3.

Aliter : (Prime Factorisation)

15 : 45

First Prime factorise of both terms

= $3 \times 5 : 3 \times 3 \times 5$

Divide both terms with common factors $3 \times 5 = 15$, we have

= 1 : 3

Aliter : 15 : 45

Firstly divide both terms by 3.

= 5 : 15

Now divide both terms by 5.

= 1 : 3 (Which is simplest form)

(iii) 36 : 28

Divide both terms by their HCF = 4

$\therefore$ 36 : 28

= $(36 \div 4) : (28 \div 4) = 9 : 7$

Hence, the simplest form is 9 : 7

Aliter : 36: 28

= $2 \times 2 \times 3 \times 3 : 2 \times 2 \times 7$

= 9 : 7 [Divide both term with common factor $2 \times 2 = 4$]

Aliter : 36 : 28

Divide both terms by 2, We have

= 18 : 14

Again divide both terms by 2, we have

= 9 : 7 (Which is the simplest form)

Example 2 : Find the ratio in the simplest form:

(i) 15 minutes to 40 minutes **(ii) 3kg to 800g**

(iii) 150cm to 6m **(iv) 2 dozen to 16pieces**

(v) 1 minute to 30 seconds

Solution : (i) 15 minutes : 40 minutes

= 15 : 40 (Divide both terms by their HCF= 5)

= 3 : 8

(ii) 3kg to 800g = 3000g to 800g (∵ 1kg = 1000g)

= 3000 : 800

= 15 : 4 (Divide both terms by their HCF=200)

(iii) 150cm to 6m = 150cm to 600cm (∵ 1m = 100cm)

= 150 : 600 (Divide both terms by their HCF=150)

= 1 : 4

(iv) 2 dozen to 16 pieces = 24 pieces to 16 pieces (∵ 1dozen = 12 units)

= 24 : 16 (Divide both terms by their HCF=8)

= 3 : 2

(v) 1 minute to 30 seconds

= 60 seconds to 30 seconds (∵ 1 minute = 60 seconds)

= 60 : 30 (Divide both terms by their HCF = 30)

= 2 :1

Example 3: There are 35 boys and 25 girls in a class. Find the ratio of

(i) The number of boys to the number of girls?

(ii) The number of girls to the total number of students in the class?

(iii) Total number of students to the number of boys?

Solution : We have number of boys = 35, number of girls = 25

∴ Total number of students in the class = 35 + 25 = 60

(i) Ratio of number of boys to the number of girls

= 35 : 25 (Divide both terms by their HCF = 5)

= 7 : 5

(ii) Ratio of the number of girls to the total number of students in the class.

= 25 : 60 (Divide both terms by their HCF = 5)

= 5 :12

(iii) Ratio of total number of students to the number of boys.

= 60 : 35 (Divide both terms by their HCF = 5)

= 12 : 7

Example 4: In a year, Neha earns ₹80,000 and saves ₹30,000. Find the ratio of money.

(i) She saves to the money she earns.

(ii) She earns to the money she spends.

(iii) She saves to the money she spends.

Solution : Neha's income = ₹80,000

Neha's savings = ₹30,000

Neha's spendings = ₹80,000 – ₹30000 = ₹50,000

(i) Ratio of Neha's savings to Neha's earnings.

= 30,000 : 80,000 (Divide both terms by their HCF = 10,000)

= 3 : 8

(ii) Ratio of Neha's earnings to Neha's spendings

= 80,000 : 50,000 (Divide both terms by their HCF = 10,000)

= 8 : 5

(iii) Ratio of Neha's savings to Neha's spendings

= 30,000 : 50,000 (Divide both terms by their HCF = 10,000)

= 3 : 5

Example 5: **In a class test 42 out of 56 students passed. Find the ratio between the**

(i) Number of passed students to the number of failed students.

(ii) The number of failed students to the total number of students.

Solution : Total students in the class = 56

Number of passed students = 42

So, Number of failed students = 56 – 42 = 14

(i) Ratio of the number of passed to the number of failed students

= 42 : 14 (Divide both terms by their HCF = 14)

= 3 : 1

(ii) Ratio of the number of failed students to the total number of students in the class.

= 14 : 56 (Divide both terms by their HCF=14)

= 1 :4

Example 6: **The present age of mother is 48 years and that of son is 20 years. Find the ratio of.**

(i) Present age of son to the present age of mother.

(ii) Age of mother to the age of son 10 years ago.

(iii) Age of son to the age of mother after 8 years.

Solution : (i) Ratio of present age of son to the present age of mother

= 20 : 48 (Divide both terms by their HCF = 4)

= 5 : 12

(ii) 10 years ago, son's age = 20 – 10 = 10 years

Mother age = 48 – 10 = 38 years.

Ratio of age of mother to the age of son, 10 years ago.

= 38 : 10 (Divide both terms by their HCF=2)

= 19 : 5

(iii) After 8 years, Son's Age = 20 + 8 = 28 years

Mother's Age = 48 + 8 = 56 years

Ratio of son's age to mother's age after 8 years

$= 28 : 56$ (Divide both terms by their HCF=28)

$= 1 : 2$

Example 7: **Find an equivalent ratio of**

(i) 5:7 (ii) 10:3

Solution : (i) To find an equivalent ratio of 5 : 7,

Multiply both terms by same natural number except 1(suppose 2)

$\therefore$ $5:7 = (5\times2):(7\times2) = 10:14$

(ii) To find an equivalent ratio of 10 : 3,

Multiply both terms by same natural number except 1 (Suppose 3)

$\therefore$ $10 : 3 = (10\times3) : (3\times3) = 30 : 9$

Example 8: **Divide ₹100 in ratio 2:3 between Aslam and Harpreet.**

Solution : Since. Aslam and Harpreet's share given in the ratio 2:3, which is in simplest form. To find their exact share, we have to multiply it with a variable (say x).

Let Aslam's share = 2x and Harpreet's share = 3x

A.T.Q (Aslam's share) + (Harpreet Share) = 100

$\Rightarrow$ $2x + 3x = 100$

$\Rightarrow$ $5x = 100$

$\Rightarrow$ $x = \frac{100}{5} = 20$

$\therefore$ Aslam's share = $2x = 2\times20 =$ ₹40

and Harpreet's share = $3x = 3\times20 =$ ₹60

Aliter : ₹100 is divided in total $2 + 3 = 5$ parts

Aslam's share = $\frac{2}{5}$ of ₹100 = $\frac{2}{5} \times 100 =$ ₹40

Harpreet's share = $\frac{3}{5}$ of ₹100 = $\frac{3}{5} \times 100 =$ ₹60

Example 9: **The ratio of number of boys and girls in the class is 5:7. Find the number of boys and girls in the class if the total number of the students in the class is 72.**

Solution : Let the number of boys = 5x

and the number of girls = 7x

Given, Total number of students = 72

i.e. (Number of boys) + (Number of girls) = 72

$\Rightarrow$ $5x + 7x = 72$

$\Rightarrow$ $12x = 72$

$\Rightarrow$ $x = \frac{72}{12} = 6$

$\therefore$ Number of boys $= 5x = 5 \times 6 = 30$

and number of girls $= 7x = 7 \times 6 = 42$

Aliter :

Ratio of number of boys and girls = 5:7

Sum of terms of the ratio= $5 + 7 = 12$

$\therefore$ Number of boys $= \frac{5}{12} \times 72 = 30$

and number of girls $= \frac{7}{12} \times 72 = 42$

Exercise 11.1

1. Express the following ratios in the simplest form:

   (i) 12:32 (ii) 45:25 (iii) 91:104 (iv) 60:72 (v) 375:125

2. Write the ratio in the simplest form:

   (i) ₹20 to ₹55 (ii) 18m to 63m (iii) 40 paise to ₹2

   (iv) One hour to 36 minutes (v) 5kg to 1200g

3. Simplify the following ratios:

   (i) 2 years : 14 months (ii) 28 min: 2 hours

   (iii) 125mℓ : 2ℓ (iv) 4m 20cm : 80cm

   (v) 3 dozen : 12 pieces

4. Find two equivalent ratios for each given ratio:

   (i) 4:1 (ii) 3:5 (iii) 5:12

5. The number of boys and girls in a class are 60 and 52 respectively. Find the ratio of number of boys to the number of girls.

6. Pankaj has 23 pens and 42 pencils. Find the ratio of pens to pencils.

7. In a year, Harjot earns ₹2,80,000 and saves ₹60,000. Find the ratio of money:

   (i) He saves to the money he spends.

   (ii) He earns to the money he saves.

   (iii) He spends to the money he earns.

8. In a school, there are 175 boys, 205 girl students and 20 teachers. Find the ratio of the number of

   (i) Number of boys to the number of teachers.

   (ii) girls to the number of boys.

   (iii) Teachers to the number of total persons in the school.

9. Out of 144 students in a school, 48 play cricket, 28 play kabaddi, 40 play volley ball and the remaining play kho-kho. Find the ratio of

   (i) Number of students play kabaddi to the number of students play kho-kho.

   (ii) Number of students play cricket to the number of students play volleyball.

   (iii) Number of students who play kho-kho to the total students of school.

10. The present age of Kush and Shelly are 22 years and 16 years respectively. Find the ratio of

   (i) Their present ages.

   (ii) Kush's age to Shelly's age after 4 years.

   (iii) Shelly's age to Kush's age 5 years ago.

   (iv) Kush's present age to Shelly's age after 6 years.

11. In a pencil box there are 150 pencils. Out of which 40 are red, 60 are black and the rest are blue pencils. Find the ratio of:

   (i) Red pencils to the black pencils.

   (ii) Blue pencils to the total number of pencils.

   (iii) Total pencils to the red pencils.

12. Divide ₹175 in ratio 4:3 between Preet and Sukhi.

13. Two numbers are in the ratio 3:7 and their sum is 140. Find the numbers.

14. The angles of a triangle are in the ratio 1:2:3. Find the measure of each angle.

15. A pipe of length 4m 16cm is cut into two pieces in ratio 3:5. Find the length of each piece of the pipe.

11.3 Proportion

When two ratios are equal then such type of equality of ratios are called proportional and their terms are said to be in proportion.

"An equality of two ratios is called a proportion."

Consider two ratios 4 : 10 and 8 : 20

We find that 4 : 10 = 2 : 5 and 8 : 20 = 2 : 5

Thus, 4 : 10 = 8 : 20 are in a proportion.

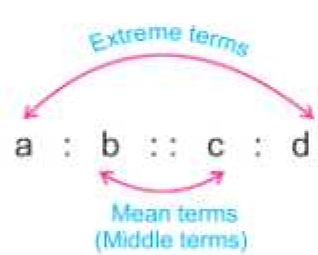
So, above proportion can also be written as 4:10::8:20. It is read as '4 to 10 as 8 to 20'.

• The symbol ': :'(double colon) or '=' is used to denote the equality of two ratios or proportion.

In general, four numbers a, b, c, d are in proportion, if a : b = c : d.

It can be represented as a:b::c:d means $\frac{a}{b} = \frac{c}{d}$ or ad = bc

Conversely, if ad = bc then $\frac{a}{b} = \frac{c}{d}$ or a: b::c:d.



Here a, b, c, d are called the first, second, third and fourth terms. The first and fourth terms of a proportion are called **extreme terms** and second, third terms are called the **middle terms** or **mean terms**.

We observe that,

Product of Extremes = Product of Means

- If product of extremes ≠ Product of Means, then terms are not in proportion.

11.3.1 CONTINUED PROPORTION

In a proportion, if second and third terms are equal then the proportion is called a continued proportion.

i.e. if a : b : : b : c then we say that a, b, c are in continued proportion.

For example : If 3:6: : 6:12 then 3, 6, 12 are in continued proportion.

Generally, If a, b,c are in continued proportion then

- 'a' is called first proportion.
- 'b' is called mean proportion.
- 'c' is called third proportion.

Let's consider some examples for making the concept of proportion more clear.

Example 11: Do the following ratios forms a proportion or not:

(i) 2:7 and 6:21 (ii) 12:10 and 48:40 (iii) 12:16 and 24:28

Solution : (i) Product of Extremes = $2\times21 = 42$

Product of Means = $7 \times 6 = 42$

∴ Product of extremes = Product of Means

Hence, 2:7 and 6:21 are in proportion.

Aliter :

First ratio = 2:7

Second ratio = 6:21

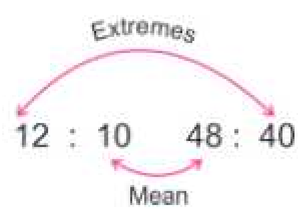
= 2 : 7 (Dividing both terms by 3)

∴ Both ratios are equal

Hence 2:7 and 6:21 are in proportion.

(ii) Product of Extremes = $12 \times 40 = 480$

Product of Means = $10 \times 48 = 480$



∴ Product of extremes = Product of Means

Hence, 12 : 10 and 48 : 40 are in proportion.

Aliter :

First ratio = 12 :: 10

= 6 : 5 (Dividing both terms by 2)

Second ratio = 48 : 40

= 6 :5 (Dividing both terms by 8)

∴ Both ratios are equal

Hence 12:10 and 48:40 are in proportion.

(iii) Product of extremes = $12 \times 28 = 336$

Product of means = $16 \times 24 = 384$

$\therefore$ Product of extremes $\neq$ Product of means

Hence 12:16 and 24:28 are not in proportion.

Aliter :

First Ratio = 12:16

= 3 : 4 (Dividing both terms by 4)

Second ratio = 24 : 28

= 6 : 7 (Dividing both terms by 4)

$\therefore$ Both ratios are not equal.

Hence 12: 16 and 24: 28 are not in proportion.

Example 12: Do the following ratios form a proportion?

(i) 15kg : 25kg and 45kg : 75kg

(ii) 40 minutes: 2 hours and 20 min : 1 hour

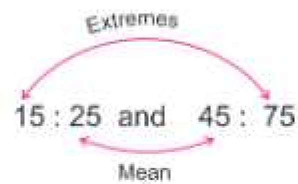
(iii) 600mℓ : 1ℓ and 1ℓ 200mℓ = 2ℓ

Solution : (i) Here units are same in both ratios.

Product of Extremes = $15 \times 75 = 1125$

Product of Means = $25 \times 45 = 1125$

$\therefore$ Product of Extremes = Product of Means



Hence. 15kg: 25kg and 45kg : 75kg are in proportion.

(ii) Here units are different so making units same

First convert all units in minutes, we have

40 minutes : 2 hours and 20 minutes : 1 hour [$\because$ 1 hour = 60 min]

i.e. 40 minutes : 120 minutes and 20 minutes : 60 minutes

Product of Extremes = $40 \times 60 = 2400$

Product of Means = $120 \times 20 = 2400$

$\therefore$ Product of Extremes = Product of Means

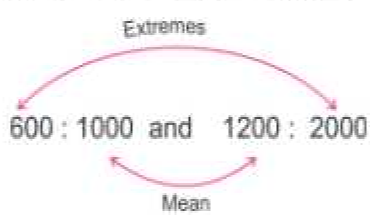
Hence 40 minutes : 2 hours and 20 minutes : 1 hour are in proportion.

(iii) Here units are different, so first make units same by converting into mℓ.

$\therefore$ 600mℓ : 1ℓ and 1ℓ 200mℓ : 2ℓ

= 600mℓ : 1000mℓ and 1200mℓ : 2000mℓ ($\because$ 1ℓ = 1000mℓ)

Now



Product of Extremes = 600×2000

= 1200000

Prouct of Means = $1000 \times 1200 = 1200000$

∴ Product of Extremes = Product of Means

Hence, 600mℓ : 1ℓ and 1ℓ 200 mℓ : 2ℓ are in proportion.

Example 13: **Find the value of 'a' in each case:**

(i) 9: a : : 45 : 40 (ii) 21 : 28 : : a : 32

Solution : (i) 9: a : : 45 : 40

Since, Given terms are in proportion.

∴ Product of Extremes = Product of Means

⇒ $9 \times 40 = a \times 45$

⇒ $\frac{9 \times 40}{45} = a$ ⇒ a = 8

(ii) Given 21:28 : : a : 32

Since, Given terms are in proportion.

∴ Product of Extremes = Product of Means

⇒ $21 \times 32 = 28 \times a$

⇒ $\frac{21 \times 32}{28} = a$ ⇒ a = 24

Exampl 14: **Show that 4, 8, 16 are in continued proportion.**

Solution : For continued proportion, 4, 8, 16 can be written as 4, 8, 8, 16

∴ Product of Extremes = $4 \times 16 = 64$

Product of Means = $8 \times 8 = 64$

∴ Product of Extremes = Product of Means

Hence, 4, 8, 16 are in continued proportion.

Exercise 11.2

1. Determine if the following are in proportion.

(i) 20, 40, 25, 50 (ii) 35, 49, 55, 78 (iii) 24, 30, 36, 45

(iv) 10, 22, 45, 99 (v) 32, 48, 70, 210

2. Do the following ratios forms a proportion:

(i) 5 : 9 and 20:36 (ii) 24:36 and 32:48 (iii) 32:40 and 36:42

(iv) 27 : 18 and 3:2 (v) 35:28 and 77:44

3. State true or false of the following:

(i) 4 : 3 : : 36 : 37 (ii) 16:4 : : 20 : 5 (iii) 19 : 43 : : 8 : 21

4. Determine if the following ratios form a proportion.

(i) 40cm : 1m and ₹12 : ₹30.

(ii) 25min : 1 hour and 40km : 96km.

(iii) ₹4 : 35 paise and 8kg : 9kg.

5. Find the value of 'x' in each case:

(i) 25 : x : : 15 : 6 (ii) 28 : 49 : : x : 56 (iii) 8 : 20 : : 10 : x

6. Check if the following terms are in continued proportion:

(i) 1, 4, 16 (ii) 3, 9, 27 (iii) 5, 10, 20

11.4 Unitary Method

In our daily life, we come across many situations like cost of 12 pencils is ₹60, what is cost of 5 pencils. or cost of a dozen bananas is ₹72, What is cost of 4 bananas etc. How we find their costs?

The method of finding the value of unit quantity of an item on the basis of the given information and then finding the value of the desired quantity of the same item is called a unitary method.

From the above discussion:

$$\text{Cost of one Article} = \frac{\text{Cost of given number of articles}}{\text{Number of articles}}$$

Thus, if cost of any number of articles is given, the cost of 1 article can be obtained by dividing the cost of given number of articles with the number of articles.

Then, Cost of required number of articles = (Cost of one article) × (Required number of articles)

Thus, to find cost of required number of articles can be obtained by multiplying cost of one article with required number of articles.

Let's illustrate some examples:

Example 15: The cost of 1 pen is ₹8. Find the cost of 12 such pens.

Solution : Cost of 1 pen = ₹8

$\therefore$ Cost of 12 pens = ₹8 × 12 = ₹96

Example 16: The cost of 9m cloth is ₹225. Find the cost of 1m cloth.

Solution : Cost of 9m cloth = ₹225

$\therefore$ Cost of 1m cloth = ₹ $\frac{225}{9}$ = ₹25

Example 17: The cost of 15 notebooks is ₹180. Find the cost of 8 such notebooks.

Solution : Cost of 15 notebooks = ₹180

\ Cost of 1 notebook = ₹ $\frac{180}{15}$ = ₹12

Hence, cost of 8 notebooks = ₹12 × 8 = ₹96

Aliter :

We can solve this example by an alternate method called a **Proportion Method**.

If two quantities are related to each other in such a way that when one increases or decreases there is corresponding increase or decrease in other quantity such that ratio of two remains same, then those quantities are said to be in direct proportion.

According to Question:

Notebooks	Cost
15	180
8	x

By Cross product, we have

$$15 \times x = 180 \times 8$$

$$\therefore \quad x = \frac{180 \times 8}{15} = 96$$

$\therefore$ Cost of 8 notebooks in ₹96.

Example 18: If a car uses 18 litres petrol to cover 360km, what is the distance that the car can cover in 24 litres?

Solution : In 18ℓ, car covers distance = 360km

$\therefore$ In 1ℓ, car covers distance = $\frac{360}{18} = 20$km

Hence, In 24ℓ, car covers distance = $20 \times 24 = 480$ km

Aliter :

Petrol (litres)	Distance
18	360
24	x

By cross product, we have

$$18 \times x = 24 \times 360$$

$$x = \frac{24 \times 360}{18} = 480$$

$\therefore$ Car cover 480 km distance in 24 litres.

Example 19: Raman purchased 15 chocolates for 375. How many chocolates can be purchased for ₹525?

Solution : For ₹375, Raman purchased chocolates = 15

$\therefore$ For ₹1, Raman purchased chocolates = $\frac{15}{375}$

Hence For ₹525, Raman purchased chocolates = $\frac{15}{375} \times 525 = 21$

Aliter :

Chocolates	Amount
15	375
x	525

By cross Product, we have

$$15 \times 525 = x \times 375$$

$$\Rightarrow \quad \frac{15 \times 525}{375} = x \qquad \Rightarrow \qquad x = 21$$

Hence, Raman purchased 21 chocolates for ₹525

Example 20: Paras earns ₹1960 in a week.

(i) How much will he earn in 18 days?

(ii) In how many days, he will earn ₹7000

Solution : (i) In a week (7 days), Paras earns = ₹1960

$$\therefore \quad \text{In a day, Paras earn} = ₹\frac{1960}{7} = ₹280$$

Hence, In 18 days, Paras will earn = ₹280 × 18 = ₹5040

Aliter :

Days	Earning
7	1960
18	x

By cross product, we have

$$7 \times x = 1960 \times 18$$

$$\therefore \quad x = \frac{1960 \times 18}{7} = ₹5040$$

Hence, Paras will earn ₹5040 in 18 days.

(ii) ₹1960 is earned in days = 7

$$\therefore \quad ₹1 \text{ is earned in days} = \frac{7}{1960}$$

$$\text{Hence, } ₹7000 \text{ is earned in days} = \frac{7}{1960} \times 7000$$

$$= 25$$

Aliter :

Days	Earning
7	1960
x	7000

By cross product, we have

$$7 \times 7000 = 1960 \times x$$

$$\Rightarrow \quad \frac{7 \times 7000}{1960} = x \qquad \Rightarrow \quad x = 25$$

Hence, Paras will earn ₹7000 in 25 days.

Exercise 11.3

1. The cost of 1kg apples is ₹45. What is the cost of 7kg apples?
2. A car travels 224km in 7 litres of petrol. How much distance will it cover in 1 litre?
3. A pipe can fill 10 water tanks in 12 hours. How much time will it take to fill 15 such water tanks ?
4. The cost of 18m cloth is ₹810. What is the cost of 25m cloth?
5. The weight of 24 books is 6kg. What is the weight of 36 such books?
6. A Train runs 28 km in 5 hours. How many kilometres does it run in 9 hours?
7. A 12m high pole casts a shadow of 30m. Find the height of the pole that casts a shadow of 45m.
8. A man earns ₹ 11200 in 7 months.
   (i) How much will he earn in 18 months?
   (ii) In how many months will he earn ₹40,000?
9. If the cost of a dozen soaps is ₹153.60 . What will be the cost of 16 such soaps?
10. Cost of 105 envelops is ₹35. How many envelops can be purchased for ₹10?
11. A bus travels 90km in $2\frac{1}{2}$ hours.
    (i) How much time is required to cover 54km with the same speed?
    (ii) Find the distance covered in 4 hours with the same speed?
12. Anshul made 57 runs in 6 overs. In how many overs he made 95 runs with same strike rate?
13. Cost of 5kg rice is ₹32.50.
    (i) What will be the cost of 14kg such rice?
    (ii) What quantity of rice can be purchased in ₹162.50 ?
14. If a cow grazes 21sq.m of a field in 6 days. How much area will it graze in 27 days?

Multiple Choice Questions

1. The ratio of 24 seconds to 1 minute is

   (a) 2:5 (b) 24:1 (c) 5:2 (d) 1:24

2. The ratio of 2m to 75cm is

   (a) 2:75 (b) 75:2 (c) 8:3 (d) 3:8

3. The ratio of 1 year to 8 months is

   (a) 2:3 (b) 3:2 (c) 1:8 (d) 8:1

4. Divide ₹40 in 2:3.

   (a) ₹20, ₹30 (b) ₹24, ₹16 (c) ₹30, ₹20 (d) ₹16, ₹24

5. Which of the following is equivalent ratio of 4:7.

   (a) 28:42 (b) 28:49 (c) 20:49 (d) 20:42

6. Find a , if 8, a, 40, 65 are in proportion.

   (a) 26 (b) 12 (c) 13 (d) 9

7. Find x if 12, 25, x, 75 are in proportion.

   (a) 36 (b) 40 (c) 30 (d) 38

8. The cost of 12 pens is ₹108. Find the cost of 18 such pens.

   (a) ₹152 (b) ₹216 (c) ₹ 162 (d) ₹144

9. Aslam earns ₹1680 in a week. In how many days, he will earn ₹2400?

   (a) 10 (b) 8 (c) 12 (d) 9

10. A bus travels 90km in $2\frac{1}{2}$ hours. How much distance it cover in 5 hours?

    (a) 100km (b) 180km (c) 150km (d) 120km

Learning Outcomes

After completion of this chapter the students are now able to

- Compare different quantities using ratio.
- Use ratio in different daily situations.
- Know about proportion.
- Use proportion in different situations.
- Apply unitary method to solve real life situations.

ANSWER KEY

Exercise 11.1

1. (i) 3:8 (ii) 9:5 (iii) 7:8 (iv) 5:6 (v) 3:1
2. (i) 4:11 (ii) 2:7 (iii) 1:5 (iv) 5:3 (v) 25:6
3. (i) 12:7 (ii) 7:30 (iii) 1:16 (iv) 21:4 (v) 3:1
4. (i) 8:2, 12:3 (ii) 6:10, 9:15 (iii) 10:24, 15:36
5. 15:13
6. 23:42
7. (i) 3:11 (ii) 14:3 (iii) 11:14
8. (i) 35:4 (ii) 41:35 (iii) 1:20
9. (i) 1:1 (ii) 6:5 (iii) 7:36
10. (i) 11:8 (ii) 13:10 (iii) 11:17 (iv) 1:1
11. (i) 2:3 (ii) 1:3 (iii) 15:4
12. 100.75
13. 42, 98
14. 30°, 60°, 90°
15. 1m 56cm, 2m 60cm

Exercise 11.2

1. (i) Yes (ii) No (iii) Yes (iv) Yes (v) No
2. (i) Yes (ii) Yes (iii) No (iv) Yes (v) No
3. (i) False (ii) True (iii) False
4. (i) Yes (ii) Yes (iii) No
5. (i) 10 (ii) 32 (iii) 25
6. (i) Yes (ii) Yes (iii) Yes

Exercise 11.3

1. ₹315
2. 32km
3. 18 hours
4. ₹1125
5. 9kg
6. 50.4 km
7. 18m
8. (i) ₹28800 (ii) 25months
9. ₹204.80
10. 30
11. (i) $1\frac{1}{2}$ hours (ii) 144km
12. 10 overs
13. (i) ₹91 (ii) 25kg (iii) 94.5 sq.m

Multiple Choice Questions

(1) a (2) c (3) b (4) d (5) b

(6) c (7) a (8) c (9) a (10) b

12

PERIMETER & AREA

Objectives

In this chapter you will learn

- About concept of perimeter.
- To find out the perimeter of the surroundings like fencing, photo-frame etc.
- About concept of area.
- To find out the area of the surroundings like floor, carpet etc.

12.1 Introduction

We have already learnt about a point, a line and line segment. A point has only position, line has infinite length but not breadth, where as length of line segment can be measured. The line segments whether straight or curved form plane figures. When we talk about plane figures, we think about their regions and their boundaries. So we need some measures to compare them. In this chapter, we shall learn about the concept of perimeter and area of plane figures. These two concepts are of great practical utility in our day-to-day life. We shall also develop formulae for finding the perimeters and areas of a rectangle and a square.

12.2 Perimeter

We all know about fields/gardens. Suppose we want to fence it with barbed wire to make it safe from animals. To measure the length of the wire needed, we started from a point on the boundary of the field/garden and keeps moving the measuring tape along the boundary line, we reach the initial point again. i.e. starting point that means we have made a complete round of the field/garden and the length of the measuring tape is equal to the distance covered in one full round. This length of the tape is called the perimeter of the garden.

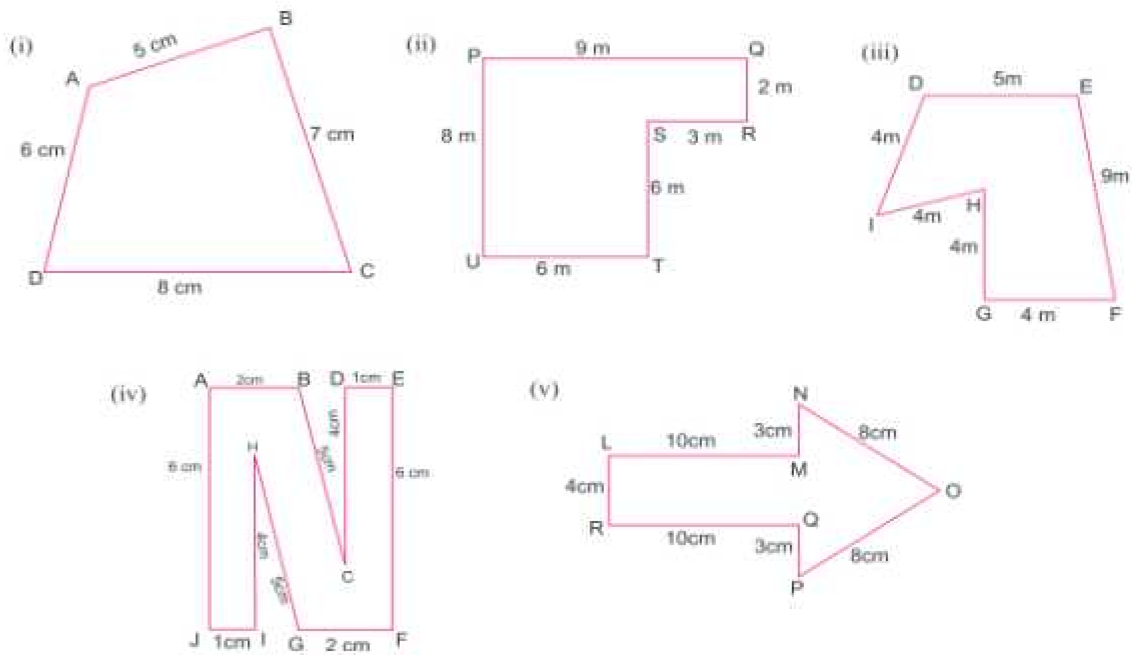
Perimeter of a figure is the total length of its boundary.

We know that all polygons (triangls, rectangle, square, hexagon etc.) are rectilinear figures.

∴ Perimeter = Sum of the lengths of its all sides.

Let's consider some examples :

Example 1: Find the perimeter of each figure:



Solution : (i) Perimeter = Sum of all sides = AB + BC + CD + DA

= (5 + 7 + 8+ 6) cm = 26cm

(ii) Perimeter = Sum of all sides

= PQ + QR + RS + ST + TU + UP

= (9 + 2 + 3 + 6 + 6 + 8)m = 34m

(iii) Perimeter = Sum of all sides

= DE + EF + FG + GH + HI + ID

= (5 + 9 + 4 + 4 + 4 + 4)m = 30m

(iv) Perimeter = Sum of all sides

= AB + BC + CD + DE + EF + FG + GH + HI + IJ + JA

= (2 + 5 + 4 + 1 + 6 + 2 + 5 + 4 + 1 + 6) cm = 36cm

(v) Perimeter = Sum of all sides

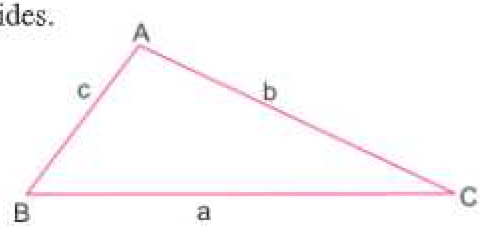
= LM + MN + NO + OP + PQ + QR + RL

= (10 + 3 + 8 + 8 + 3 + 10 + 4)cm

= 46cm.

12.2.1. Perimeter of a Triangle

Perimeter of ΔABC is the sum of the lengths of its sides.



i.e. Perimeter of ΔABC = AB + BC + CA

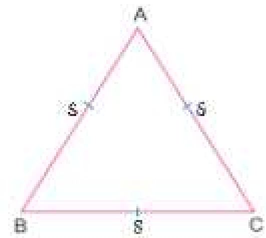
= c + a + b

or a + b + c

*** Perimeter of an Equilateral triangle :**

We know that a triangle having all sides are equal is called an equilateral triangle.

Let 's' be the length of each sides.



Perimeter of ΔABC = AB + BC + CA

= s + s + s

= 3s

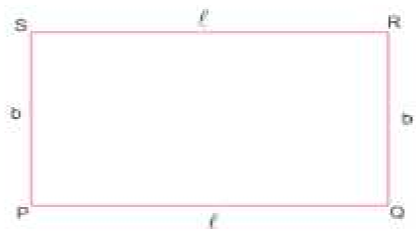
= 3 × (Length of a side of ΔABC)

Perimeter of an equilateral triangle = 3 × Lenght of a side of the triangle

12.2.2.Perimeter of A Rectangle

Perimeter of a rectangle PQRS is the sum of all its sides.

Let 'ℓ' and 'b' denote its length and breadth respectively.



∴ Perimeter of rectangle PQRS = PQ + QR + RS + SP

$= \ell + b + \ell + b$

$= 2\ell + 2b = 2(\ell + b)$

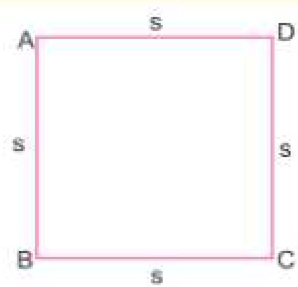
Perimeter of a rectangle = 2 × (length + Breadth)

From this formula, we can obtain the length or breadth of the rectangle.

∴ $\text{Length} = \dfrac{\text{Perimeter}}{2} - \text{Breadth}$ and $\text{Breadth} = \dfrac{\text{Perimeter}}{2} - \text{Length}$

Note:- Before finding perimeter, It must be noticed that **units of length and breadth are same.**

12.2.3. Perimeter of a Square



We know that in a square, all sides are equal.

Let 's' be each side of square.

∴ Perimeter of square ABCD = AB + BC + CD + DA

= s + s + s + s = 4s

= 4 × (side of square)

Perimeter of a square = 4 × (side of a square)

From this formula, we can obtain the side of the square.

$$\therefore \text{ side} = \frac{\textbf{Perimeter}}{\textbf{4}} \textbf{ or Perimeter} \div \textbf{4}$$

12.2.4. Perimeter of a Regular Pentagon

A regular pentagon is a polygon with 5 equal sides.

$\therefore$ Perimeter of a Pentagon = 5 × Length of a side of regular pentagon

$$\Rightarrow \text{Length of a side of a regular Pentagon} = \frac{\text{Perimeter of a regular pentagon}}{5}$$

12.2.5. Perimeter of a Regular Hexagon :

A regular hexagon is a polygon with 6 equal sides.

$\therefore$ Perimeter of a hexagon = 6 × Length of a side of regular Hexagon

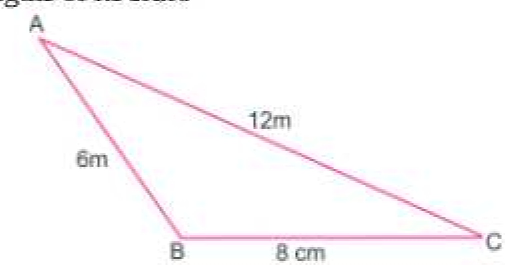
$$\Rightarrow \text{Length of a side of a regular Hexagon} = \frac{\text{Perimeter of a regular Hexagon}}{6}$$

Let's consider some examples.

Example 2: **Find the perimeter of triangle ABC with sides AB = 6 m, BC = 8 m and AC = 12 m**

Solution : Perimeter of triangles ABC = Sum of lengths of its sides

= AB + BC + CA = (6 + 8 + 12)m = 26m

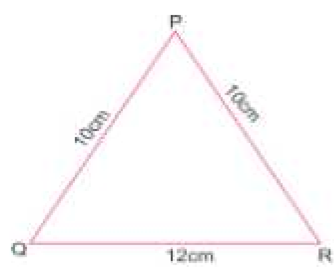


Example 3: **Find the perimeter of an isosceles triangle PQR with PQ = PR = 10cm as length of equal side and QR = 12cm as base.**

Solution : Sides of isosceles triangle = 10cm, 10cm, 12cm

$\therefore$ Perimeter = Sum of lengths of its sides

= (10 + 10 + 12)cm = 32cm



Example 4: **Find the perimeter of the following rectangles having**

(i) **Length 15m and breadth 12m**

(ii) **Length 10.3cm and breadth 14.8cm**

(iii) **Length 128cm and 115cm**

Solution : (i) Length of rectangle = 15m, Breadth of rectangle = 12m

$\therefore$ Perimeter of a rectangle = 2 × (length + breadth)

= 2 × (15 + 12) = 2 × 27 = 54m

(ii) Length of rectangle = 10.3cm, Breadth of rectangle = 14.8cm

$\therefore$ Perimeter of rectangle = 2 × (length + breadth)

= 2 × (10.3 + 14.8) = 2 × 25.1

= 50.2cm

(iii) Length of rectangle = 128cm, Breadth of rectangle = 115cm

$\therefore$ Perimeter of rectangle = 2 × (length + breadth)

= 2 × (128 + 115) = 2 × 243

= 486cm

Example 5: Find the perimeter of a square with

(i) Side = 12m (ii) Side = 6.3cm (iii) Side = 85cm

Solution : (i) Side of square = 12m

$\therefore$ Perimeter of square = 4 × side

= 4 × 12 = 48m

(ii) Side of square = 6.3cm

$\therefore$ Perimeter of square = 4 × side

= 4 × 6.3 = 25.2cm

(iii) Side of square = 85cm

$\therefore$ Perimeter of a square = 4 × side

= 4 × 85 = 340cm

Example 6: Find the Perimeter of a regular hexagon with side 15cm

Solution : Side of hexagon = 15cm

Perimeter of a regular hexagon = 6 × side

= 6 × 15 = 90cm

Example 7: Two sides of a triangle are 12cm and 15cm and the perimeter of the triangle is 40cm. What is the length of the third side?

Solution : Sum of length of two sides = (12 + 15) cm

= 27cm

Perimeter = 40cm

\ third side = Perimeter of triangle – (sum of two sides)

= 40cm – 27cm = 13cm

Aliter :- Let sides are a = 12cm, b = 15cm, c = ?

Perimeter of a triangle = Sum of length of all sides

then

40 = a + b + c

40 = 12 + 15 + c

$$40 = 27 + c$$
$$c = 40 - 27$$
$$= 13cm$$

Example 8 : If the Perimeter of an equilateral triangle is 45cm. Find the length of each side of the triangle.

Solution : Given, Perimeter of an equilateral triangle = 45cm

Perimeter of an equilateral triangle = 3 × (Side of the triangle)

$$\Rightarrow 45 = 3 \times \text{side} \Rightarrow \frac{45}{3} = \text{side}$$

$$\Rightarrow \text{Side} = 15cm$$

Example 9 : If the perimeter of a square lawn is 72m. Find the side of the square lawn.

Solution : Given Perimeter of square lawn = 72m

$\therefore$ Perimeter of square = 4 × (side)

$$\Rightarrow 72m = 4 \times (\text{side})$$

$$\Rightarrow \text{side} = \frac{72}{4} m = 18m$$

Example 10 : The perimeter of a rectangle is 80cm. If length of the rectangle is 25cm. Find the breadth of the rectangle.

Solution : Given, Perimeter of Rectangle = 80cm and

length of the rectangle = 25cm

$\therefore$ Perimeter of a Rectangle = 2 × (length + breadth)

$$\Rightarrow 80 = 2 \times (25 + \text{breadth})$$

$$\Rightarrow \frac{80}{2} = 25 + \text{breadth}$$

$$\Rightarrow \text{breadth} + 25 = 40 \Rightarrow \text{breadth} = 40 - 25 = 15cm$$

Applications of Perimeter in Daily life

Example 11: Kanwar takes 5 rounds of square park of side 135m. Find the distance covered by Kanwar?

Solution : Distance covered in 1 round of square park = Perimeter of square park

$\therefore$ Perimeter of the square = 4 × side

$$= 4 \times 135 = 540m$$

Distance covered in 5 rounds of square park

$$= 5 \times \text{Perimeter of the square}$$
$$= 5 \times 540$$
$$= 2700m$$

$$\text{or } \frac{2700}{1000} km = 2.7km \ (\because 1km = 1000m)$$

Example 12: **A gardener wants to fence his rectangular garden of length 180m and breadth 150m. If he wants to use three layers of wire to fence the garden, find the length of the wire required by him.**

Solution : The gardener wants to fence his rectangular garden so we have to find perimeter of the garden

Given, length of the rectangular garden = 180m

breadth of the rectangular garden = 150m

∴ Perimeter of the rectangular garden = $2 \times (\text{length} + \text{breadth})$

$= 2 \times (180 + 150)$

$= 2 \times 330 = 660\text{m}$

∴ One layer of fencing = Perimeter of the garden

Three layers of fencing = $3 \times$ Perimeter of the garden

$= 3 \times 660 = 1980\text{m}$

So he needs 1980m of wire to fence his garden.

Example 13: **Find the cost of constructing wall around a square park of side 30m at the rate of ₹ 500 per running metre**

Solution : Given side of square park = 30m

∴ Perimeter of a square = $4 \times$ side of the square

$= 4 \times 30 = 120\text{m}$

Also cost of constructing wall per metre = ₹ 5

∴ cost of constructing wall around a square park = $120 \times 500 =$ ₹ 60000

Example 14 : **If the length of a rectangle is x units and breadth is 3 units. Find the perim eter of the rectangle**

Solution : Given, length of the rectangle = x units

Breadth of the rectangle = 3 units

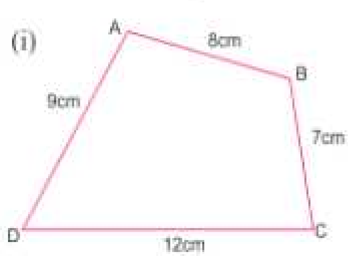
∴ Perimeter of the rectangle = 2 (length + breadth)

$= 2\,(x + 3)$ units

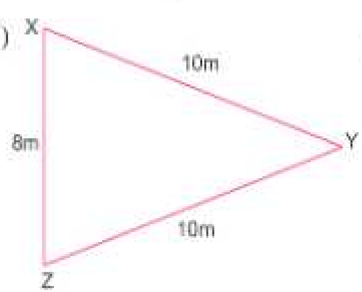
Exercise 12.1

1. Find the perimeter of the following shapes:-

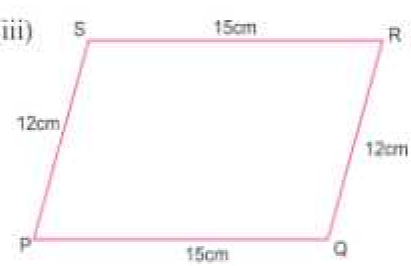
(i)



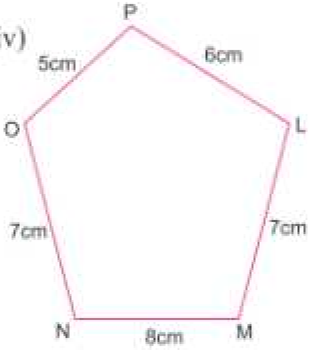
(ii)



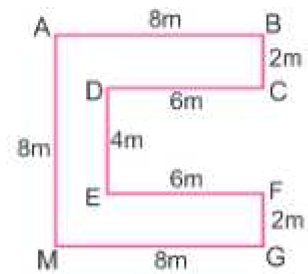
(iii)



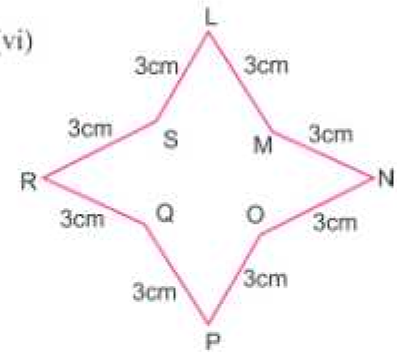
(iv)



(v)



(vi)



2. Find the perimeter of the triangle with sides :

(i) 5cm, 6cm and 7cm (ii) 10m, 12m, 18m (iii) 4.6cm, 3.2cm and 5.8cm

3. Find the perimeter of an isosceles triangle with 15cm as length of equal side and 18cm as base.

4. Find the perimeter of a square with side :

(i) 16cm (ii) 4.8mm (iii) 125cm (iv) 45m (v) 39cm

5. Find the perimeter of a rectangle with

(i) Length 20m and breadth 15m

(ii) Length 25m and breadth 35m

(iii) Length 40cm and breadth 28cm

(iv) Length 18.3cm and breadth 6.8cm

(v) Length 12.5 cm and breadth 15cm.

6. Find the perimeter of a regualar hexagon with side :

(i) 5cm (ii) 12cm (iii) 7.2cm

7. Find the perimeter of an equilateral triangle with side

(i) 10cm (ii) 8m (iii) 24m (iv) 5.6m (v) 12.1cm

8. If the perimeter of a triangle is 48cm and two sides are 12cm and 17cm. Find the third side.

9. Find the side of an equilateral triangle, if the perimeter is :

(i) 45cm (ii) 69mm (iii) 117cm

10. Find the side of a square if the perimeter is:

(i) 52cm (ii) 60cm (iii)112cm

11. (i) The perimeter of rectangular field is 260m. If its length is 80m then find its breadth.

(ii) The perimeter of a rectangualr garden is 140m. If its breadth is 45m then find its length.

(iii) The perimeter of a rectangle is 114cm. If its length is 32cm then find its breadth in metres

12. The sides of a triangular field are 15m, 20m and 18m. Find the total distance travelled by a boy in taking 2 complete rounds of this field.

13. Find the cost of fencing a square field of side 26m at the rate of ₹ 3 per metre.

14. Mani runs around a square park of side 75m. Kush runs around a rectangular park of length 60m and breadth 45m. Who covers less distance?

15. Find the cost of framing a rectangular white board with length 240 cm and breadth 150 cm at the rate of ₹ 6 per cm.

16. If length of a rectangle is 'a' units and breadth is 5 units. Find the perimeter of the rectangle.

17. Fill in the blanks:-

(i) The sum of lengths of all sides of a polygon is called ....

(ii) Perimeter of Square = ... × side

(iii) Perimeter of Rectangle = 2 × (.................. + ...)

(iv) Side of a Square = (..................) ÷ 4

(v) Perimeter of an equilateral triangle = ... × side

12.3. Area

We have learnt in previous classes that the area of the portion of the plane or a shape can be defined as the amount of stuff required to cover it. For finding the area of a polygon, we consider the enclosed region of the polygon.

Let us consider an example to clear the idea. Pritpal buys a piece of land which is 80 metres long and 65 metres wide and his friend Aslam buys a piece of land which is 60 metres long and 75 metres wide at the same rate.

Who will pay the more price?

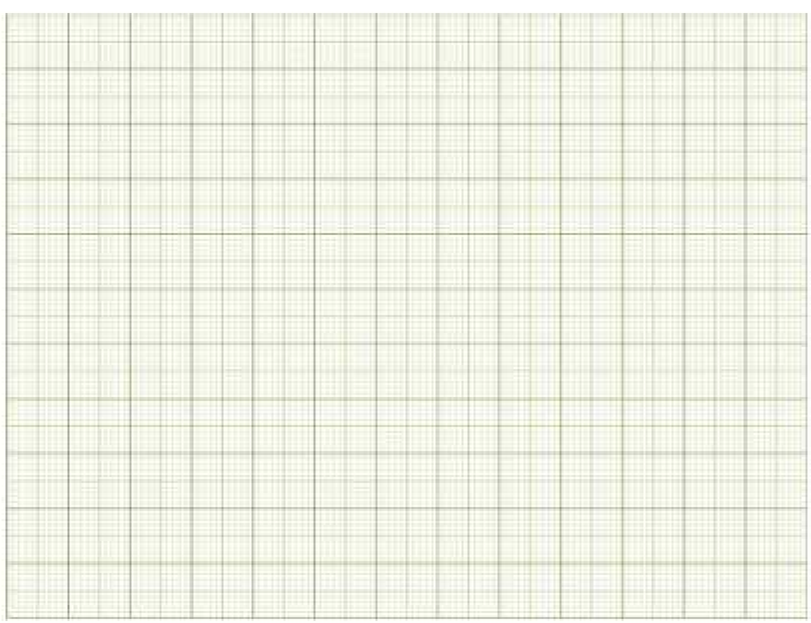
Obviously, the one who has more land will pay more. In this case, we shall find the areas of both the lands to know who has more land.

The measurement of the region enclosed by a close plane figure is called its area.

Unit of Area:- Let a square of side 1 unit. It covers the region 1 square units. So we always denote the area in square units.

12.3.1 Finding Area by the use of squared paper (Graph paper) :

The squared paper or a graph paper is a convenient method for finding the approximate area of a region enclosed by any simple closed curve.



- If the figure encloses an exact number of complete squares, then count the number of squares.
- If the figure consists more than half or half squares then use the following formula :

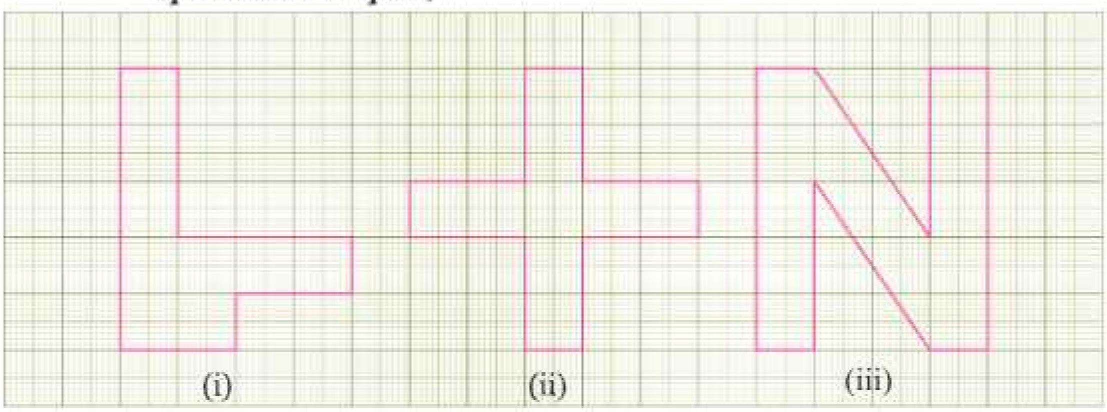
Area of the plane figure = $\left(m + n + \frac{1}{2}p\right)$

where m = Number of complete squares

n = Number of squares more than half part enclosed

p = Number of squares exactly half part enclosed.

Example 15:- Find the approximate area of each of the following figures by counting the number of squares - complete, more than half and exactly half. (Area of 1 square box = 1sq.cm)



Solution : (i) Number of complete squares m = 9

Here we do not have any half square or more than half

$\therefore$ n = 0, p = 0

$\therefore$ Area of plane figure = $m + n + \frac{1}{2}p$

$= 9 + 0 + 0 = 9$ sq.cm.

(ii) Number of complete squares, m = 9

Here n = 0, p = 0

$\therefore$ Area plane figure = 9 sq.cm.

(iii) Number of complete squares, m = 10

Number of squares exactly half, p = 0

Number of squares more than half (n) = 4

$\therefore$ Area of plane figures = $m + n + \frac{1}{2}p$

$= 10 + 4 + \frac{1}{2} \times (0)$

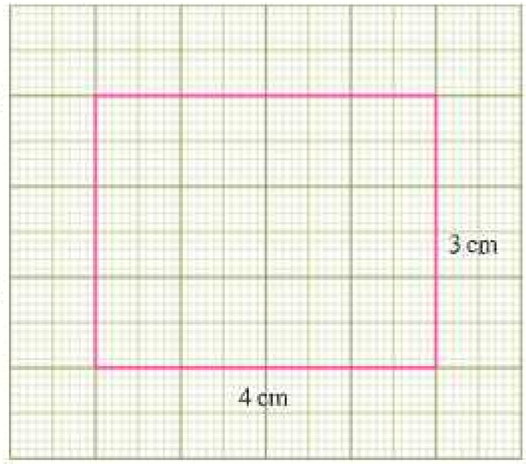
$= 14 + 0$

$= 14$ sq.cm.

12.3.2 Measurement of Area by Using Formula

We have learnt in previous class about the area of a rectangle and a square. In this class, we shall discuss it in detail.

- **Area of a Rectangle :**



Let us draw a rectangle of length 4cm and breadth 3cm on a graph paper with 1cm × 1cm squares.

We find that it covers 12 squares completely.

∴ Area of the rectangle = 12 sq. cm

We observe that, there are 4 squares in a row and there are 3 such rows.

∴ Total number of squares

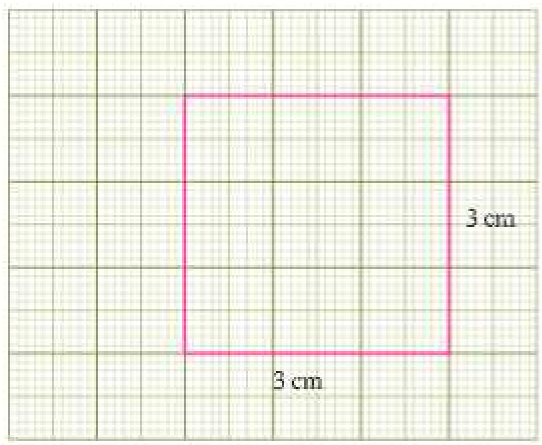
= 4 × 3 = length × breadth

Area of rectangle = length × breadth

from the above formula, we can derive :

$$\textbf{Length} = \frac{\textbf{Area}}{\textbf{Breadth}} \quad \text{or} \quad \textbf{Breadth} = \frac{\textbf{Area}}{\textbf{Length}}$$

- **Area of a Square**



Let us draw a square of side 3cm on a graph paper with 1cm × 1cm squares.

We find that it covers 9 squares completely.

∴ Area of the square = 9 sq. cm

We observe that, there are 3 squares in a row and there are 3 such rows.

∴ The Total number of squares

= 3 × 3 = side × side

Area of square = side × side

Note:- To find the area of a figure, all its dimension must be expressed in same units.

Example 16: Find the area of the rectangle with

(i) length = 15cm and breadth = 12cm

(ii) length = 18m and breadth = 24m

(iii) length = 5cm and breadth = 12mm

Solution : (i) Length of rectangle = 15cm and breadth of rectangle = 12cm

∴ Area of Rectangle = Length × Breadth

= 15cm × 12cm

= 180 sq.cm.

(ii) Length of rectangle = 18m and breadth of rectangle = 24m

$\therefore$ Area of Rectangle = Length $\times$ Breadth

= 18m $\times$ 24m

= 432 sq.m.

(iii) Here units of length and breadth are different. First convert them in same units

Length = 5cm = 5 $\times$ 10 mm = 50mm (∵ 1cm = 10mm)

Breadth = 12mm

$\therefore$ Area of Rectangle = Length $\times$ Breadth

= 50mm $\times$ 12mm

= 600 sq.mm.

Example 17: Find the area of square with side

(i) 5cm (ii) 4.1mm (iii) 18m

Solution : (i) Side of a square = 5cm

$\therefore$ Area of Rectangle = side $\times$ side

= 5cm $\times$ 5cm

= 25 sq.cm

(ii) Side of a square = 4.1mm

$\therefore$ Area of Rectangle = side $\times$ side

= 4.1mm $\times$ 4.1mm

= 16.81 sq.mm

(iii) Side of a square = 18m

$\therefore$ Area of Rectangle = side $\times$ side

= 18m $\times$ 18m

= 324 sq.m

Example 18: The length of a rectangular plot is 90m and its area is 1800 sq.m. Find the breadth of the plot.

Solution : Length of rectangle plot = 90m

Area of Rectangle plot = 1800 sq.m

$\therefore$ Area = Length $\times$ Breadth

$\Rightarrow$ 1800 = 90 $\times$ Breadth

$$\Rightarrow \frac{1800}{90} = \text{Breadth}$$

$\Rightarrow$ Breadth = 20m.

Example 19: The side of a square plot of land is 35m. Find the cost of levelling the plot, if the rate is ₹ 4 per square metre

Solution : Side of Square plot = 35m

Area of Square plot = side $\times$ side

= 35m $\times$ 35m

= 1225 sq.m.

Now levelling cost of 1 sq.m = ₹ 4

$\therefore$ Levelling cost of 1225 sq.m = ₹ 4 × 1225

= ₹ 4900

Example 20: The area of a rectangle is 380 sq. cm and its breadth is 20cm. Find the perimeter of the rectangle.

Solution : Given area of rectangle = 380 sq. cm

and breadth of rectangle = 20cm

Area of rectangle = length × breadth

380 = length × 20

$\Rightarrow$ length of rectangle $= \frac{380}{20} = 19$cm

Now, Perimeter of Rectangle = 2 × (length + breadth)

= 2 × (19 + 20)

= 2 × 39 = 78cm

Example 21: How many envelops can be made out of a sheet of paper measuring 108cm by 105cm, if each evelope requires a piece of paper of size 9cm by 15cm.

Solution : Given length of sheet of paper = 108cm

and breadth of sheet of paper = 105cm

Area of sheet of paper = length × breadth

= (108 × 105) sq.cm

Area of paper required for one envelope = (9 × 15) sq. cm

$\therefore$ Number of envelops that can be made $= \frac{\text{Area of Sheet of paper}}{\text{Area of one envelope}}$

$= \frac{108 \times 105}{9 \times 15} = 84$

Hence, 84 envelops can be made.

Example 22: The perimeter of the square is 68cm. Find its area.

Solution : Perimeter of square = 68cm

$\Rightarrow$ 4 × side of square = 68cm

$\Rightarrow$ Side of square $= \frac{68}{4} = 17$cm

$\therefore$ Area of square = side × side

= 17cm × 17cm

= 289 sq.cm

Example 23: A marble tile measures 25cm by 20cm. How many tiles will be required to cover a floor of size 4.5m by 3m?

Solution : Area of the floor $= 4.5 \times 3$ sq. m

$= 13.5$ sq.m

$= 13.5 \times 10000$

$= 135000$ sq.cm

1m	= 100cm
1 sq. m	= 100cm × 100cm
	= 10000 sq. cm

Area of one tile $= (25 \times 20)$ sq cm

$= 500$ sq.cm

$\therefore$ Number of tiles required to cover the floor

$$= \frac{\text{Area of the floor}}{\text{Area of one tile}}$$

$$= \frac{135000}{500} = 270$$

Hence, 270 tiles will be required to cover the floor.

Example 24: The floor is 5m long and 4m wide. A square carpet of side 3m is laid on the floor. Find the area of the floor that is not carpeted.

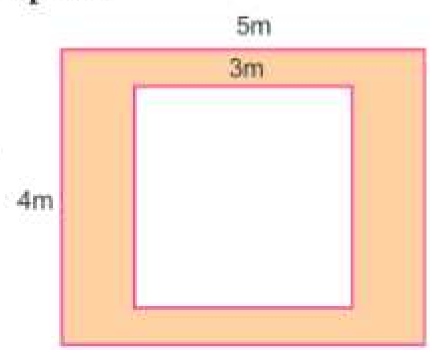
Solution : Given length of floor $=$ 5m,

Breadth of floor $=$ 4m

Area of the floor $=$ length $\times$ breadth

$= 5\text{m} \times 4\text{m}$

$= 20$ sq. m



Area of the square carpet $=$ side $\times$ side

$= 3\text{m} \times 3\text{m} = 9$ sq. m

$\therefore$ Area that is not carpeted $=$ (Area of floor) – (Area of the carpet)

$= (20 - 9)$ sq.m $= 11$ sq.m

Example 25: (i) **What will happen to the area of a square if its side is doubled?**

(ii) **What will happen to the area of a rectangle if its length is doubled and breadth is trebled (Tripled)?**

Solution : (i) Let the side of the square $= x$ cm

$\therefore$ Area of the square $= (x \times x)$ sq. cm

Now, If the side is doubled, then

side of new square $= 2x$ cm

$\therefore$ Area of the new square $= [(2x) \times (2x)]$ sq.cm

$= (2 \times 2 \times x \times x)$ sq.cm

$= (4 \times x \times x)$ sq.cm

$= 4\,(x \times x)$ sq.cm

$= 4 \times$ (Area of Original square)

$\therefore$ If side is doubled, then the area becomes 4 times of original area

(ii) Let ℓ cm and b cm be the length and breadth of the rectangle respectively.

$\therefore$ Area of the rectangle $= \ell \times b$

Now, If length is double and breadth is trebled.

$\therefore$ Now length $= 2\ell$

and new breadth $= 3b$

Thus, area of new rectangle $=$ length $\times$ breadth

$= 2\ell \times 3b$

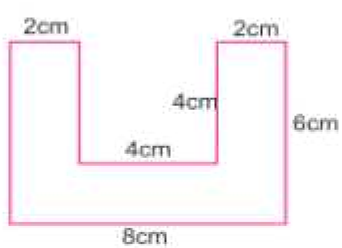
$= 6 \times (\ell \times b)$

$= 6 \times$ (Area of Original Rectangle)

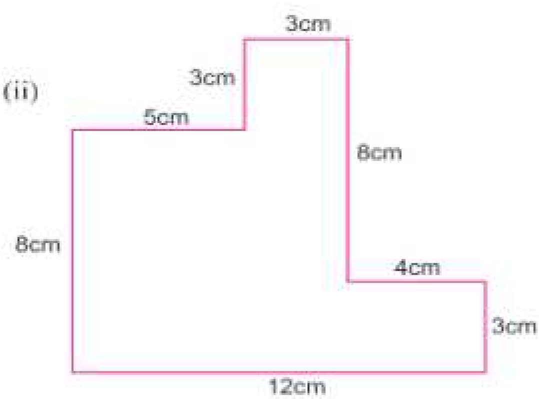
Hence, Area becomes 6 times of original area.

Example 26: **Find the area of the following figures by splitting it into rectangles and squares :**

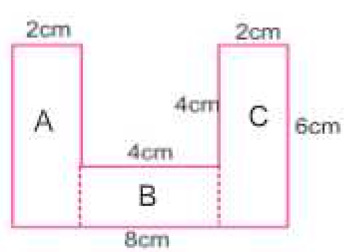
(i)



(ii)



Solution : (i) The given figures cna be divided into 3 parts



- Rectangle A of size 2cm × 6cm
- Rectangle B of size 4cm × 2cm
- Rectangle C of size 2cm × 6cm

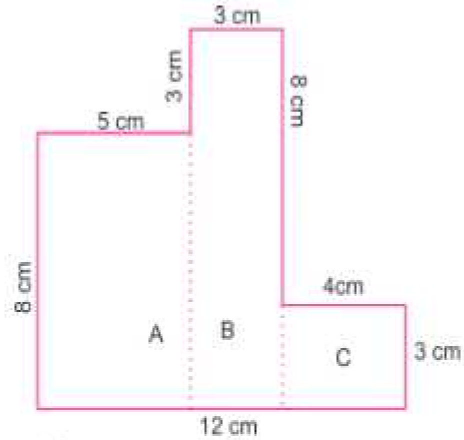
$\therefore$ Area of rectangle A = 2cm × 6cm = 12 sq. cm.

Area of rectangle B = 4cm × 2cm = 8 sq. cm

Area of rectangle C = 2cm × 6cm = 12 sq.cm

$\Rightarrow$ Total area of the figures = 12 + 8 + 12 = 32 sq. cm

(ii) The figure can be divided into 3 parts.



- Rectangle A of size 5cm × 8cm
- Rectangle B of size 3cm × 11cm
- Rectangle C of size 4cm × 3cm

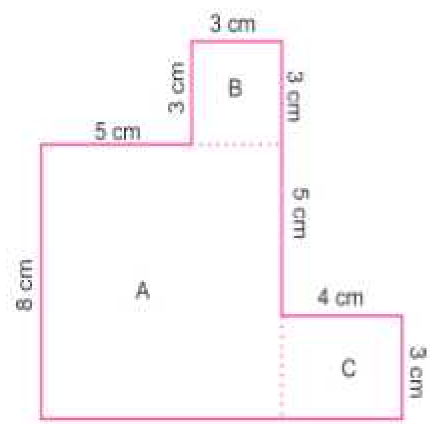
∴ Area of rectangle A = 5cm × 8cm = 40 sq. cm.

Area of rectangle B = 3cm × 11cm = 33 sq. cm

Area of rectangle C = 4cm × 3cm = 12 sq.cm

∴ Total area of the figures = 40 + 33 + 12 = 85 sq. cm

Alter :- The figure can be divided into 3 parts.



- Square A of size 8cm × 8cm
- Square B of size 3cm × 3cm
- Rectanlge C of size 4cm × 3cm

∴ Area of rectangle A = 8cm × 8cm = 64 sq. cm.

Area of rectangle B = 3cm × 3cm = 9 sq. cm

Area of rectangle C = 4cm × 3cm = 12 sq.cm

⇒ Total area of the figures = 64 + 9 + 12 = 85 sq. cm

Example 27: If the length of the rectangle x units and breadth of the rectangle is 5 units. Find the area of the rectangle

Solution : Length of the rectangle = x units

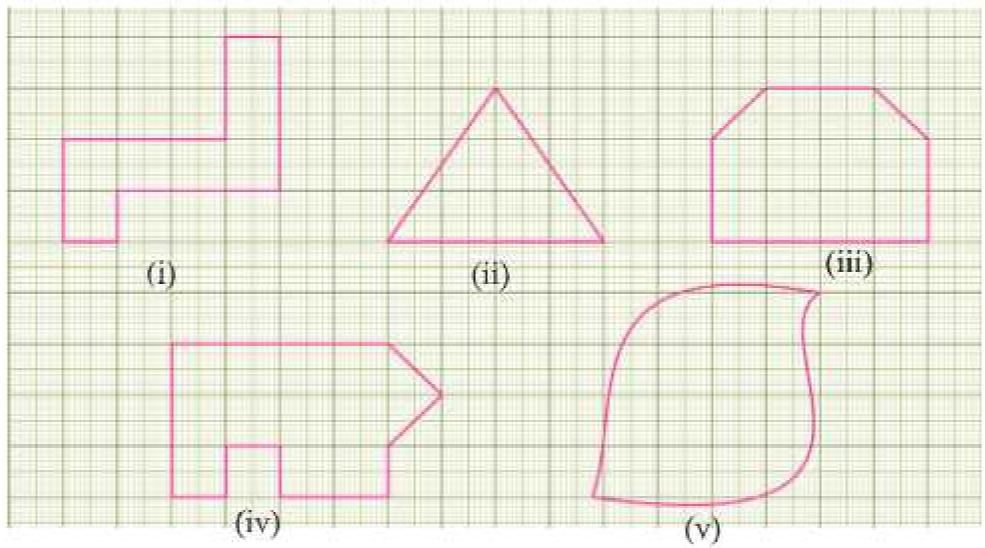
Breadth of the rectangle = 5 units

Area of the rectangle = Length × Breadth

= x × 5 = 5x sq. units.

Exercise 12.2

1. Find the approximate area of each of the following figures by counting the number of squares – complete, more than half and exactly half.



2. Find the area of a rectangle whose

   (i) length = 12cm, breadth = 16cm (ii) length = 25m, breadth = 18m

   (iii) length = 2.7m, breadth = 45cm (iv) length = 4.2cm, breadth = 1.5cm

   (v) length = 3.8mm, breadth = 4mm

3. Find the area of the square with side :

   (i) 19cm (ii) 24mm (iii) 3.5cm (iv) 2.6cm (v) 8.2cm

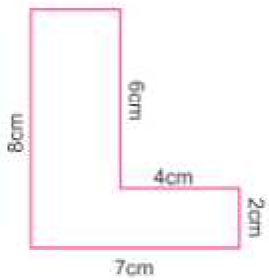
4. The area of a rectangle is 216 sq.cm and its length is 12cm. Find its breadth.
5. The area of a rectangle is 225 sq. m and its breadth is 9m. Find its length.
6. The length and breadth of a ground are 32m and 24m respectively. Find the cost of levelling the ground at the rate of 3 per sq. m
7. Find the perimeter of a rectangle whose area is 324 sq.cm and its one side is 36cm.
8. The perimeter of a square field is 100m. Find its area.
9. Area of a rectangle of length 20cm is 340 sq. cm. Find its perimeter.
10. A marble tile measure 15cm × 20cm. How many tiles will be required to cover a wall of size 4m × 6m?
11. Find the cost of levelling the square field of side 75m at the rate of ₹ 5 per square metre.
12. How many stamps of size 2cm × 1.5cm can be pasted on a sheet of paper of size 6cm × 12cm?
13. (i) What will happen to the area of a square if its side is trebled (tripled)?

    (ii) What will happen to the area of a rectangle if its length is halved and breadth is doubled?

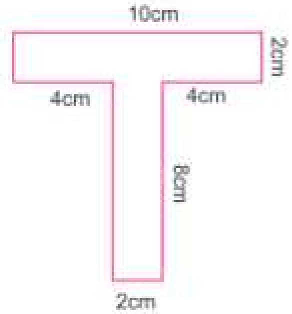
    (iii) What will happen to the area of a square if its side is halved?

14. Find the area of the following figures by splitting it into rectangles and squares :

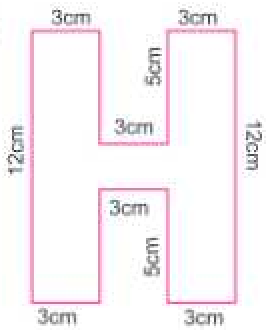
(i)



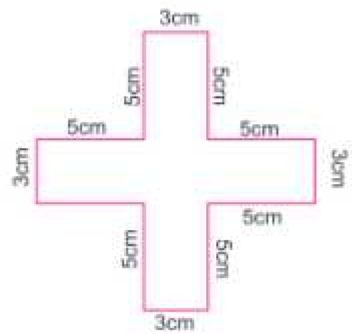
(ii)



(iii)



(iv)



15. Fill in the blanks:-

(i) 1 square metre = ... sq. cm.

(ii) 1 square cm = ... sq. mm.

(iii) Area of Rectangle = ... × ....

(iv) Length of Rectangle = ... ÷ breadth

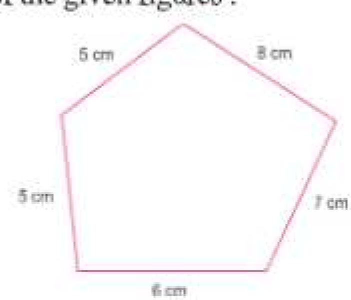
(v) Area of square = ... × ....

Multiple Choice Questions

1. The outer boundary of a closed figure is called ....

(a) Perimeter (b) Region (c) Area (d) Curve

2. Find the perimeter of the given figures :



(a) 30cm (b) 31cm (c) 32cm (d) 33cm

3. Perimeter of an equilateral triangle = ...
(a) 3 + Side (b) Side × Side (c) Side + Side (d) 3 × Side

4. Perimeter of Retangle = ...
(a) 2l + b (b) 2 (l + b) (c) l + 2b (d) l × b

5. If side of an equilateral triangle is 4cm then perimeter = ...
(a) 8cm (b) 7cm (c) 12cm (d) 16cm

6. If length and breadth of a rectangle are 2.4cm and 1.9cm respectively then its perimeter is ....
(a) 4.3cm (b) 8.2cm (c) 4.2cm (d) 8.6cm

7. The perimeter of a square is 16cm then its side is ....
(a) 4cm (b) 64cm (c) 24cm (d) 32cm

8. The perimeter of a rectangle is 50cm and its length is 12cm then breadth is ....
(a) 38cm (b) 13cm (c) 62cm (d) 18cm

9. Two sides of a triangle are 4.8cm and 3.9cm. The perimeter of the triangle is 12cm. Find the third side.
(a) 3.3cm (b) 4.3cm (c) 20.7cm (d) 3.7cm

10. Samandeep takes 3 rounds of square park of side125m. Find the distance covered by her.
(a) 1.5km (b) 1500km (c) 500m (d) 375m

11. The measurement of the region enclosed by a closed plane figure is called its ....
(a) Circumference (b) Curve (c) Perimeter (d) Area

12. If the length of a rectangle is x units and breadth is 5 units then its perimeter is ....
(a) 5x (b) 2 (x + 5) (c) 10x (d) 10 + x

13. Find the area of the given rectangle whose length is 16m and breadth is 8m.
(a) 42 sq.m (b) 128 sq.m (c) 72 sq.m (d) 21 sq.m

14. The area of a rectangle is $144m^2$. If its breadth is 9m then find its length.
(a) 16sq.m (b) 12m (c) 16m (d) 18m

15. 1 sq.m = ... sq.cm
(a) 100 (b) 10000 (c) 1000 (d) 1

16. Find the area of a square having side 3.6cm.
(a) 14.4 cm (b) 12.96 cm (c) 1.29 sq.cm (d) 12.96 sq.cm

17. The perimeter of a square is 68m. Find its area.
(a) 289 sq.m (b) 329 sq.m (c) 279 sq.m (d) 249 sq.m

18. A marble tile is of side 25cm by 25cm. How many tiles will be required to cover a floor of 4m by 3m?
(a) 216 (b) 192 (c) 188 (d) 196

19. What will happen to the area of a square, if side is doubled?
(a) Double (b) Half (c) Four times (d) No change

20. Find the perimeter of a rectangle whose area is 234sq.cm and its one side is 13cm.
(a) 31cm (b) 62cm (c) 18cm (d) 24cm

Learning Outcomes

After completion of this chapter the students are now able to

- Know about the concept of perimeter.
- Find the perimeter from their surrounding and use it in its practical life.
- Know about the concept of area.
- Use the concept of area in its daily use..

ANSWER KEY

Exercise 12.1

1. (i) 36cm (ii) 28m (iii) 54cm (iv) 33cm (v) 44m (vi) 24cm

2. (i) 18cm (ii) 40m (iii) 13.6cm **3.** 48cm

4. (i) 64cm (ii) 19.2mm (iii) 500cm (iv) 180m (v) 156cm

5. (i) 70m (ii) 120m (iii) 136cm (iv) 50.2cm (v) 55cm

6. (i) 30cm (ii) 72cm (iii) 43.2cm

7. (i) 30cm (ii) 24m (iii) 72m (iv) 16.8m (v) 36.3cm

8. 19cm **9.** (i) 15cm (ii) 23mm (iii) 39cm

10. (i) 13cm (ii) 15cm (iii) 28cm **11.** (i) 50m (ii) 25 m (iii) 25cm

12. 106m **13.** ₹ 312 **14.** Kush **15.** ₹ 4680 **16.** 2 (a + 5)

17. (i) Perimeter (ii) 4 (iii) Length, breadth (iv) Perimeter (v) 3

Exercise 12.2

1. (i) 7 units (ii) 6 units (iii) 11 units (iv) 12 units (v) 13 units

2. (i) 192 sq cm (ii) 450 sq m (iii) 12150 sq cm (iv) 6.3 sq cm (v) 15.2 sq mm

3. (i) 361 sq cm (ii) 576 sq mm (iii) 12.25 sq cm (iv) 6.76 sq cm (v) 67.24 sq.cm

4. 18cm **5.** 25m **6.** ₹ 2304 **7.** 90cm **8.** 625 sq m **9.** 74cm

10. 800 **11.** ₹ 28125 **12.** 24 **13.** (i) 9 times (ii) No effect (iii) One fourth times

14. (i) 32 sq cm (ii) 36 sq.cm (iii) 78 sqcm (iv) 69 sq units

15. (i) 10000 (ii) 100 (iii) length, breadth (iv) Area (v) Side, Side

Multiple Choice Questions

(1) a **(2)** b **(3)** d **(4)** b **(5)** c **(6)** d **(7)** a **(8)** b **(9)** a **(10)** a

(11) d **(12)** b **(13)** b **(14)** c **(15)** b **(16)** d **(17)** a **(18)** b **(19)** c **(20)** b

A8H4Q2

13

SYMMETRY

Objectives

In this chapter you will learn

- About symmetrical figures.
- About symmetrical lines.
- To identify the symmetrical figures in daily life.
- About the reflection of different objects.

13.1 Introduction

Symmetry is an important geometrical concept exhibited in nature and is widely used in engineering, architecture, textile designing, pottery and many other fields. We can observe symmetry in nature such as flowers, leaves and insects. Around us, we find symmetry on bed sheets, shawls and also in monuments like Taj Mahal and Eiffel Tower.

Symmetry around us

When a picture is folded from the middle and the two halves match exactly, then we say that the picture is symmetrical. This phenomenon is called **Symmetry.**

13.2 Making Symmetric Figures

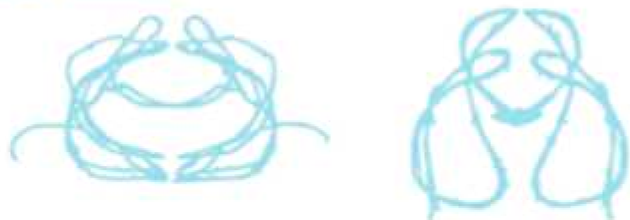
13.2.1 Ink Blot Devils:-

Take a piece of paper. Fold it in half. Spill a few drops of ink on one half Side. Now press the both halves together and see the resulting figure. Is it symmetric? If yes then find its line of symmetry.

Is there any other line along which it can be folded in two identical parts?

Try more such patterns

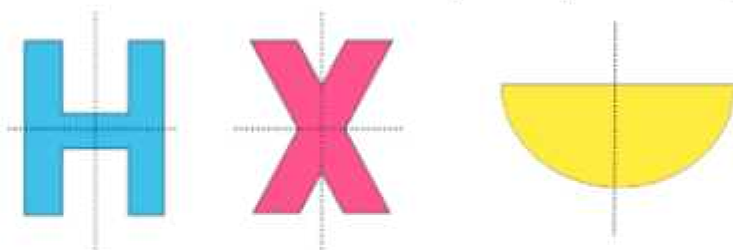
13.2.2 Inked String Patterns



Fold a paper in half. On one half portion, arrange short lengths of thread dropped in coloured inks or paints. Now press the two halves. observe the figure you obtain. Is it Symmetric?

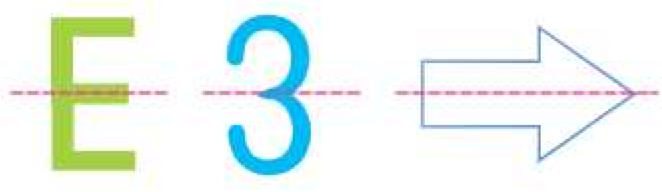
13.3. Lines of Symmetry

If you observe the symmetrical figure, there are some lines which divide the figure into two exactly identical halves. These **lines are called lines of symmetry or axis of Symmetry.**



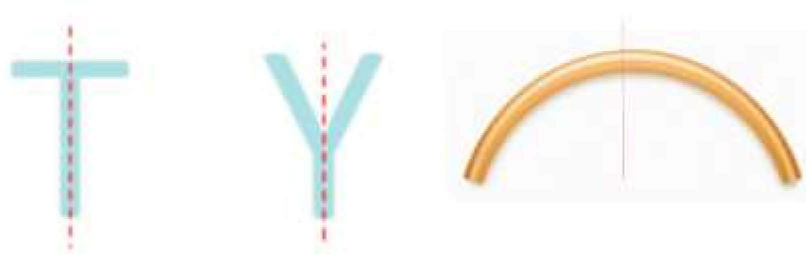
Lines of Symmetry.

Observe the pictures shown in Fig. The line of symmetry in these pictures is horizontal. This type of symmetry is called **horizontal symmetry.**



Horizontal Symmetry

Observe the pictures shown in Fig. The line of symmetry in these pictures is vertical. This type of symmetry is called **vertical symmetry.**



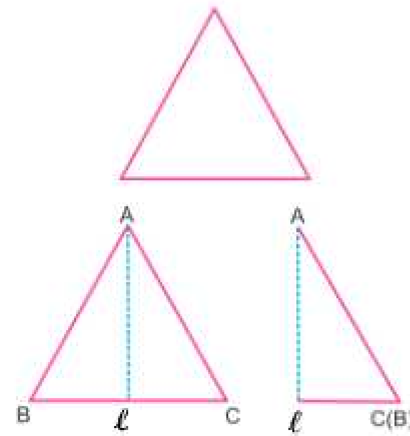
Vertical Symmetry

ACTIVITY

Determine the number of lines of symmetry of different shapes by paper folding.

Material Required : Chart Paper, Scale etc.

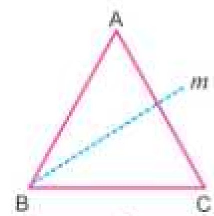
Procedure : 1. Equilateral Triangle:



Case I :

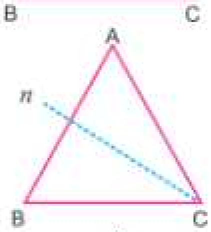
- Cut an equilateral ΔABC from the chart paper.
- Now fold the triangle in such a way so that B and C coincides.
- You will observe that after unfolding, the crease made by folding, passes through A.
- This crease divides the triangle in two identical parts.
- So there is one line of symmetry (ℓ) through A.

Case II



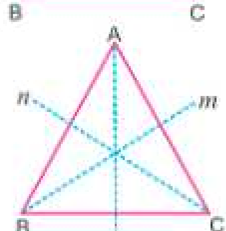
- Now fold the triangle in such a way that A and C coincides.
- You will observe that after unfolding crease made by folding passes through B.
- This crease divides the triangle in two identical parts.
- So there is one more line of symmetry (m) through B.

Case III



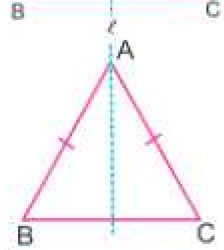
- Now fold the triangle in such a way that A and B coincides.
- You will observe that after unfolding, crease made by folding passing through C.
- This crease divides the triangle in two identical parts.
- So there is again one more line of symmetry (n) through C.

Thus, An equilateral triangle has 3 lines of symmetry



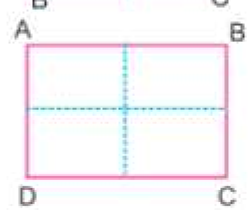
(2) Isosceles Triangle:

Follow the process as discussed above you will find, there is only one line of symmetry in an Isosceles Triangle. (Which is between the vertex of equal sides)



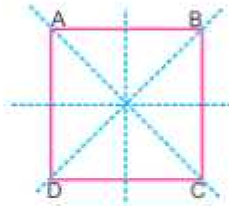
(3) Rectangle:

Follow the process as discussed above, you will find there are 2 lines of symmetry in a rectangle.



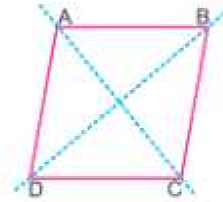
(4) Square:

Follow the process as discussed above, you will find, there are 4 lines of symmetry in a square.



(5) Rhombus:

Follow the process as discussed above, you will find there are 2 lines of symmetry in a rhombus.



* Lines of Symmetry for Regular Polygons

A polygon is a closed figure made of line segments. A triangle is a polygon with the least number of sides (three sides).

A regular polygon has all its sides equal and all its angles equal.

For example : (1) An equilateral triangle has three lines of symmetry.

(2) A square has four lines of symmetry.

(3) A regular pentagon has five lines of symmetry.

Therefore we can say that regular polygons are symmetrical figures having as many lines of symmetry as they have sides or vertices.

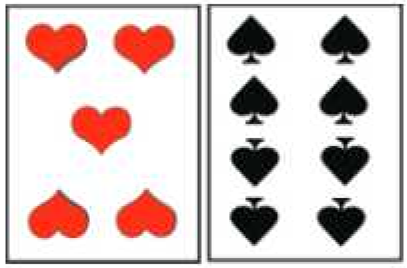
Symmetry, Symmetry Everywhere!

- Many road signs you see everyday have lines of symmetry. Here, are a few.

Identify a few more symmetric road signs and draw them. Do not forget to mark the lines of symmetry.

- The nature has plenty of things having symmetry in their shapes: look at these:

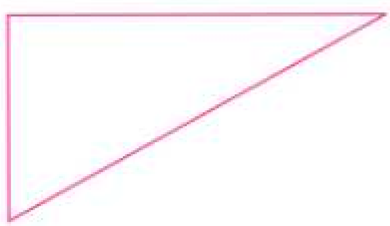
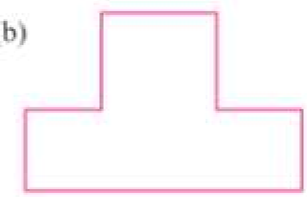
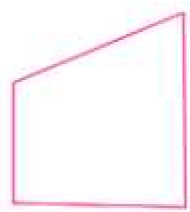
- The designs on some playing cards have line symmetry. Identify them for the following cards.

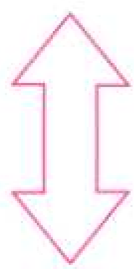


- Let us take scissor !

  How many lines of symmetry does it have?

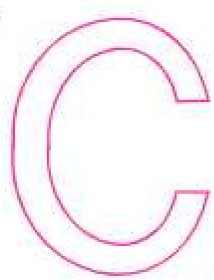
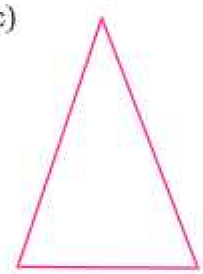
Example 1:- Identify the figures which are symmterical :

(a)  (b)  (c) 

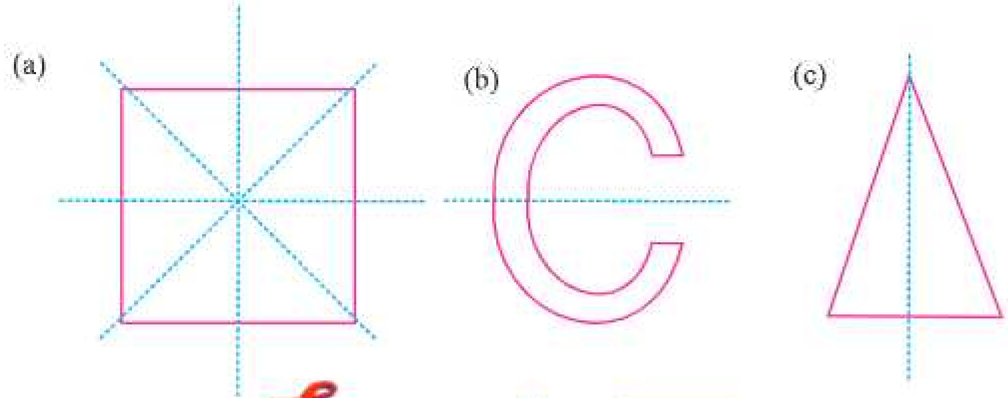
(d)  (e) 

Sol. : Figure (b) and (e) are symmetrical

Example 2: Draw line/lines of symmetry in the following figure

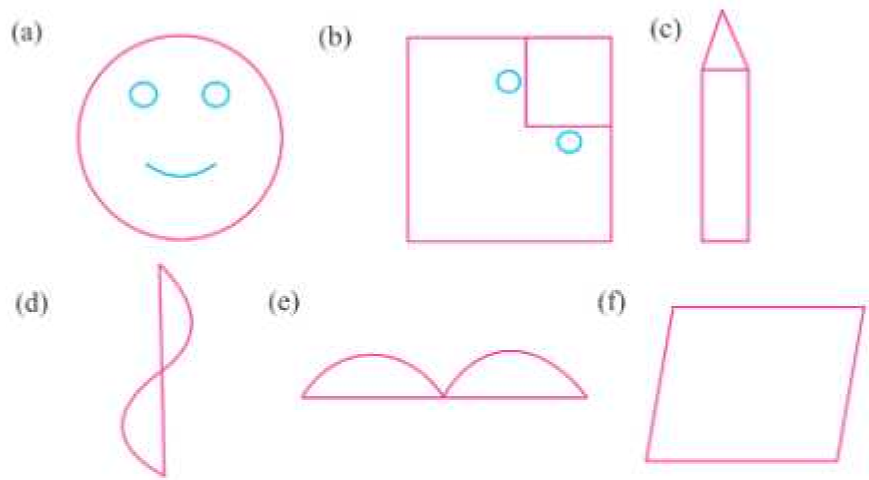
(a)  (b)  (c) 

Sol.



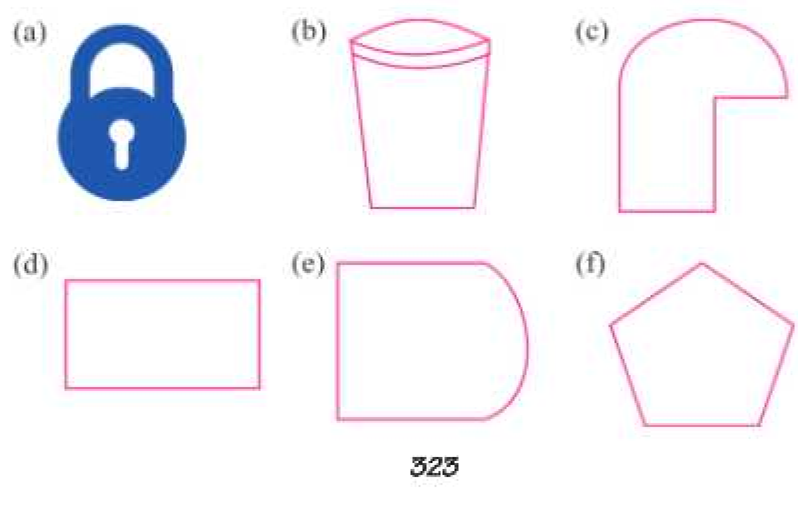
Exercise 13.1

1. Classify the figure as symmetrical or non-symmetrical. Also draw the line/ lines of symmetry (if any).



2. Which Capital letter of english alphabet have:
   (i) No line of symmetry.
   (ii) 1 line of symmetry.
   (iii) 2 lines of symmetry.

3. Find the numbers of line/ lines of symmetry for the following :

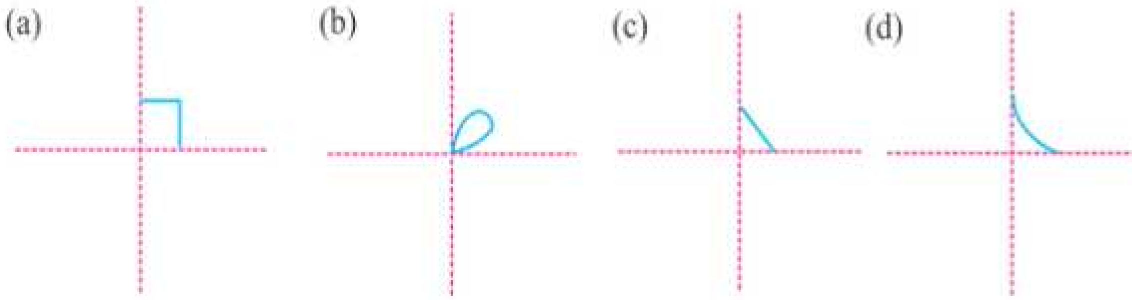


4. Draw the line (s) of symmetry in the following figures:

(a) Rhombus (b) Scalene Triangle (c) Parallelogram

(d) Rectangle (e) Square (f) Regular Pentagon

5. Complete each of the figure using both lines of symmetry:



6. Draw a triangle which has:

(i) No line of symmetry

(ii) Exactly one line of symmetry

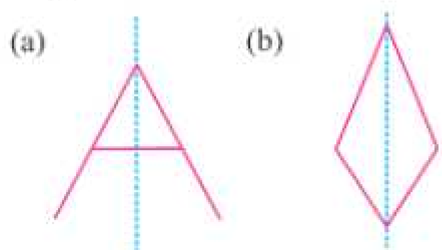
(iii) Exactly three lines of symmetry.

7. List any three symmetrical objects from your day-to-day life.

13.4 Reflection Symmetry

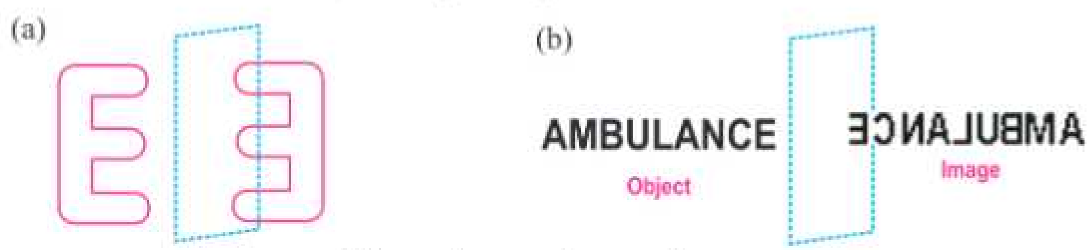
In figures with line of symmetry, the two identical parts are mirror images of each other. If a mirror is placed on the line of symmetry, then the image of one half of the figure/objects will fall exactly on the other half. Then, this line becomes the mirror line.

Look at the adjoining figures. We consider the dotted line to be a mirror, and each part is a **mirror image** of the other. Here, the mirror line act as a line of symmetry and the object along with its image form a symmetrical shape.



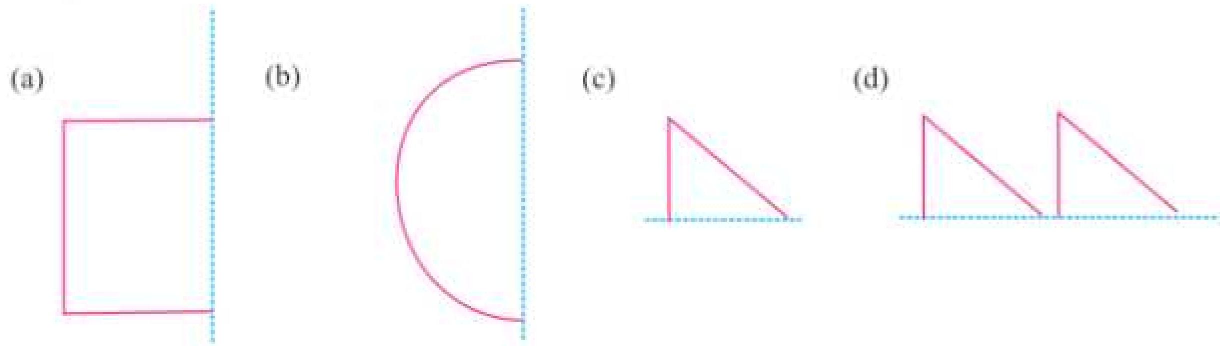
Reflection Symmetry

In reflection symmetry, one part is the object and the other part is the image and they are at an equal distance from the mirror line (line of symmetry).

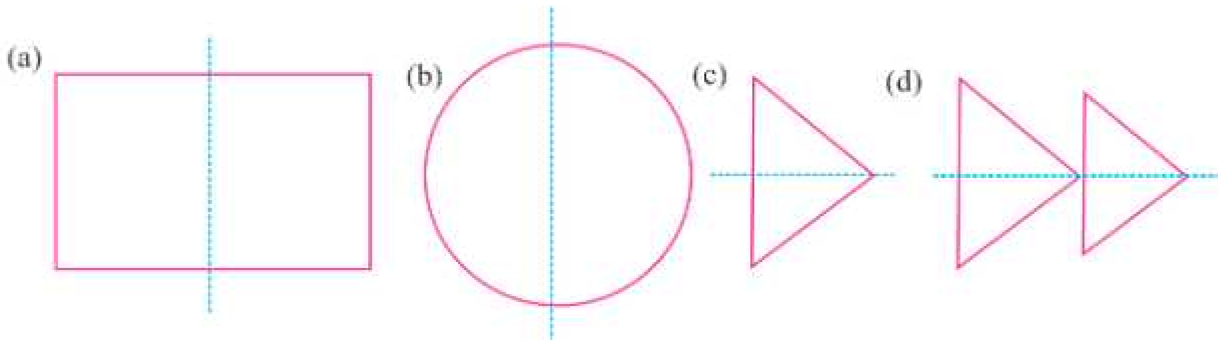


Mirror images of some Figures

Example 3: Reflect each of the given figures in the dotted line (mirror line) and draw the image:-



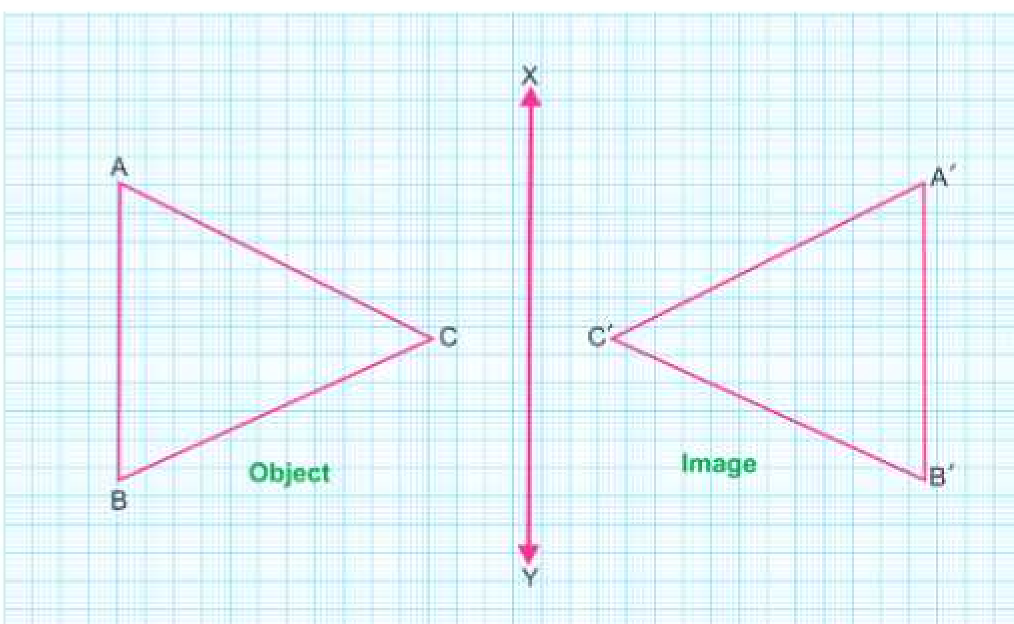
Sol. The reflections of the given figures are as follows:-



* To Draw the reflection of a figure using a graph paper

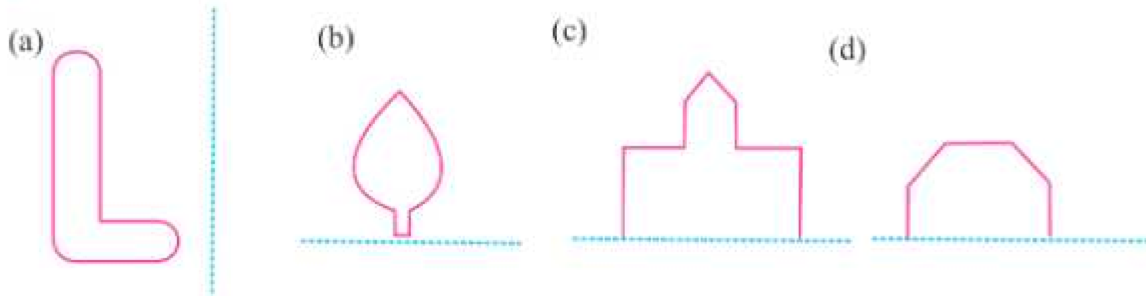
We can draw reflection of a figure using a graph paper.

Take a graph paper. Draw a triangle ΔABC. Then, draw its reflection $\Delta A'B'C'$, where A and A', B and B', C and C' are equidistant from the line of symmetry.

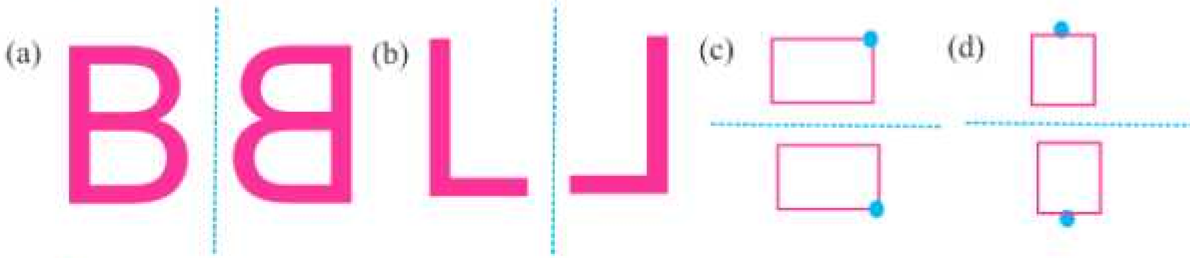


Exercise 13.2

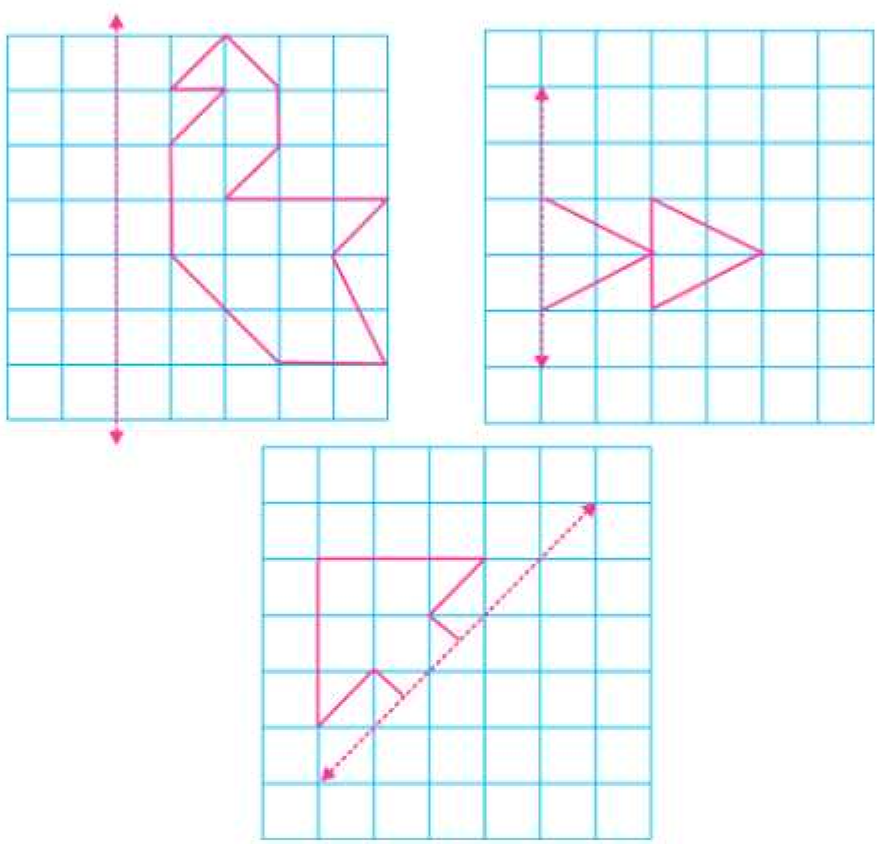
1. Draw the reflection of following figures along the dotted line :-



2. Write 'yes' for right reflection and 'no' for wrong reflection:-



3. Trace the figures on the graph paper and draw the reflections. The dotted line is the line of symmetry:-



* Key Points

- A line of symmetry divides a figure into two equal parts.
- If two parts of an object or a figure are identical then it is said to be symmetrical.
- A symmetrical figure can have more than one line of symmetry.
- A regular polygon has as many lines of symmetry as the number of sides and vertices.
- A line of symmetry is very closely related to reflection symmetry.

Multiple Choice Questions

1. An equilateral triangle has .... lines of symmetry.

(a) 1 (b) 3 (c) 2 (d) 4

2. A rectangle has .... lines of symmetry

(a) 2 (b) 3 (c) 4 (d) 1

3. A square has .... lines of symmetry.

(a) 1 (b) 2 (c) 3 (d) 4

4. An isosceles triangle has .... line (s) of symmetry.

(a) 1 (b) 2 (c) 3 (d) 4

5. A circle has .... lines of symmetry.

(a) 1 (b) 2 (c) 4 (d) Infinite

6. A rhombus has .... lines of symmetry.

(a) 1 (b) 2 (c) 3 (d) 4

7. A regular hexagon has .... lines of symmetry.

(a) 3 (b) 2 (c) 6 (d) 5

8. The mirror reflection of name ARUN is

(a) NUSA (b) ИUЯA (c) NURA (d) ARUN

Learning Outcomes

After completion of this chapter the students are now able to

- Recognise the symmetrical figures.
- Draw symmetrical lines of different shapes.
- Recognise the symmetrical designs in daily life.
- To know about reflection of mirror images.

ANSWER KEY

Exercise 13.1

1. (a) Symmetric (b) Symmetric (c) Symmetric
   (d) Non-Symmetric (e) Symmetric (f) Non-Symmetric

2. (i) F, G, J, L. N, P, Q, R. S, Z
   (ii) A, B, C, D, E, K, M, T, U, V, W, Y
   (iii) O, X, H, I

3. (a) 1 (b) 1 (c) 0 (d) 2 (e) 1
   (f) 1

Exercise 13.2

2. (a) Yes (b) Yes (c) Yes (d) Yes

Multiple Choice Questions

(1) b (2) a (3) d (4) a (5) d (6) b (7) c (8) b

P5J3U7

14

DATA HANDLING

Objectives

In this chapter you will learn

- To collect and arrange the data.
- To represent the data in pictograph.
- To interpret the pictograph and bar graph.
- To Use the graphs in daily life situation.

14.1 Introduction

In our daily life, we come across many situations when decisions have to be made based on the data. Our decisions are based on the collection, organisation, analysis and interpretation of the facts gathered.

You must have observed that your teacher records the attendance of students in your class everyday or records marks obtained by you after every test or examination.

14.2 Collection of Data

The initial step of any investigation is the collection of data. It may be collection of numbers, figures, facts or symbols. So the collection of facts gathered in the form of numerical values is called **data** which gives meaningful information. There are two types of data.

Primary Data : The data collected directly from the source is called the primary data. For Example attendance recorded by your teacher is primary data.

Secondary Data : When the data is collected from an external source is called secondary data . For Example the data collected from newspapers, magazines, internet etc. is secondary data.

Let us consider an example.

A teacher collects the data of the choice of the sweets of 25 students of class 6th which is as follows:

Ladoo, Barfi, Ladoo, Jalebi, Rasgulla, Ladoo, Jalebi, Jalebi, Ladoo, Barfi, Rasgulla, Rasgulla, Barfi, Jalebi, Ladoo, Ladoo, Barfi, Barfi, Jalebi, Rasgulla, Rasgulla, Barfi, Ladoo, Jalebi, Jalebi

The teacher wants to know the number of students who like different sweets. He starts counting one by one. This process is very time consuming and he has to repeat the same for every sweet. If there are 100 students then it becomes difficult. To make this process easy we organise the data in different ways.

14.3 Organisation of Data

Organisation of data helps in bringing out meaningful conclusion from the data. To make the above data meaningful, we have to arrange the data in a tabular form.

When the number of observations is larger, then to minimise the number of errors and to make tabulation easier, we can use **tally marks.**

Tally marks are always recorded in the bunches of five. Fifth tally mark is drawn diagonally across the first four to make a group of five i.e. 1 = I, 2 = II 3 = III, 4 = IIII, 5 = ~~IIII~~, 6 = ~~IIII~~I so on.

Sweet name	Tally marks	Number of Students
Ladoo	~~IIII~~ II	7
Barfi	~~IIII~~ I	6
Jalebi	~~IIII~~ II	7
Rasgulla	~~IIII~~	5

This is the better way to understand and analyse the data.

Let's consider some examples.

Example 1: **In a Mathematics Test, the following marks were obtained by 40 students. Arrange these marks in a table using tally marks.**

8	1	3	7	6	5	5	4	4	2
4	9	5	3	7	1	6	5	2	7
7	3	8	4	2	8	9	5	8	6
7	4	5	6	9	6	4	4	6	6

(i) Find how many students obtained marks equal to or more than 7?

(ii) How many students obtained marks below 4?

Solution :

Marks	Tally Marks	Number of Students
1	II	2
2	III	3
3	III	3
4	~~IIII~~ II	7
5	~~IIII~~ I	6
6	~~IIII~~ II	7
7	~~IIII~~	5
8	IIII	4
9	III	3

(i) Number of students obtained marks equal to or more than 7 = 5 + 4 + 3 = 12

(ii) Number of students obtained marks below 4 = 2 + 3 + 3 = 8

Exercise 14.1

1. The heights (in cm) of students of a class 6^{th} were recorded as below:

116, 117, 125, 116, 118, 120, 125, 121, 124, 117, 116, 115, 119, 121, 124, 117, 116, 119, 123, 120, 116, 121, 119, 116, 118, 125, 116, 119, 123, 122, 121, 120

Arrange the data in a tabular form using tally marks.

2. The weight of 25 students (in kg) are given below:

25, 34, 32, 28, 25, 28, 34, 32, 32, 34, 32, 25, 28, 34, 34, 28, 28, 25, 32, 33, 32, 34, 33, 32, 25

Arrange the data in a tabular form using tally marks.

3. Ekta is asked to collect data for size of shoes of students in her class 6^{th}. Her finding are recorded in the manner shown below:

5	4	7	5	6	7	6	5	6	6	5
4	5	6	8	7	4	6	5	6	4	6
5	7	6	7	5	7	6	4	8	7	

Arrange the data in a tabular form using tally marks.

4. Shweta threw a dice 40 times and noted the number appearing each time as shown below:

1	3	5	6	6	3	5	4	1	6
2	5	3	4	6	1	5	5	6	1
1	2	2	3	5	2	4	5	5	6
5	1	6	2	3	5	2	4	1	5

Make a table and enter the data using tally marks. Find the number that appeared :

(i) The minimum number of times.

(ii) The maximum number of times.

5. The students of class 6^{th} had a Maths test and scored marks out of 10, which are listed below:

3	7	6	2	5	9	10	8	7	1
8	4	3	5	6	7	8	7	6	5
3	6	9	8	7	5	9	6	7	8

(i) Organise the data using tally marks.

(ii) How many students scored less than or equal to 6?

(iii) How many students scored more than 7?

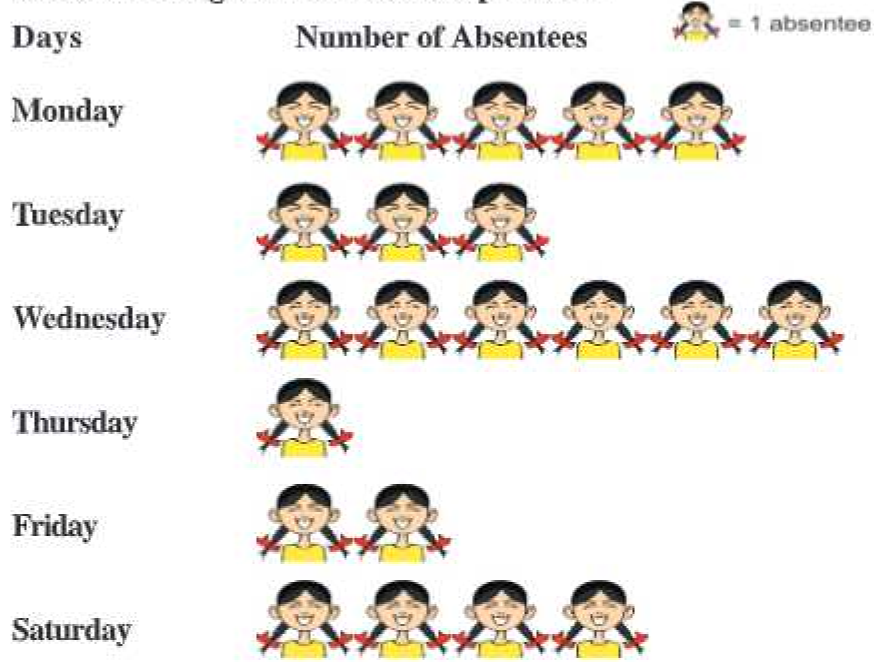
14.4 Representation of Data

There are many ways to represent numerical data. Such as pictographs, bar graphs etc. These graphs help us in the suitable representation of the data.

14.4.1 Pictograph

A pictograph is a way of representing data using pictures or symbols to match the frequencies of different information or events. The picture visually helps to understand and analyse the data.

Example 1: **The following pictograph shows the number of absentees in a class of 21 students during the first week of April 2018.**



(i) **On which day were the maximum number of students absent?**

(ii) **What was the total number of absentees in that week?**

(iii) **On which day were the minimum number of students absent?**

Solution : (i) There are 6 pictures against Wednesday. Thus, maximum number of students were absent on Wednesday.

(ii) There are 21 pictures in all, so the total number of absentees in that week was 21.

(iii) There is 1 picture against Thursday. Thus, minimum number of students were absent on Thursday.

Example 2. **Total number of dogs in five villages are as follows:**

Village A : 30 **Village D = 40**

Village B : 20 **Village E = 60**

Village C : 50

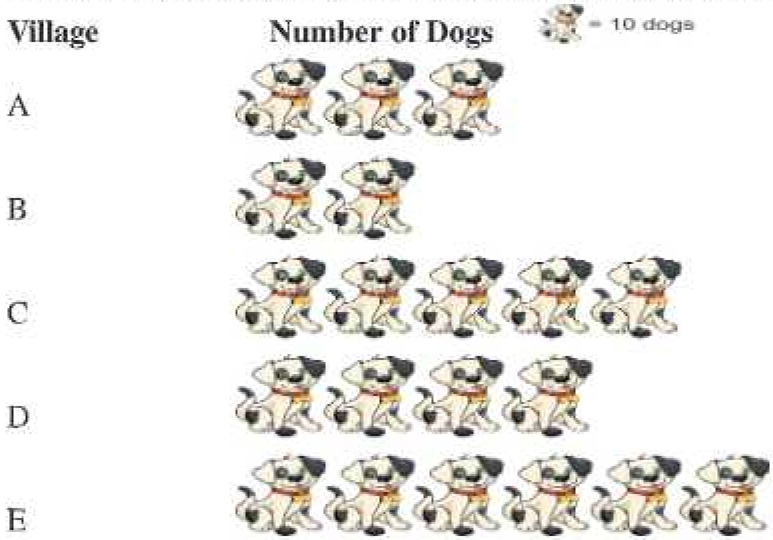
Prepare a pictograph of these animals using one symbol ([dog symbol] = 10 dogs) and answer the following.

Questions:

(i) How many symbols are used to represent dogs of village E?

(ii) Which village has maximum number of dogs?

Solution : Taking the scale (as given), we may draw the pictograph as shown below:



(i) There are 6 symbols for village E.

(ii) Village E has maximum number of dogs.

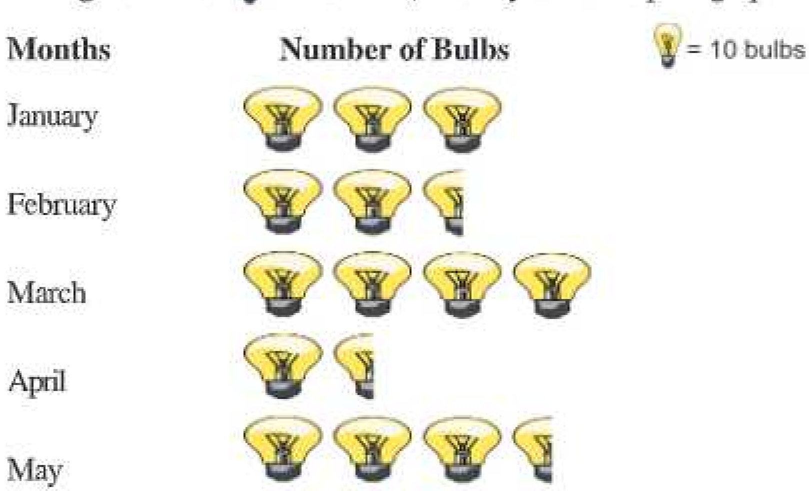
Example 3. **The following number of electric bulbs were purchased for a school in the first five months of a year.**

January - 30, **February - 25** **March - 40**

April - 15 **May - 35**

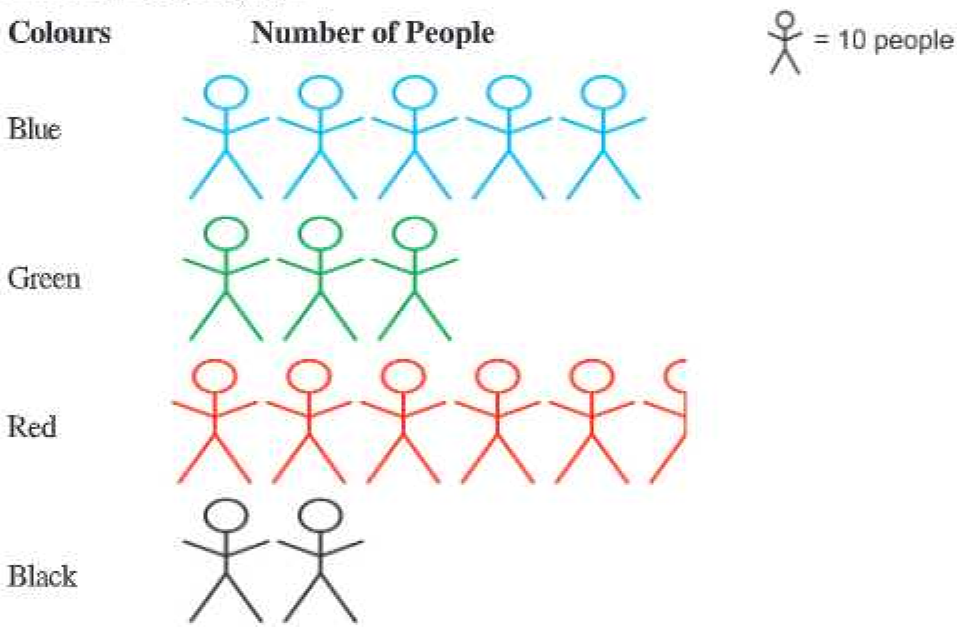
Represent the above information by pictograph.

Solution : Taking the scale [bulb symbol] = 10 bulbs, we may draw the pictograph as shown below:



Exercise 14.2

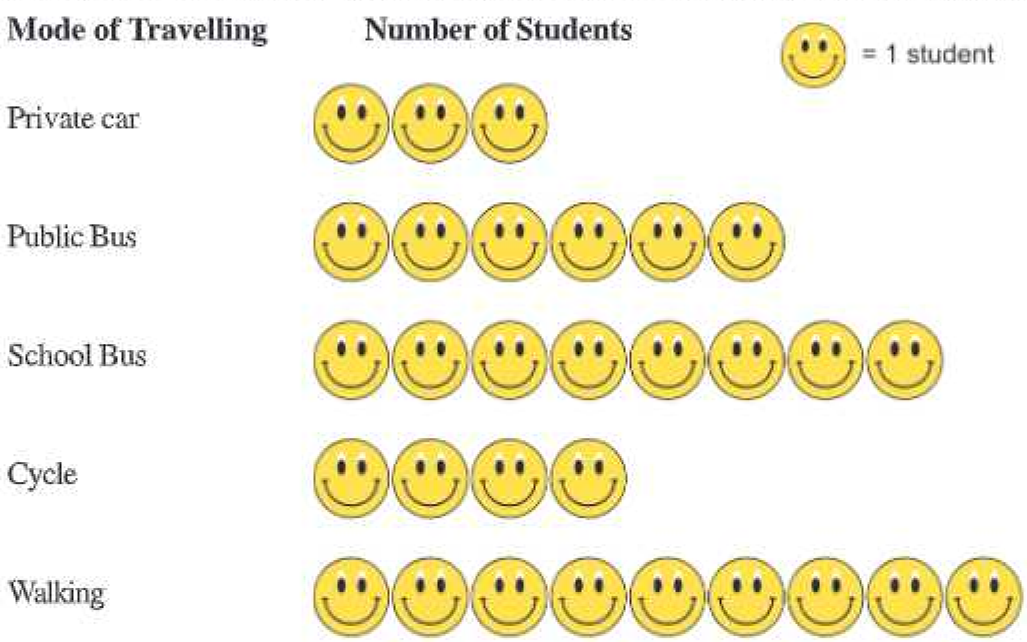
1. The colour of refrigerators preferred by number of people living in a locality are shown by the following pictograph:



(i) Find the number of people preferring blue colour?

(ii) How many people liked red colour?'

2. A survey was carried out on 30 students of class VI in a school. Data about different modes of transport used by them to travel to school was displayed through pictograph.

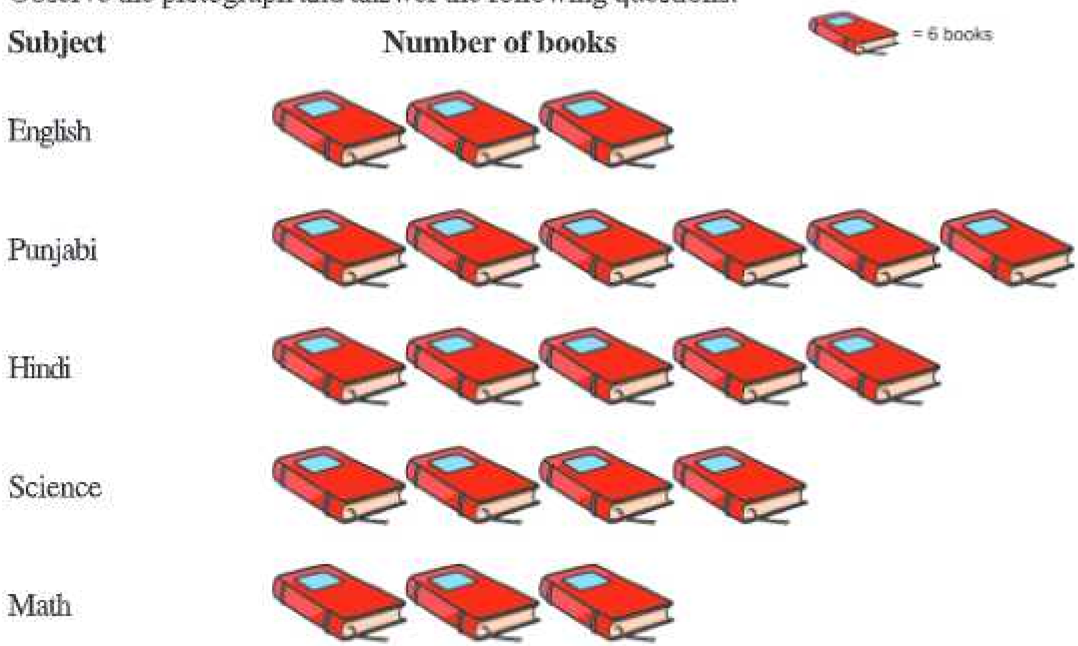


Observe this pictograph and answer these questions?

(i) How many students come on foot?

(ii) By which mode of transport, less students come?

3. Following is the pictograph showing books on different subjects kept in school library. Observe the pictograph and answer the following questions:



(i) How many math books are there in the library?

(ii) Which books are minimum in number?

(iii) Which books are maximum in number?

4. The number of desks in rooms of classes VI to X are given below:

Class	VI	VII	VIII	IX	X
No. of desks	30	50	40	35	45

Draw the pictograph by using any suitable scale.

5. In the half yearly examination, the marks scored by a student in each subject out of 100 are given below:

Subject	English	Hindi	Maths	Science	Social Science
Marks scored	70	85	80	65	75

Draw a pictograph taking the scale in which one picture = 10 marks and answer the following questions:

(i) In which subject the maximum marks are obtained?

(ii) In which subject the student has to do more hard work?

(iii) What is the difference between the maximum and minimum marks?

Read the following information and choose the correct answer for given questions:

6. The marks received by Harjeet in different subjects are as follows:

Subjects	Punjabi	English	Hindi	Math	Science
Marks	43	40	45	48	37

(i) In which subject, Harjeet scored maximum?

(a) Science (b) Hindi (c) Math (d) Punjabi

(ii) In which subject, Harjeet scored minimum?

(a) Science (b) Hindi (c) Math (d) Punjabi

(iii) How many marks he scored in Math?

(a) 40 (b) 45 (c) 48 (d) 43

(iv) How many marks he scored in English?

(a) 40 (b) 45 (c) 48 (d) 43

(v) In how many subjects, he scored more than 45?

(a) 2 (b) 3 (c) 4 (d) 1

7. The number of books sold by a shopkeeper on the different days are shown below:

Days	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
No. of books sold	250	280	190	175	220	300

(i) On which day, the sale is maximum?

(a) Saturday (b) Friday (c) Thursday (d) Wednesday

(ii) On which day, the sale is minimum?

(a) Saturday (b) Friday (c) Thursday (d) Wednesday

(iii) On Friday, how many books are sold?

(a) 280 (b) 220 (c) 175 (d) 300

(iv) On Tuesday, how many books are sold?

(a) 220 (b) 175 (c) 280 (d) 300

(v) How many books are sold on Saturday?

(a) 220 (b) 175 (c) 280 (d) 300

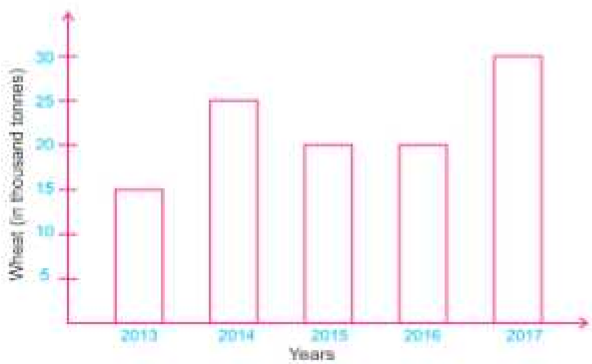
14.4.2 Bar Graph

We have observed that drawing pictograph is very time consuming. Interpretations of pictograph may differ from person to person while using half or quarter pictures as data may be any number like 4, 51, 87 etc. Pictograph of this type of data is quite difficult.

So, we use another method to represent the data visually in a simple manner called **Bar Graph.** Graphs communicate information quite effectively. You usually see bar graphs in newspapers and magazines.

A bar graph is a chart with rectangular bars of equal width and lengths of bars are, proportional to the values that they represent. The bars can be horizontal or vertical with equal spacing between them.

Example 1: **The bar graph given below shows the amount of wheat purchased by government during the year 2013-17.**



Read the bar graph and write down your observation. In which year was

(i) The wheat production maximum?

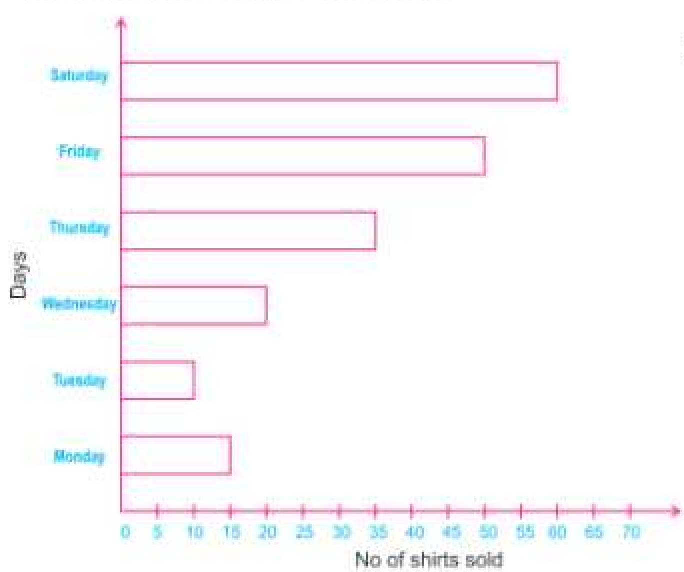
(ii) The wheat production minimum?

Solution : (i) It is observed that wheat production in year 2017 is maximum.

(ii) It is observed that wheat production in year 2013 is minimum.

Example 2: **Observe this bar graph which is showing the sale of shirts in a ready made shop from Monday to Saturday.**

1 unit length = 5 shirts



Now answer the following questions:

(i) What information does the above bar graph provide?

(ii) What is the scale chosen on the horizontal line representing number of shirts?

(iii) On which day were the maximum number of shirts sold? How many shirts were sold on that day?

(iv) On which day were the minimum number of shirts sold?

(v) How many shirts were sold on thursday?

Solution : (i) The above bar graph represent the sale of shirts on different days.

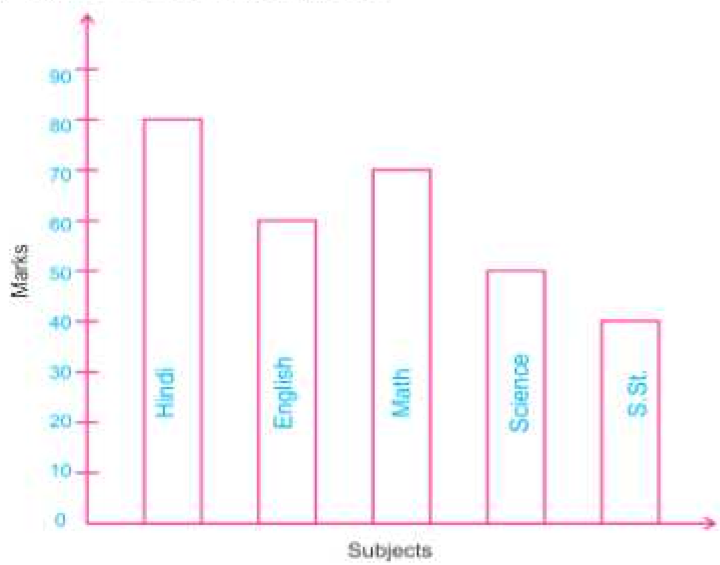
(ii) 1 unit length = 5 shirts

(iii) The maximum number of shirts sold on Saturday. There are $60 \times 5 = 300$ shirts sold on Saturday.

(iv) The minimum number of shirts sold on Tuesday.

(v) $30 \times 5 = 150$ shirts sold on Thursday.

Example 3: **Observe this Bar Graph which shows the marks obtained by Aniza in half yearly exam in different subjects.**



Answer the following questions:

(i) What information does the Bar graph provide?

(ii) State the name of subjects and marks obtained in each of them?

(iii) Name the subject in which Aniza scored maximum marks?

(iv) Name the subject in which she scored minimum marks?

Solution : (i) The bar graph gives information of marks obtained by Aniza in different subjects.

(ii) Hindi = 80, English = 60, Maths = 70, Science = 50, S.St = 40

(iii) She has scored maximum marks in Hindi.

(iv) She has scored minimum marks in S.St.

Construction of a Bar Graph

To draw a bar graph, we draw two mutually perpendicular lines on a plane paper. The horizontal line is called **x-axis** and the vertical line is called **y-axis**. If the rectangular bars are drawn on a horizontal line (x–axis), the scale of heights of bars is shown along the y-axis or vice-versa.

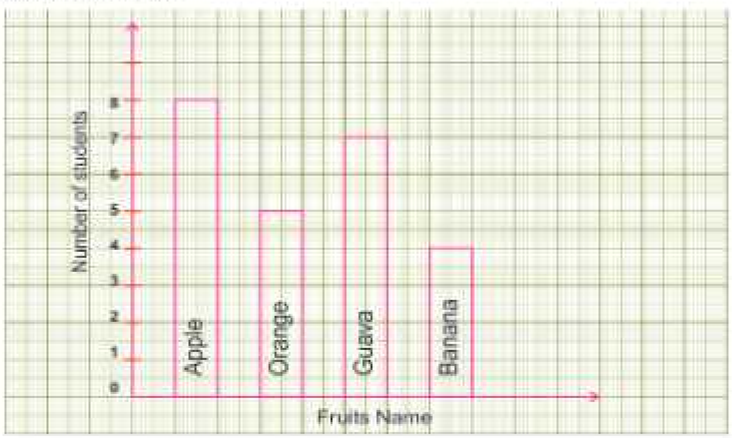
For drawing bar graphs, the following points should be kept in mind:

- The width of the bar should be uniform throughout.
- The gap/space between the bars should be uniform throughout.
- Bars may be horizontal or vertical.
- Choose a suitable scale for determining the height of bars.

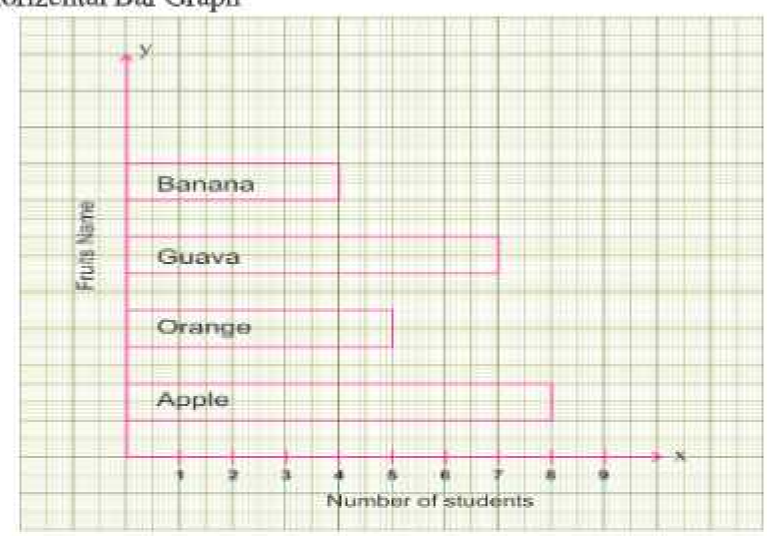
Example 4 : The following table gives the information of choice of fruits of class 6th students. Draw a bar graph for this data.

Fruit	Apple	Orange	Guava	Banana
No. of students	8	5	7	4

Solution : Vertical Bar Graph



Horizental Bar Graph



Note :- Students can draw any type of Bar Graph as their Choice

It is observed that above information represented through a vertical bar graph (Graph 1)

It can also be represented through a horizontal bar graph (Graph 2)

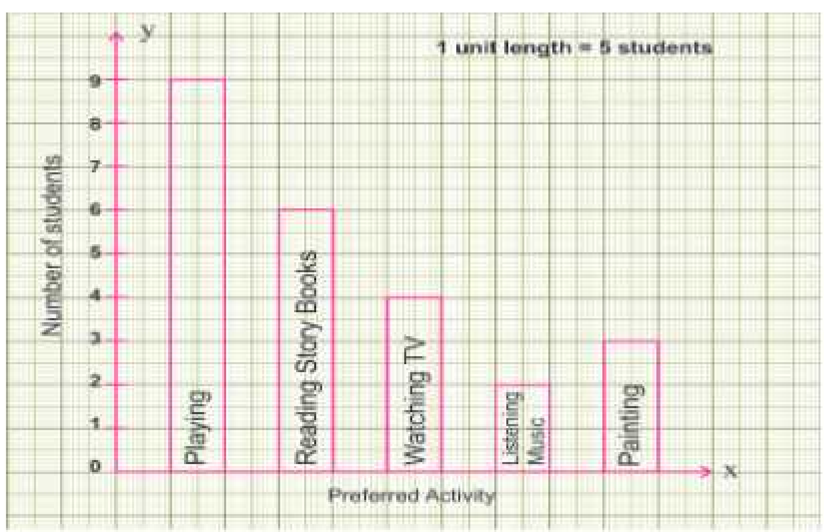
Usually we draw vertical Bar graph.

Example 5 : A survey of 120 school students was done to find the activity they prefer to do in their free time.

Preferred activity	Number of Students
Playing	45
Reading story books	30
Watching TV	20
Listening Music	10
Painting	15

Draw a bar graph to illustrate the above data taking scale of 1 unit length = 5 students. Which activity is preferred by most of the students other than playing?

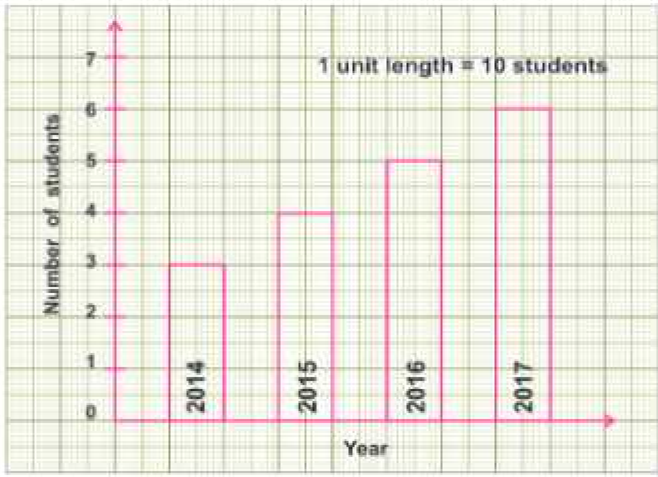
Solution :



- Reading story book is preffered most by the students other than playing.

Exercise 14.3

1. Read the adjoining bar graph showing the number of students in a particular class of a school:



Answer the following questions:

1. What is the scale of this graph?
2. How many students are there in 2016?
3. Is the number of students in the year 2017 twice the year 2014?

2. Read the bar graph and answer the following questions:

(i) What is the information given by the bar graph?

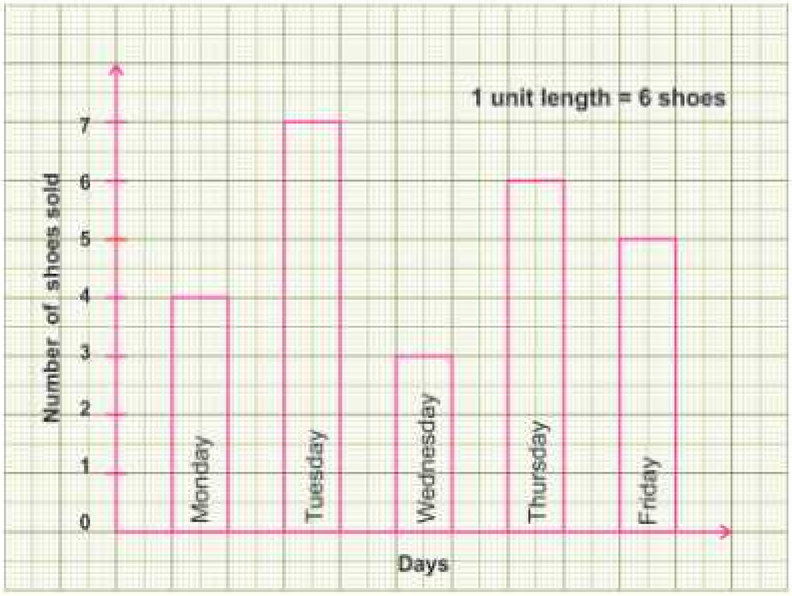
(ii) Which scale is used in this bar graph?

(iii) What is the maximum age? How many students have maximum age?

(iv) How many students have minimum age?

(v) How many students are 13 years old?

3. Read the given bar graph and answer the following questions:



(i) What information does the bar graph represent?

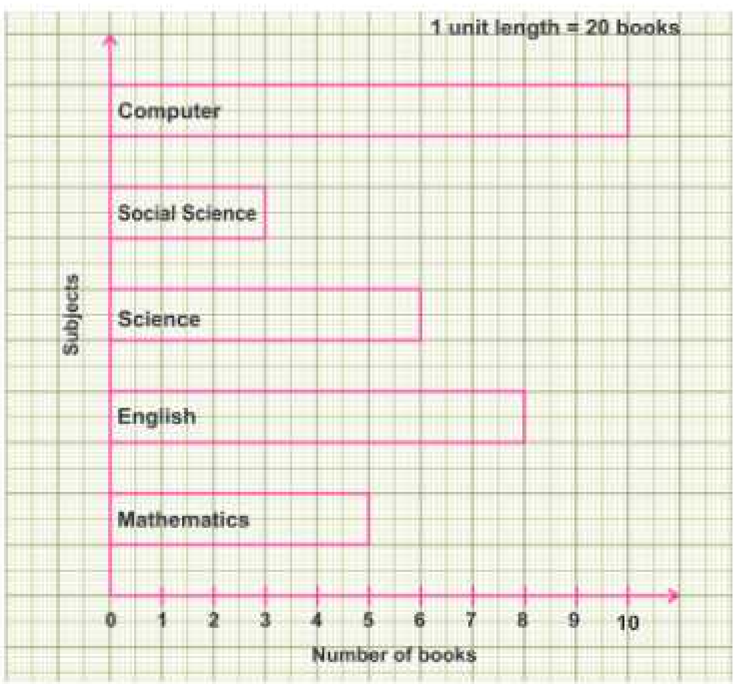
(ii) What is the scale chosen for this graph?

(iii) On which day were the maximum number of shoes sold and how many?

(iv) On which day were the minimum number of shoes sold and how many?

(v) How many shoes were sold on Thursday?

4. Read the bar grpah which shows the number of books of different subjects in a library:



Answer the following questions:

(i) What information does the bar graph gives?

(ii) What is the scale chosen for this graph?

(iii) Which subject has maximum number of books and how many?

(iv) Which subject has minimum number of books and how many?

5 : The number of Mathematics books sold by a shopkeeper on the different days are shown below:

Days	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday
Number of books sold	65	40	30	50	20	70

Draw a bar graph to represent the above information choosing the scale of your choice.

Multiple Choice Questions

1. If ☆ represents 10 flowers then how many flowers does ☆☆☆☆☆ represent?

(a) 30 (b) 40 (c) 50 (d) 5

2. If ☺ = 7 children then what does ☺ ☺ ☺ represent?

(a) 1 (b) 14 (c) 21 (d) 28

3. What is value of 卌 II

(a) 6 (b) 7 (c) 5 (d) 8

4. If □□□□ represents 400, then what does □□ stand for?

(a) 200 (b) 2000 (c) 20 (d) 2

5. .... represents data through picture of objects.

(a) Bar Graph (b) Histogram

(c) Pictograph (d) None of these

6. Which tally marks represents 14?

(a) 卌 IIII (b) 卌 卌 IIII

(c) IN IN IIII (d) IIII IIII IIII II

7. If ☆ represents 4 balls, No. of ☆ to be drawn to represents 40 balls.

(a) 5 (b) 10 (c) 12 (d) 160

8. .... is method of representing the data in uniform width size horizontal or vertical box with equal spacing.

(a) Histogram (b) Bar Graph

(c) Pictograph (d) Tally Marks

9. A .... is a collection of numbers gathered to give some information.

(a) Frequency (b) Data

(c) Tally mark (d) None of these

10. If on a scale 1 unit = 200 then how much quantity does 5 units will represent?

(a) 100 (b) 1000 (c) 300 (d) 600

Learning Outcomes

After completion of this chapter the students are now able to

(i) Collect and arrange the different type of data.

(ii) Represent the data in pictograph and bar graph.

(iii) Interpret the pictograph & bar graph.

(iv) Use the graphs in daily life situation.

ANSWER KEY

Exercise 14.1

1.

Height (in cm)	Tally marks	Frequency
115	I	1
116	卌 II	7
117	III	3
118	II	2
119	IIII	4
120	III	3
121	IIII	4
122	I	1
123	II	2
124	II	2
125	III	3

2.

Weight (in kg)	Tally marks	Frequency
25	卌	5
28	卌	5
32	卌 II	7
33	II	2
34	卌 I	6

3.

Size of shoes	Tally Marks	Frequency
4	~~IIII~~	5
5	~~IIII~~ III	8
6	~~IIII~~ ~~IIII~~	10
7	~~IIII~~ II	7
8	II	2

4.

Number of dice	Tally marks	Frequency
1	~~IIII~~ II	7
2	~~IIII~~ I	6
3	~~IIII~~	5
4	IIII	4
5	~~IIII~~ ~~IIII~~ I	11
6	~~IIII~~ II	7

(i) 4 (ii) 5

5. (i)

Marks	Tally Marks	Frequency
1	I	1
2	I	1
3	III	3
4	I	1
5	IIII	4
6	~~IIII~~	5
7	~~IIII~~ I	6
8	~~IIII~~	5
9	III	3
10	I	1

(ii) 15 (iii) 9

Exercise 14.2

1. (i) 50 (ii) 55
2. (i) 9 (ii) Private car
3. (i) 18 (ii) Math and English (iii) Punjabi
5. (i) Hindi (ii) Science (iii) 20
6. (i) c (ii) a (iii) c (iv) a (v) d
7. (i) a (ii) c (iii) b (iv) c (v) d

Exercise 14.3

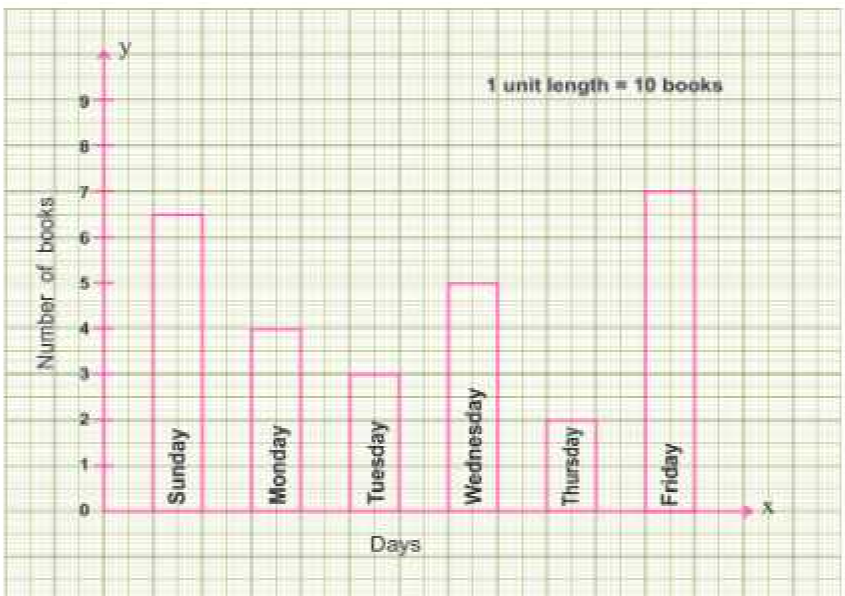
1. (i) 1 unit = 10 students (ii) 50 (iii) Yes

2. (i) Age of different students (ii) 1 unit = 4 students
   (iii) 15, 24 students (iv) 16 (v) 28

3. (i) Number of shoes sold in different days.
   (ii) 1 unit = 6 shoes sold (iii) Tuesday, 42 shoes
   (iv) Wednesday, 18 shoes (v) 36 shoes

4. (i) Number of books of different subjects in the library.
   (ii) 1 unit = 20 books (iii) Computer, 200 books
   (iv) Social Science, 60 books

5.



Multiple Choice Questions

1. (c)	2. (c)	3. (b)	4. (a)	5. (c)
6. (b)	7. (b)	8. (b)	9. (b)	10. (b)